Now, if we neglect the term $q^{-2}T_{\alpha\beta}dx^\alpha dx^\beta$ in the expression of the metric of the space-time, we can write

$$d\tau^2 = -\left(1+\frac{\kappa_0}{4\pi}\int\frac{\sigma(r')}{|r-r'|}dV'\right)(dx^2+dy^2+dz^2) + c^2\left(1-\frac{\kappa_0}{4\pi}\int\frac{\sigma(r')dV'}{|r-r'|}\right)dt^2. \quad (8.12)$$

This result is the same as the one obtained by Einstein in general relativity. The required deflection of light passing near a strong gravitational field follows from

$$d\tau^2=0.$$

Hence, if L is the velocity of light, we have approximately

$$L=c\left(1-\frac{\kappa_0}{4\pi}\int\frac{\sigma(r')}{|r-r'|}dV'\right), \quad (8.13)$$

so that the deflection of light is

$$\alpha=\int_{-\infty}^{\infty}\frac{1}{L}\frac{\partial L}{\partial x}dz=\frac{\kappa_0 M}{2\pi R}=1.75', \quad (8.14)$$

in complete agreement with the result of general relativity.[18]

The author is not aware of any other unified field theory comprising the results obtained in the foregoing. We think that these implications of the theory are important enough to warrant our confidence in its validity as a correct physical theory.

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[18] It can easily be seen that for $\kappa=0$ the above theory reduces to Einstein's version of the unified field theory.

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Mobility in High Electric Fields

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An extension of conductivity theory to high fields, subject to the usual simplifying assumptions, is carried out for the cases in which the change of energy of an electron in a collision can be neglected. This yields a relationship between mobility and relaxation time which is valid over a wide range of fields.

THE formal theory of conduction can be extended simply to cover the case of high fields when the scattering processes which give rise to the resistance are to a good approximation elastic. This condition is satisfied in monatomic solids for the range of temperature and field strength in which collisions are mainly with acoustical modes of lattice vibrations and impurities. Experiments indicate that in germanium at room temperature, for example, this is the case up to about 4000 volts per cm.[1] This condition is also satisfied for electrons in a gas when the collisions are nonionizing ones with atoms or ions.

The distribution function for the electrons in an electric field will be denoted by $f(\mathbf{k}, \mathbf{E})$, where $\mathbf{k}$ is the wave vector and $\mathbf{E}$ is the electric field intensity. If we neglect crystal anisotropy, since the scattering is elastic the distribution function in the presence of the field will be nearly isotropic in k space. It can be shown that it is a good approximation to take

$$f(\mathbf{k}, \mathbf{E})=f_0(k, E)-f_1(k, E)\cos\theta, \quad (1)$$

where θ is the angle between $\mathbf{k}$ and the field direction, chosen as the z axis, and f_1 is much smaller than f_0. In low fields, of course, f_0 will be the zero field equilibrium distribution. In the steady state, the rate of change of f due to the field must be balanced by the rate of change due to collisions, or

$$(eE/\hbar)(\partial f/\partial k_z)+(\partial f/\partial t)_c=0. \quad (2)$$

It has been shown that probabilities of scattering by lattice vibrations or imperfections are independent of electric field intensity up to fields of the order of 6×10^5 volts per cm.[2] In the approximation that the scattering is elastic, transitions will take place to states on the constant energy surface. Let the probability of transition per unit time from a state near $\mathbf{k}$ to one of a group of states in area dS' of the constant energy surface be denoted by $P(\mathbf{k}, \mathbf{k}')dS'$. Then

$$(\partial f/\partial t)_c=-\int_{S'}[f(\mathbf{k}, \mathbf{E})P(\mathbf{k}, \mathbf{k}') - f(\mathbf{k}', \mathbf{E})P(\mathbf{k}', \mathbf{k})]dS'. \quad (3)$$

To carry this further it is necessary to assume that $P(\mathbf{k}, \mathbf{k}')=P(\mathbf{k}', \mathbf{k})$ and depends only on the angle between $\mathbf{k}$ and $\mathbf{k}'$. Since collisions can only redistribute electrons around the constant energy surface, f_0 does not contribute to this term. The rate of change of f_1 may be found by considering an element of phase

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[1] W. Shockley, Bell System Tech. J. 30, 990 (1951).
[2] J. Bardeen and W. Shockley, Phys. Rev. 80, 69 (1950).

space along the $\mathbf{k}$ vector parallel to $\mathbf{E}$. Using (1), we obtain

$$(\partial f_1/\partial t)_c = +\int_{S'} f_1(k, E)(1-\cos\theta)P(\theta)dS', \quad (4)$$

$$(\partial f_1/\partial t)_c = +[f_1(k, E)]/[\tau(k)], \quad (5)$$

where τ is the zero field relaxation time. Equation (2) then takes the form

$$(eE/\hbar)(df/dk)+(f_1/\tau)=0, \quad (6)$$

for high as well as low fields.[3]

Since the isotropic part of the distribution will not contribute to the current, the current density is given by

$$j=\int\int\int e\frac{\hbar k}{m}\cos\theta(-f_1\cos\theta)\frac{2}{(2\pi)^3}k^2\sin\theta d\theta d\phi dk, \quad (7)$$

where the integration is over all k space. Using (6) to eliminate f_1, we obtain

$$j=\frac{2}{(2\pi)^3}\frac{e^2}{m}E\int\int\int \tau\frac{df}{dk}k^3\cos^2\theta\sin\theta d\theta d\phi dk. \quad (8)$$

We now make the assumption that the energy surfaces are spherical. The integration can then be carried out, giving

$$j=\frac{1}{3\pi^2}\frac{e^2}{m}E\left[(k^3\tau f)\Big|_0^\infty-\int_0^\infty f\frac{d}{dk}(k^3\tau)dk\right]. \quad (9)$$

For the mechanisms of scattering under consideration, $k^3\tau$ vanishes at the lower limit, and the first term makes no contribution to j. The expression can then be written

$$j=-\frac{e^2}{3m}E\int_0^\infty \frac{1}{k^2}\frac{d}{dk}(k^3\tau)f(k)\frac{1}{\pi^2}k^2dk$$

$$=\frac{e^2}{3m}En\left\langle\frac{1}{k^2}\frac{d}{dk}(k^3\tau)\right\rangle, \quad (10)$$

where n is the number of electrons per unit volume, and the quantity in brackets is to be averaged over all electrons. In terms of the mobility, this result is

$$\mu=\frac{e}{3m}\left\langle\frac{1}{k^2}\frac{d}{dk}(k^3\tau)\right\rangle. \quad (11)$$

In this form the result is valid for low fields and high within the limits that the scattering is nearly elastic. It yields a field-dependent mobility at high fields because the electron distribution is field-dependent.

It is easily checked that this gives correct results in all familiar low field cases. For example, in the case of lattice or impurity scattering in nondegenerate semiconductors at low fields it gives the same mobility as $(e/m)\langle v^2\tau\rangle/\langle v^2\rangle$, which is valid for a Maxwell-Boltzmann distribution.[4] At high fields it gives the correct drift velocity for electrons in a gas, for which the high field distribution is known.[5] It has been pointed out by Wannier that this expression for the mobility is derivable in the high field case from the equation system (17) of his paper.[6]

[3] Essentially this equation is derived for electrons in a gas in high fields in S. Chapman and T. G. Cowling, *Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, Cambridge, 1939), p. 346.

[4] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950), p. 276.

[5] See reference 3, p. 351.

[6] G. Wannier, Phys. Rev. 83, 281 (1951).

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The Comparison between the Longitudinal Correlation and the Time Correlation in a Turbulent Flow*

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Some preliminary applications of high speed computing to the analysis of experimental data on turbulence are reported. A time correlation curve is determined from a recording of velocity fluctuations measured with a single hot-wire anemometer. The shape of the time correlation curve differs from the shape of the longitudinal correlation curve measured in the same fluid flow. One must, therefore, be cautious in reaching conclusions from experiments based on the assumption that time and space spectra of turbulence are identical. A more extensive study of the relation between time and space characteristics of turbulence is in process.

WHEN the relation between a spectrum of turbulence and a correlation coefficient was first found by Taylor,[1] an assumption was made that the turbulence pattern moves in the fluid stream without changing. This assumption was necessary, since Taylor's spectrum described a statistical characteristic of velocity fluctuations at a fixed point of the fluid flow while his correlation coefficient referred to simultaneous velocity fluctuations along the mean velocity direction. Taylor's

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[1] G. I. Taylor, Proc. Roy. Soc. (London) A164, 476 (1938).