

line structure of the material. To determine the influence of the crystalline state of material on the specific heat, K. Lark-Horovitz suggested measuring the atomic heat of a very pure single crystal of Ag, for comparison with the results of earlier measurements on polycrystalline Ag by Keesom and Kok.<sup>1</sup>

We have therefore measured the atomic heat of a single crystal of Ag<sup>2</sup> at liquid helium temperatures. The results are identical, within experimental error, with those obtained earlier by Keesom and Kok.<sup>1</sup> On a plot of  $C_v/T$  against  $T^2$  (see Fig. 1), it is not

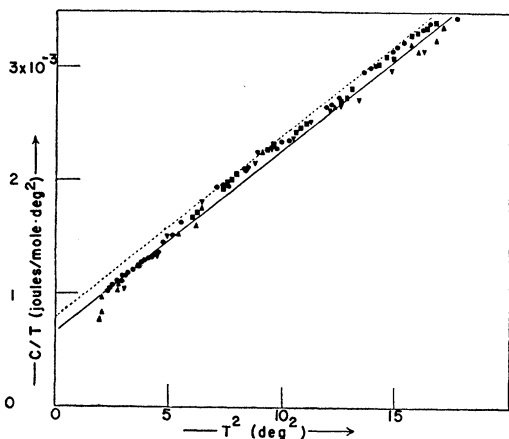


FIG. 1.  $C_v/T$  vs  $T^2$  for Ag;  $\nabla$ —Keesom and Kok, 1932;  $\Delta$ —Keesom and Kok, 1933;  $\square$ —this measurement, 1/25/52;  $\circ$ —1/30/52.

possible to fit all the data below 4.2°K with one straight line. Below 2.2°K, both sets of data can be represented by the solid line, which has the equation

$$C_v = 1.619 \times 10^{-4} T^3 + 6.45 \times 10^{-4} T \text{ joules/mole degree,} \quad (1)$$

while above 2.5°K, the dashed line, which has the equation:

$$C_v = 1.598 \times 10^{-4} T^3 + 7.82 \times 10^{-4} T \text{ joules/mole degree,} \quad (2)$$

fits both sets of data.

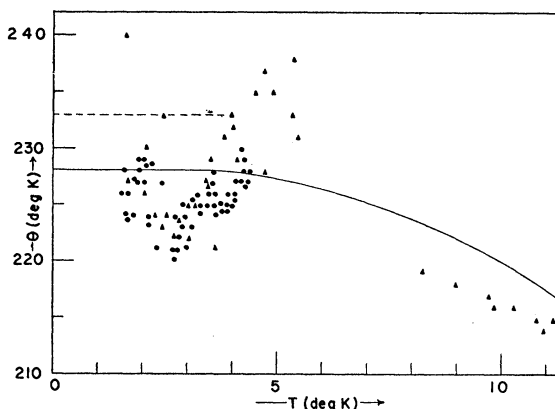


FIG. 2. Debye  $\theta$  vs  $T$  for Ag;  $\Delta$ —Keesom and Kok;  $\circ$ —this measurement; —Leighton's calculated curve; - -  $\theta(E)$  calculated from elastic constants (this calculation is valid only in the true  $T^3$  region).

While this anomaly is also evident in the data of Keesom and Kok, it becomes obvious only in a plot of  $C_v/T$  vs  $T^2$ . However, as was common at the time of their experiments (which provided the first proof of the existence of electronic heat capacity in metals), they analyzed their results only with a plot of  $\theta$  vs  $T$ .

Several alternatives suggest themselves for the explanation of this effect. Gerritsen and Korringa<sup>3</sup> have suggested that the rise of resistance observed in some metals at very low temperatures is due to the presence of a very small concentration of paramagnetic

impurity atoms, such as Mn. They also predict an additional contribution to the atomic heat in the form of a Schottky curve<sup>4</sup> arising from electronic transitions between states whose degeneracy has been removed by the paramagnetic atoms. Their predicted effect, however, would be much too small compared with that observed, if our single crystal had the purity stated for it (99.999 percent). It also seems unlikely that both our sample and that of Keesom and Kok would contain the same type and amount of impurities.

Katz<sup>5</sup> has suggested that lattice perturbations (due to work hardening, mosaic structure, impurities, etc.) can result in deviations from the Debye vibration spectrum at very low temperatures and provide anomalous contributions to the atomic heat. He has discussed in this way apparent irregularities in the curve of  $\theta$  vs  $T$  (see Fig. 2) for Ag, but Keesom and Kok do not believe them to be significant in view of their experimental error. Such perturbations, moreover, must have a considerable degree of order themselves, so that they may form a "superlattice" capable of affecting the spectrum in this way. It is hard to imagine a perturbation of this type which would be identical in Keesom and Kok's polycrystalline material and our single crystal.

Leighton<sup>6</sup> has calculated the elastic spectrum for face-centered cubic crystals and finds that for  $T$  less than 4°K,  $\theta$  for Ag is practically constant at 228°K. This agrees very well with the experimental values, 229°K for the solid line and 230°K for the dashed line, and with the value  $\theta(E) = 233$ °K calculated from elastic constants<sup>7</sup> by the method of Hopf and Lechner.<sup>8</sup> It is therefore unlikely<sup>6</sup> that deviations from the Debye vibration spectrum due to the nature of the Ag lattice itself are responsible for the anomaly.

It seems reasonable to interpret the behavior shown in Fig. 1 as a change in  $\gamma$ , the coefficient of the linear term, occurring between 2.2°K and 2.5°K, with the cubic term remaining constant. A general expression for  $\gamma$  per mole is<sup>9</sup>

$$\gamma = (\pi^2 k^2 / 3) V f(\zeta), \quad (3)$$

where  $k$  is Boltzmann's constant,  $V$  is the atomic volume, and  $f(\zeta)$  is the density of electronic states at the Fermi level. An increase in  $\gamma$  could correspond to an increase in  $f(\zeta)$ . This increase could come about as a result of overlapping of allowed bands, and the position of the overlap and the density of levels in the higher band should be derivable from the observed increase in  $\gamma$  (21 percent) and the temperature at which it occurs. Unfortunately, however, no detailed calculation of the band structure of Ag on the basis of electronic wave functions is available for comparison. If such a calculation were made, it would be of great interest to compare the electronic heat capacity predicted from it with our experimental results.

\* Work performed under contract with the Signal Corps.

<sup>1</sup> W. K. Keesom and J. A. Kok, Commun. Kamerlingh Onnes Laboratory, University of Leiden, No. 219d (1932); No. 232d (1933).

<sup>2</sup> The single crystal weighed about 90 grams and was kindly supplied to K. Lark-Horovitz by Dr. R. Maddin of The Johns Hopkins University.

<sup>3</sup> A. N. Gerritsen and J. Korringa, Phys. Rev. **84**, 604 (1951).

<sup>4</sup> W. Schottky, Physik. Z. **23**, 448 (1922).

<sup>5</sup> E. Katz, J. Chem. Phys. **19**, 488 (1951).

<sup>6</sup> R. B. Leighton, Revs. Modern Phys. **20**, 165 (1948).

<sup>7</sup> Landolt-Bornstein, *Physikalisch-Chemische Tabellen* (Verlag Julius Springer, Berlin, 1927), fifth edition.

<sup>8</sup> L. Hopf and G. Lechner, Verhandl. deut. physik. Ges. **16**, 643 (1914).

<sup>9</sup> F. Seitz, *The Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), pp. 130-151.

### A Further Test of the Shell Model\*

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AS an additional test of the accuracy of the shell model in ascribing definite orbital angular momentum states to nucleons in a nucleus<sup>1,2</sup> we have measured the angular distribution of the protons associated with the ground state in the reaction  $\text{Cl}^{36}(d, p)\text{Cl}^{36}$ . As pointed out by Bethe and Butler, the selection

rules for this reaction allow the neutron to carry into the residual nucleus an orbital angular momentum of 0, 2, or 4 units, while the shell model requires that  $l_n=2$ , corresponding to the addition of a  $d_{3/2}$  neutron to the configuration of  $\text{Cl}^{35}$ .

The experimental data are plotted in Fig. 1 along with the angular distribution of the protons predicted by the Butler<sup>3</sup>

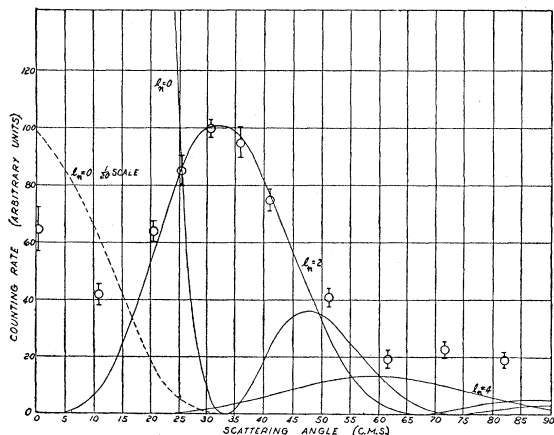


FIG. 1. The angular distribution of the protons associated with the ground state for the reaction  $\text{Cl}^{35}(d, p)\text{Cl}^{36}$ . The solid curves are calculated from the Butler theory.

theory for values of  $l_n=0, 2$ , and 4. The theoretical curves were computed using values of  $r_0=5.5 \times 10^{-13}$  cm,  $a=0.23 \times 10^{13}$  cm<sup>-1</sup>, and  $b=1.4 \times 10^{13}$  cm<sup>-1</sup>. The incident deuteron energy was 6.90 Mev (c.m. system) and the  $Q$  of the reaction was measured to be 6.3 Mev in good agreement with Shrader and Pollard<sup>4</sup> and with Ennis.<sup>5</sup>

The target, a foil of silver chloride 0.0003 inch thick, was rolled from a pure silver chloride crystal. The background counting rate was obtained by substituting for the silver chloride target a foil of pure silver containing very nearly the same amount of silver per square centimeter as that of the target. It was negligible at all angles except 0°, where it accounted for 35 percent of the total counting rate. In Fig. 1 the vertical lines through the experimental points represent the standard deviations as determined from the

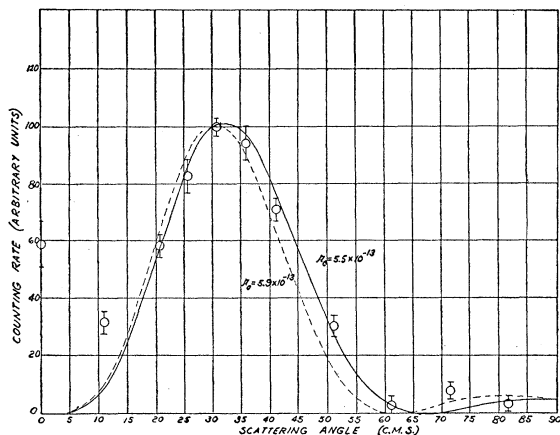


FIG. 2. The angular distribution of Fig. 1 replotted after subtracting from each point the constant "background" counting rate at 65°. The solid curves are calculated from the Butler theory for two values of  $r_0$ .

total number of counts and do not represent the over-all uncertainty of the data.

Although  $(d, p)$  reactions proceed mainly by means of a stripping process, other processes, such as compound nucleus formation, offer competition. These, however, do not show such marked angular dependence.<sup>6</sup> Because of the competitive processes the

measured values do not become zero where indicated by the Butler curves. Since it is difficult to obtain a reliable estimate of the relative cross sections, we subtracted from each point of Fig. 1 an amount equal to the counting rate at 65° and replotted the renormalized data in Fig. 2. The theoretical curves for  $l_n=2$  for two values of  $r_0$  are included. Much better agreement is obtained for angles of 20° and larger.

The results of the measurement indicate that the neutron carries mainly two units of orbital angular momentum into the residual nucleus and is presumably a  $d_{3/2}$  nucleon as required by the shell model. Assuming that the ground state is a single level (there is some evidence of a doublet structure as in the case of  $\text{P}^{32}$ ), the amount of admixture of  $l_n=0$  appears to be less than 4 percent. It is difficult to set a limit on the  $l_n=4$  admixture, particularly because of the uncertainty in the "background." It is, however, presumably small.

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<sup>3</sup> S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

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<sup>5</sup> W. W. Ennis, Phys. Rev. **82**, 304 (1951).

<sup>6</sup> L. Wolfenstein, Phys. Rev. **82**, 690 (1951).

## Nuclear Spin of $\text{V}^{50}$ by Paramagnetic Resonance\*

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WE have investigated the hfs in the paramagnetic resonance absorption spectrum of a Tutton salt of vanadium, enriched to 22.83 percent of  $\text{V}^{50}$ , and find evidence that the spin of  $\text{V}^{50}$  is 6. This nucleus, consisting of 23 protons and 27 neutrons, was expected on the basis of Nordheim's rule<sup>1</sup> to have a spin of 7, or near this maximum value. Hitchcock<sup>2</sup> has recently made shell-theoretical calculations based on various potential functions and finds that the state  $I=6$  lies lowest for a  $\delta$ -function potential.

The crystals of  $\text{V}(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$  diluted in the corresponding zinc salt were grown from a solution prepared by using 10 mg of  $\text{V}_2\text{O}_5$  with a V:Zn ratio of about 1.5:100. A sample of  $\text{V}_2\text{O}_5$  enriched in  $\text{V}^{50}$  was supplied by the Stable Isotopes Research and Production division of the AEC. Measurements were performed on several specimens, each approximately 5 mm long and 2 mm in diameter, inserted axially into a cylindrical transmission cavity operating in the  $TE_{011}$  mode at approximately 23,000 Mc/sec. The rf was kept constant in frequency while the magnetic field was modulated at 60 cps over a range of approximately 80 gauss.

The observed spectrum consisted of 8 strong lines, corresponding to  $I(\text{V}^{51})=7/2$ ,<sup>3</sup> and a number of weak components which have been assigned to  $\text{V}^{50}$ . The intensity ratio of weak to strong lines was approximately 1:5 and their relative splittings 0.39. The spectrum can be calculated from the known  $g$ -values<sup>4,5</sup> and isotopic abundance for various  $\text{V}^{50}$  spin values. Figure 1 shows the spectrum that is expected for  $I(\text{V}^{50})=6$  and 7. The calculated relative splitting is 0.38 and the intensity ratio, for  $I=6$ , is 1:5.5.

In all samples used we observed the components corresponding to  $-6 \leq m \leq 6$ , excepting those for  $m=\pm 4$ , which, according to

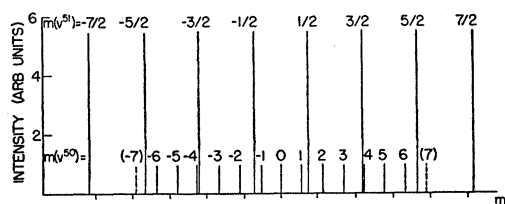


FIG. 1. Theoretical hfs spectrum of  $\text{V}^{50}$  and  $\text{V}^{51}$ . The solid  $\text{V}^{50}$  lines are those to be expected for  $I=6$ . For  $I=7$ , the additional dotted lines would appear.