Proton-Deuteron Scattering at 5.1 Mev and Deuteron-Proton Scattering at 10.2 Mev*

K. B. MATHER
Commonwealth Scientific and Industrial Research Organization, Physics Department, University of Melbourne, Melbourne, Australia

(Received July 21, 1952)

A photographic scattering chamber has been used to study the elastic scattering of 5.1-Mev protons by deuterium and 10.2-Mev deuterons by hydrogen, these being the same process in the center-of-mass system. Differential cross sections were obtained from 16.4° to 172.9°. These are in good agreement with other experimental data at about the same energy. Comparison is made with the theoretical angular distribution due to Buckingham, Hubbard, and Massey, based on a symmetrical exchange force. Agreement is close at this energy, favoring exchange rather than ordinary force theories. However, at higher energies (∼10-Mev $p-d$) the BHM calculations do not represent the interaction satisfactorily.

I. INTRODUCTION

THE proton-deuteron interaction has now been studied at various energies from 250 kev to ∼10 Mev, including three studies at approximately 5 Mev.[1] The present report contributes further data at 5.1 Mev over a somewhat wider range of angles. The data were acquired by scattering both 5.1-Mev protons from deuterium and 10.2-Mev deuterons from hydrogen, the combination of the techniques having obvious advantages in revealing special errors to which either one is liable.

II. METHOD

The apparatus employed consisted of the photographic scattering chamber described in an earlier publication[2] on proton-proton scattering, and its manner of use was essentially unchanged in the present work. However, in $p-d$ scattering, scattered and recoil particles are different and distinguishable and were counted separately, leading to the differential scattering cross section in the laboratory system by the following formula:[3]

$$\sigma(\Theta)=n_aRh\sin\Theta/N_{cc}N_i2b\sin\Phi.$$

Conversion to center-of-mass cross section $\sigma(\theta)$ at center-of-mass angle θ is achieved by formulas:

$$\sigma(\theta)=\sigma(\Theta_1)\cdot(\sin\Theta_1/\sin\theta)^2\cdot\cos(\theta-\Theta_1)$$

if *scattered* tracks at angle Θ_1 are counted, and

$$\sigma(\theta)=\sigma(\Theta_2)/4\cos\Theta_2$$

if *recoil* tracks at angle Θ_2 are counted, the angle relationship being:

$$\theta=(\pi-2\Theta_2).$$

In $p-d$ scattering the proton may be scattered at all laboratory angles from 0 to 180° the deuteron recoiling at angle from 0 to 90°. In $d-p$ scattering the deuteron has a maximum angle of scatter of 30°, the proton recoiling at angles from 0 to 90°. Owing to the fact that there are two kinds of collision (hard and soft) which can scatter the deuteron at any angle less than 30°, three distinct track lengths appear in the photographic emulsion at all angles below 30°—a short deuteron track, a long deuteron track, and a recoil proton track.

Approximate ranges in Nuclear Research Emulsions (Ilford C2) are listed in column 4 of Tables I and II, for $p-d$ and $d-p$ cases, respectively. It will be observed that it is always possible and generally easy to distinguish, by their ranges, the several particles which appear at any angle. The fact that one plate can, for $d-p$ scattering below 30°, yield three scattering cross sections endows the method with special merit. These three values serve as related "triplets" since they derive from the same slit unit and precisely the same geometry is used in computing them. They should therefore be very reliable in relative value. Similarly, in $p-d$ scattering below 90° there are related "doublets."

III. EXPERIMENTAL DETAILS

The source of protons and deuterons was the Washington University 45-inch cyclotron. The beam energy was redetermined during every run by measuring ranges in a photographic plate placed at the 45° position at a small angle, $\Phi=3°$. The range-energy data of Rotblat[4] were used, leading to average energy values at the scattering volume of 5.1±0.1 Mev for the protons and 10.2±0.2 Mev for the deuterons, in the laboratory system.

The energy of each individual run could be stated rather more precisely than this but as the beam energy appeared to vary slightly from run to run, average

---

* The experimental portion of this work was conducted at Washington University, St. Louis, Missouri, assisted by the ONR and AEC.

[1] R. F. Taschek, Phys. Rev. **61**, 13 (1942), 250 and 275 kev; Tuve, Heydenberg, and Hafstad, Phys. Rev. **50**, 806 (1936), 830 kev; Sherr, Blair, Kratz, Bailey, and Taschek, Phys. Rev. **72**, 662 (1947), 1.51, 2.08, 2.53, 3.00, and 3.49 Mev; Heitler, May, and Powell, Proc. Roy. Soc. (London) **A190**, 180 (1947), 4.2 Mev; Rodgers, Leiter, and Kruger, Phys. Rev. **78**, 656 (1950), 4.97 Mev; Karr, Bondelid, and Mather, Phys. Rev. **81**, 37 (1950), 5.0 Mev; L. Rosen and J. C. Allred, Phys. Rev. **82**, 777 (1951), 5.2 Mev; Armstrong, Allred, Bondelid, and Rosen, Phys. Rev. **83**, 218 (1951), 9.7 Mev.

[2] K. B. Mather, Phys. Rev. **82**, 133 (1951).

[3] This formula appeared incorrectly in reference 2. The symbols are defined there.

[4] J. Rotblat, Nature **167**, 550 (1951).

values have been quoted together with uncertainties which cover the extreme limits encountered.

The deuterium and hydrogen used as scattering gases were 99.5 and 99.8 percent pure, respectively. Experiments were usually conducted at a gas pressure of approximately 1.4 cm Hg which was low enough to prevent any serious multiple scattering effects.

The $p-d$ data were much "cleaner" than the $d-p$ due particularly to the heavy neutron flux which is always associated with a cyclotron accelerating deuterons. During preliminary runs it was found that the background on the plates due to knock-on protons was heavy and would have complicated the scanning unnecessarily. This was reduced to a few percent by surrounding the scattering chamber with a concrete fort 16 inches thick on the side facing the cyclotron and reinforced about the collimator with timber blocks.

Runs were made at several different exposures so as to give optimum track density over each range of angles to facilitate scanning. The method of scanning was the same as described in reference 2. In certain cases, however, e.g., $d-p$ runs 5 and 6, for $\Theta<30°$, only short deuteron tracks were counted as these could be counted quickly. Counting long deuterons and protons was more tedious, requiring ideal exposure conditions, and was only considered worthwhile for run 7.

IV. RESULTS AND DISCUSSION

The final values of seven runs are listed in Table I ($p-d$) and Table II ($d-p$) ranging from $\theta=16.4°$ to $172.9°$, and all data are shown in Fig. 1 as $\sigma(\theta)$ *versus* θ. The total number of tracks counted was 105,500. The number contributing to each value is given in parentheses after the value and determines the standard deviation of the value imposed by the counting. Other errors are the same as listed in Sec. VIII of reference 2. The probable error in relative values, excluding the statistical error, is ~0.4 percent.

Corrections had to be applied to the $p-d$ data as follows: (1) Scattering volume. This is not exactly the same as implied in the formula for $\sigma(\Theta)$ in Sec. II. A correction had to be applied only at $\Theta=165°$ and amounted to 0.7 percent. (2) Air leakage, degassing of plates and equipment, and impure gas. This amounted to ~5 percent at $\Theta=11°$ and declined rapidly at larger angles. (3) Slit width. This applied only where the cross section was changing rapidly, *viz.*, at small laboratory angles where it amounted in several cases to a few percent.

Similar corrections pertained to the $d-p$ runs. On the whole, the $p-d$ scanning was more straightforward. Fewer cases of ambiguous track lengths occurred and hence runs 1 and 2 should carry rather more weight.

In run 6 the tracks were so congested that only short deuteron tracks could be counted confidently and even there it was felt that some tracks might be overlooked. This may account for the tendency towards lower values in this run. The $d-p$ data show considerable scatter at $\theta=30°$ to $40°$. This is due to the difficulty of counting accurately the short proton tracks, especially at 30°. These tracks are only 7μ in mean range and, allowing for straggling, some of the tracks are too short to be established with certainty. In run 4 the value at 30° is a lower limit. About 10 percent more "probables" were recorded and it seems likely that others escaped notice altogether.

Beyond about 160° the same trouble operates. Some short deuterons, expecially at $\theta=168.6°$ and $172.9°$ may be missed. The $p-d$ data which have better statistics in this region would be expected to suffer from

Table I. Differential cross sections derived from two $p-d$ scattering runs at 5.1 Mev. The figures in parenthesis are numbers of tracks contributing to the values. In the third column, p indicates scattered proton and d recoil deuteron.

Θ	θ	Kind of track	Range in μ	$\sigma(\theta)\times10^{25}$ cm² steradian⁻¹ Run 1	Run 2
11	16.4	p	168	x3.90 (1200)	
	158.0	d	89	2.29 (1212)	
15	22.4	p	162		
	150.0	d	84		
16	23.9	p	162	2.48 (1158)	
	148.0	d	83	1.72 (1355)	
21	31.1	p	155	2.27 (966)	
	138.0	d	77	1.17 (845)	
25	37.2	p	148	2.00 (1733)	
	130.0	d	71	0.846 (1245)	
30	44.5	p	138	1.86 (735)	
	120.0	d	61	0.602 (401)	
35	51.7	p	129	1.63 (1242)	
	110.0	d	52	0.533 (680)	
40	58.7	p	118	1.47 (1294)	1.49 (1207)
	100.0	d	42	0.646 (911)	0.631 (841)
45	65.7	p	106		
	90.0	d	33		0.84 (188)
50	73.5	p	92	1.21 (968)	
	80.0	d	26	1.00 (1205)	
55	79.2	p	82	1.02 (721)	1.12 (861)
	70.0	d	19	1.25 (1294)	1.24 (1382)
60	85.6	p	71		0.862 (1200)
	60.0	d	13		1.37 (2570)
65	91.9	p	62	0.743 (611)	0.734 (919)
	50.0	d	8	1.71 (1736)	1.60 (2457)
70	98.0	p	54		0.651 (1112)
	40.0	d	5		
75	103.9	p	48	0.550 (511)	0.616 (1006)
	30.0	d	~3		
85	114.9	p	35		0.577 (1037)
	10.0	d	—		
95	124.5	p	26	0.618 (367)	0.712 (990)
105	133.8	p	19		1.01 (1121)
115	141.9	p	15	1.24 (534)	1.36 (1141)
125	149.1	p	12		1.77 (1305)
135	155.7	p	10		2.22 (1028)
145	161.6	p	8		l2.60 (1240)
155	167.2	p	7		l2.84 (1217)
165	172.4	p	~7		l2.96 (2488)

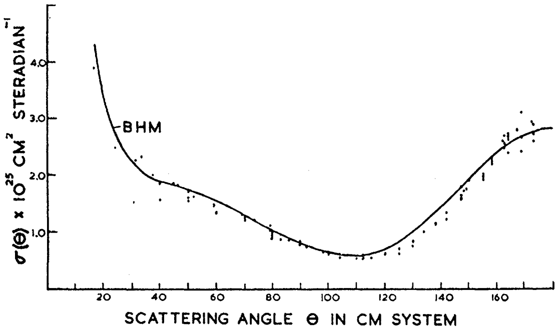

FIG. 1. Differential $p-d$ and $d-p$ cross sections in center-of-mass coordinates. All values listed in Tables I and II are plotted. The full curve (BHM) is theoretical, based on calculations for the same energy by Buckingham, Hubbard, and Massey.

the same cause but seem to be rather higher in value. Probably the curve begins to flatten off at or near 160° as predicted by the Buckingham, Hubbard, and Massey theory, but this region is difficult to study and the present data are not conclusive beyond about 160°.

Moreover, where this error exists, multiple scattering losses will also be at their worse and due to the complicated "compensating" mechanism (see Sec. VIII of reference 2) it is very difficult to estimate the consequences. Lower limit values are prefixed in the tables by l. Other values which are suspected for any reason (poor scanning conditions, heavy background, etc.) are prefixed by x.

In the present work absolute values were determined by two methods: (a) By measuring the charge collected on a standard condenser as described in the earlier publications. (b) By assuming a value for the absolute cross section of $p-p$ scattering (which is established to a higher degree of precision than any other scattering process) and determining $p-d$ and $d-p$ cross sections relative to $p-p$. The reason for introducing (b) is that as a result of a recheck by the Bureau of Standards some doubts have arisen concerning the true values of the condensers at the time this work was carried out.

Cross sections were therefore determined relative to the $p-p$ results of reference 2, which appear to be consistent with other results at or near this energy; i.e., assuming absolute $p-p$ cross sections to be in accordance with pure S-wave scattering of phase shift $\delta_0=54.5°$. The derived absolute $p-d$ and $d-p$ cross sections agreed with those measured directly (method (a)) to $\sim$2 percent. The cross sections in Tables I and II are the mean of both methods and are believed to have an accuracy in absolute value of ± 2 percent (probable error) taking account of all sources of systematic error which have been recognized but not including the effect of statistical error on the individual values.

TABLE II. Differential cross sections derived from five $d-p$ scattering runs at 10.2 Mev. In the third column ld indicates long deuteron track, sd short deuteron track and p recoil proton track. Prefix l indicates lower limit value and x doubtful value.

Θ	θ	Kind of track	Range in μ	σ(θ) ×10^25 cm² steradian⁻¹ Run 3	Run 4	Run 5	Run 6	Run 7
7	21.1	ld	325					
	166.0	p	440					
	172.9	sd	13			l2.73 (532)	l2.6 (118)	l2.9 (314)
11	33.4	ld	300					2.32 (430)
	158.0	p	425					2.31 (187)
	168.6	sd	15			l2.68 (1120)	l2.42 (996)	l3.11 (81)
15	46.1	ld	264					1.82 (1129)
	150.0	p	403					1.91 (504)
	163.9	sd	16	2.63 (889)		2.69 (1536)	2.40 (1859)	2.74 (262)
16	49.4	ld	260					
	148.0	p	396					
	162.5	sd	16	2.57 (946)		2.49 (1320)		2.70 (724)
21	66.8	ld	238					1.30 (1132)
	138.0	p	359					1.14 (390)
	155.2	sd	20			1.92 (1404)	1.99 (488)	2.04 (375)
25	82.7	ld	196		0.954 (1236)			0.875 (1709)
	130.0	p	324		0.839 (384)			0.752 (544)
	147.3	sd	25		1.80 (722)	x1.58 (994)	1.66 (1280)	1.60 (973)
30	120.0	p	276		0.639 (316)			
35	110.0	p	230	0.536 (1903)	0.563 (1070)	0.545 (2400)		0.610 (1049)
40	100.0	p	184	0.637 (2180)	0.613 (739)	0.622 (2267)		0.666 (1075)
45	90.0	p	141	0.780 (291)				
50	80.0	p	104	0.957 (2365)	0.945 (1047)	0.879 (2488)		
55	70.0	p	70			1.21 (2906)		
60	60.0	p	47			1.34 (1828)		
65	50.0	p	28	1.55 (2270)		x1.56 (2392)		
70	40.0	p	15	1.85 (1918)		1.58 (1905)		
75	30.0	p	7		l1.52 (220)			

In Fig. 1 all the experimental points of the present work have been plotted. The point at $\theta=16.4°$ is uncertain but seems to be consistent with purely Rutherford-Darwin scattering indicating that the scattering is essentially Coulombian below this angle. The minimum cross section at this energy occurs at approximately 112° in the center-of-mass system.

The full curve (BHM) represents the calculated angular distribution of 5-Mev protons due to Buckingham *et al.*[5] based on a symmetrical exchange force as in the earlier work of Buckingham and Massey, but including allowance for D-wave scattering. The new calculations are markedly superior in representing the observations at large angles (see comparison with earlier theory made by Karr *et al.*, reference 1) and appear to represent the experimental facts of $p-d$ scattering fairly satisfactorily from about 2 to 5 Mev. In absolute value at 5 Mev the theoretical curve would fit the data better if ~9 percent lower.

However, the recent $p-d$ scattering at 9.7 Mev carried out at Los Alamos by Armstrong *et al.* (see reference 1) can be compared roughly with the BHM calculations at 11.5 Mev and here the agreement is poor. The experimental curve rises much more steeply at large angles and also shows no tendency to flatten off between about 30° and 60° as predicted by BHM. These discrepancies seem to be paralleled also by high energy $n-d$ scattering.

Part of the scanning involved in this project was carried out at Birmingham University, England, assisted by an Imperial Chemical Industries Fellowship.

[5] Buckingham, Hubbard, and Massey, Proc. Roy. Soc. (London) **A211**, 183 (1952).

PHYSICAL REVIEW VOLUME 88, NUMBER 6 DECEMBER 15, 1952

The Relativistic Configuration Space Formulation of the Multi-Electron Problem*

MARIAN GÜNTHER
Institute of Theoretical Physics, University of Warsaw, Warsaw, Poland

(Received July 16, 1952)

A relativistic configuration space (many-time) presentation of quantum electrodynamics is developed, which, being equivalent to the Tomonaga-Schwinger formalism, may be expected to have the advantage of its direct applicability to bound-state problems of multi-electron systems. The main purpose of this paper is therefore to get an evidently relativistically invariant description of the interactions between individual electrons, in such a way, however, as to avoid those divergence difficulties which may be accounted for by the use of electron plane waves for building up the actual state of the system. The order of the presentation is as follows: a general relativistically invariant proof of equivalence of the Tomonaga-Schwinger formalism and the many-time formalism of Dirac, Fock, and Podolski given in Sec. I will provide a necessary starting point for further generalization in the form of Eq. (53). This equation, when eventually solve in Sec. II, gives rise to a formal extension of the multi-electron wave function concept into regions for which the latter remained hitherto undefined. In Sec. III a method of elimination of virtual processes is outlined and invariant expressions for the multi-electron interactions are derived. All explicit derivations in this paper are carried out to terms of the order of e^2. An example, a relativistic two-electron wave equation, is given. This equation accounts for the Coulomb and Breit interactions.

I. MANY-TIME AND TOMONAGA-SCHWINGER THEORIES

A FORMALISM based on the relativistic configuration space concept was introduced for the first time by Dirac, Fock, and Podolski[1] (referred to in the following as D.F.P.). This formalism is usually called the many-time theory, because an introduction of separate sets of spatial coordinates for individual electrons necessitates an introduction of a separate time variable for each electron. Our first task will be to show the equivalence of the Schwinger-Tomonaga theory[2] (referred to later on as T-Sch.) and the D.F.P. theory, if Fermi-Dirac statistics is assumed in the latter. The method employed is thoroughly invariant in form. We start with the basic equations of the D.F.P. theory:

$$[\gamma_k{}^\mu((\partial/\partial x_k{}^\mu)-(ie/\hbar c)A_\mu(x_k))+(mc/\hbar)]\times\varphi(x_1\cdots x_k\cdots x_\nu)=0;\quad k=1,2,3\cdots\nu;\quad (1)$$

$$\Box A_\mu=0;\quad (2)$$

$$[A_\mu(x),A_\nu(x')]_-=i\hbar c\delta_{\mu\nu}D(x-x');\quad (3)$$

$$[\partial A_\mu(x)/\partial x^\mu-e\sum_{k=1}^{\nu}D(x-x_k)]\varphi(x_1\cdots x_\nu)=0;\quad (4)$$

where A_μ stand for the components of the potential four-vector of the radiation field, and $\varphi(x_1\cdots x_\nu)$ is the wave function of the system containing ν-electrons. Each x, when without upper index, stands for all the

* This part of the work submitted to the Faculty of Mathematics, Physics, and Chemistry of the University of Warsaw for obtaining the "Venia Legendi" (May, 1951).

[1] Dirac, Fock, and Podolski, Physik. Z. Sowjetunion **2**, 468 (1932).

[2] S. Tomonaga, Prog. Theor. Phys. **1**, 27 (1946); J. Schwinger, Phys. Rev. **74**, 1439 (1948).