

FIG. 1. Schematic diagram of the experimental arrangement.

Figure 2 shows the angular distribution of the protons integrated over the energy spectrum. Curve A shows the distribution to be expected with our geometry for meson production from a free neutron at rest; curve B takes into account the internal momentum of the deuteron, assuming the zero-range wave function and no momentum transfer to the "spectator" proton. The cross section for meson production was assumed constant over the energy and angular intervals employed. The agreement of curve B with the experimental points is satisfactory, supporting the assumption of negligible momentum transfer to the "spectator" proton.

Figure 3 shows the energy spectrum of the recoil protons observed at  $30^\circ \pm 6^\circ$ . Curves A and B were computed as for Fig. 2. The experimental points differ significantly from the theoretical curve in this case; they appear to be shifted from 10 to 15 Mev towards higher energy.

Considerable time has been spent in an effort to resolve this discrepancy. The proton counter was calibrated (1) by measurement of the  $\text{Cs}^{137}$  photoelectric peak at 0.67 Mev; (2) by measurement of pulses produced by cosmic-ray mesons at 9.5 Mev<sup>1</sup>; and (3) by extrapolation of the number-bias curve for an inhomogeneous beam of protons to the high energy cutoff at 69 Mev.<sup>2</sup> All these methods agreed to  $\pm 5$  percent. Further checks were made by using different crystals for the proton counter and by direct range measurements on the protons.

An error in the definition of meson energy<sup>3</sup> could be produced if mesons were being scattered around the anticoincidence counter

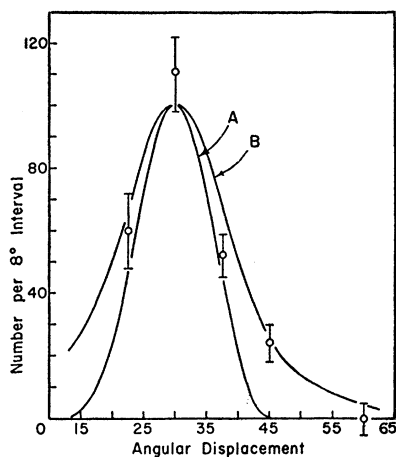


FIG. 2. Angular distribution of recoils counted in coincidence with a 56-Mev meson at  $90^\circ$ . See text for explanation of curves.

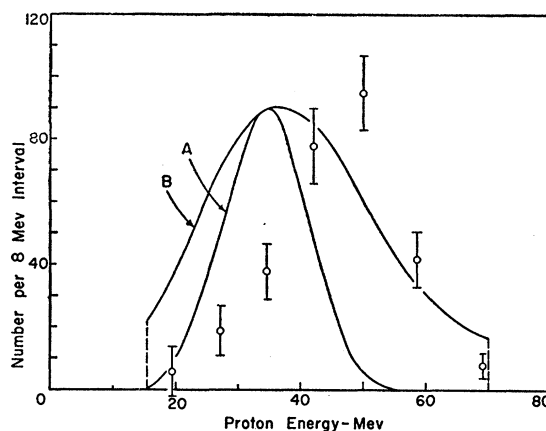


FIG. 3. Energy distribution of recoils at  $30^\circ$  counted in coincidence with a 56-Mev meson at  $90^\circ$ . See text for explanation of curves.

$\text{C}_3$ . To check this point, experiments were performed with two different telescopes, in one of which  $\text{C}_3$  had twice the area of the other. The results with the two telescopes were identical.

Assuming the angular distribution of recoils to be correctly given by curve B of Fig. 2, one can compute the differential cross section for the production of negative pions accompanied by correlated recoil protons. The value obtained is  $10.8 \pm 1.0 \mu\text{b}/\text{sterad}$  for production at  $90^\circ$  in the laboratory by 236-Mev photons. The error quoted is the standard statistical deviation. We have also determined the cross section for the production of negative pions without the requirement of a correlated recoil, using the  $\pi^-/\pi^+$  ratios of Littauer and Walker<sup>4</sup> to separate the  $\pi^-$  and  $\pi^+$  contributions in the telescope. The value so obtained is  $11.8 \pm 1.2 \mu\text{b}/\text{sterad}$ . The agreement between the two cross sections may be taken as evidence that the "spectator process" does indeed account for a substantial fraction of the meson production in the deuteron at the angles and energies here involved.

The correctness of the interpretation is, of course, still in doubt due to the failure to fit the energy distributions of the recoils. In closing, we might mention two possible explanations of this discrepancy which have been considered and found unsatisfactory. The first is that the low energy protons are preferentially removed by Coulomb scattering in the target. This is a small effect and there is a tendency for *scattering in* to cancel *scattering out*. The second is that the cross section for meson production might vary in such a way as to favor high energy recoils. Here the difficulty is that the photon energy in the c.m. system is defined within fairly narrow limits by the meson energy, regardless of the momentum state of the struck nucleon, and the cross section is unlikely to vary steeply enough within these limits.

\* Work supported by the ONR.

<sup>1</sup> W. L. Whittmore and J. C. Street, Phys. Rev. **76**, 1786 (1949).

<sup>2</sup> J. C. Keck, Phys. Rev. **85**, 410 (1952).

<sup>3</sup> In obtaining a range-energy curve for mesons from that for protons a small correction was made for the increased importance of Coulomb scattering.

<sup>4</sup> R. Littauer and D. Walker, Phys. Rev. **86**, 838 (1952).

## An Anomaly in the Low Temperature Atomic Heat of Silver\*

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IN measurements of the atomic heat of the semiconductors Ge and Si at very low temperatures, the problem arose of distinguishing between two possible sources of deviations from a cubic dependence of the atomic heat on temperature: (a) electronic heat capacity (due to the presence of impurities); (b) polycrystal-

line structure of the material. To determine the influence of the crystalline state of material on the specific heat, K. Lark-Horovitz suggested measuring the atomic heat of a very pure single crystal of Ag, for comparison with the results of earlier measurements on polycrystalline Ag by Keesom and Kok.<sup>1</sup>

We have therefore measured the atomic heat of a single crystal of Ag<sup>2</sup> at liquid helium temperatures. The results are identical, within experimental error, with those obtained earlier by Keesom and Kok.<sup>1</sup> On a plot of  $C_v/T$  against  $T^2$  (see Fig. 1), it is not

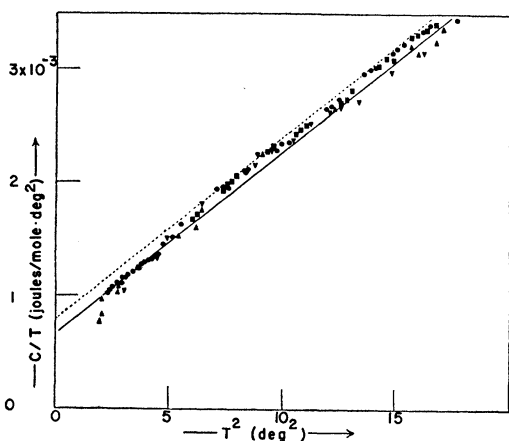


FIG. 1.  $C_v/T$  vs  $T^2$  for Ag;  $\nabla$ —Keesom and Kok, 1932;  $\Delta$ —Keesom and Kok, 1933;  $\square$ —this measurement, 1/25/52;  $\circ$ —1/30/52.

possible to fit all the data below 4.2°K with one straight line. Below 2.2°K, both sets of data can be represented by the solid line, which has the equation

$$C_v = 1.619 \times 10^{-4} T^3 + 6.45 \times 10^{-4} T \text{ joules/mole degree,} \quad (1)$$

while above 2.5°K, the dashed line, which has the equation:

$$C_v = 1.598 \times 10^{-4} T^3 + 7.82 \times 10^{-4} T \text{ joules/mole degree,} \quad (2)$$

fits both sets of data.

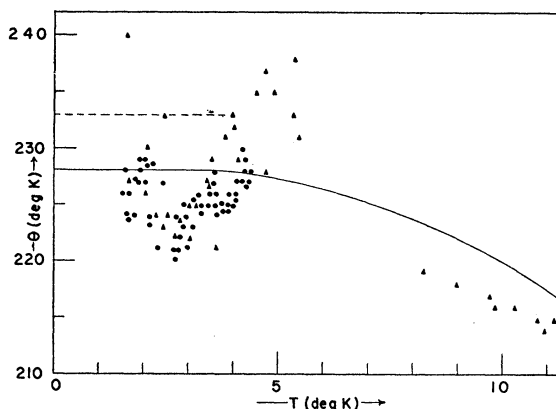


FIG. 2. Debye  $\theta$  vs  $T$  for Ag;  $\Delta$ —Keesom and Kok;  $\circ$ —this measurement; —Leighton's calculated curve; - -  $\theta(E)$  calculated from elastic constants (this calculation is valid only in the true  $T^3$  region).

While this anomaly is also evident in the data of Keesom and Kok, it becomes obvious only in a plot of  $C_v/T$  vs  $T^2$ . However, as was common at the time of their experiments (which provided the first proof of the existence of electronic heat capacity in metals), they analyzed their results only with a plot of  $\theta$  vs  $T$ .

Several alternatives suggest themselves for the explanation of this effect. Gerritsen and Korringa<sup>3</sup> have suggested that the rise of resistance observed in some metals at very low temperatures is due to the presence of a very small concentration of paramagnetic

impurity atoms, such as Mn. They also predict an additional contribution to the atomic heat in the form of a Schottky curve<sup>4</sup> arising from electronic transitions between states whose degeneracy has been removed by the paramagnetic atoms. Their predicted effect, however, would be much too small compared with that observed, if our single crystal had the purity stated for it (99.999 percent). It also seems unlikely that both our sample and that of Keesom and Kok would contain the same type and amount of impurities.

Katz<sup>5</sup> has suggested that lattice perturbations (due to work hardening, mosaic structure, impurities, etc.) can result in deviations from the Debye vibration spectrum at very low temperatures and provide anomalous contributions to the atomic heat. He has discussed in this way apparent irregularities in the curve of  $\theta$  vs  $T$  (see Fig. 2) for Ag, but Keesom and Kok do not believe them to be significant in view of their experimental error. Such perturbations, moreover, must have a considerable degree of order themselves, so that they may form a "superlattice" capable of affecting the spectrum in this way. It is hard to imagine a perturbation of this type which would be identical in Keesom and Kok's polycrystalline material and our single crystal.

Leighton<sup>6</sup> has calculated the elastic spectrum for face-centered cubic crystals and finds that for  $T$  less than 4°K,  $\theta$  for Ag is practically constant at 228°K. This agrees very well with the experimental values, 229°K for the solid line and 230°K for the dashed line, and with the value  $\theta(E) = 233$ °K calculated from elastic constants<sup>7</sup> by the method of Hopf and Lechner.<sup>8</sup> It is therefore unlikely<sup>6</sup> that deviations from the Debye vibration spectrum due to the nature of the Ag lattice itself are responsible for the anomaly.

It seems reasonable to interpret the behavior shown in Fig. 1 as a change in  $\gamma$ , the coefficient of the linear term, occurring between 2.2°K and 2.5°K, with the cubic term remaining constant. A general expression for  $\gamma$  per mole is<sup>9</sup>

$$\gamma = (\pi^2 k^2 / 3) V f(\zeta), \quad (3)$$

where  $k$  is Boltzmann's constant,  $V$  is the atomic volume, and  $f(\zeta)$  is the density of electronic states at the Fermi level. An increase in  $\gamma$  could correspond to an increase in  $f(\zeta)$ . This increase could come about as a result of overlapping of allowed bands, and the position of the overlap and the density of levels in the higher band should be derivable from the observed increase in  $\gamma$  (21 percent) and the temperature at which it occurs. Unfortunately, however, no detailed calculation of the band structure of Ag on the basis of electronic wave functions is available for comparison. If such a calculation were made, it would be of great interest to compare the electronic heat capacity predicted from it with our experimental results.

\* Work performed under contract with the Signal Corps.

<sup>1</sup> W. K. Keesom and J. A. Kok, Commun. Kamerlingh Onnes Laboratory, University of Leiden, No. 219d (1932); No. 232d (1933).

<sup>2</sup> The single crystal weighed about 90 grams and was kindly supplied to K. Lark-Horovitz by Dr. R. Maddin of The Johns Hopkins University.

<sup>3</sup> A. N. Gerritsen and J. Korringa, Phys. Rev. **84**, 604 (1951).

<sup>4</sup> W. Schottky, Physik. Z. **23**, 448 (1922).

<sup>5</sup> E. Katz, J. Chem. Phys. **19**, 488 (1951).

<sup>6</sup> R. B. Leighton, Revs. Modern Phys. **20**, 165 (1948).

<sup>7</sup> Landolt-Bornstein, *Physikalisch-Chemische Tabellen* (Verlag Julius Springer, Berlin, 1927), fifth edition.

<sup>8</sup> L. Hopf and G. Lechner, Verhandl. deut. physik. Ges. **16**, 643 (1914).

<sup>9</sup> F. Seitz, *The Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), pp. 130-151.

### A Further Test of the Shell Model\*

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AS an additional test of the accuracy of the shell model in ascribing definite orbital angular momentum states to nucleons in a nucleus<sup>1,2</sup> we have measured the angular distribution of the protons associated with the ground state in the reaction  $\text{Cl}^{36}(d, p)\text{Cl}^{36}$ . As pointed out by Bethe and Butler, the selection