obtained:[3]

$$D/\mu=0.0268\pm0.0013 \text{ ev},$$
$$T=303\pm1\text{°K},$$
$$kT/q=0.0262\pm0.0001 \text{ ev}.$$

[3] The probable error in the values of T and kT/q were not obtained using the method of least squares which would give a value much less than this. Generous allowance is made for systematic uncertainty in recording ambient temperature.

It would appear that these results verify the relation $D/\mu=kT/q$. Although there have been other experimental verifications of this relationship using colloidal particles and ions, this is the first direct experimental proof of the validity of this equation for electrons and holes of which we are aware.

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Gravitation and Electrodynamics

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In this paper Einstein's unified field theory is modified, and some of the physical implications of the new theory are examined. It entails: (1) a restriction of 4-current distribution, (2) an electromagnetic field consisting of short- and long-range parts, (3) a finite self-energy for the electron, (4) a classical description of pair production and annihilation as discussed by Feynman in his electrodynamics, (5) the Lorentz-force law for a charged particle moving in an external electromagnetic field, (6) the bending of light grazing the surface of the sun—the same as given by the general theory of relativity.

1. INTRODUCTION

THE arguments for the necessity of a unified field theory are well known, and therefore they will not be elaborated at length. The author believes that a correct and unified quantum theory of fields, without the need of the so-called renormalization of some physical constants, can be reached only through a complete classical field theory that does not exclude gravitational phenomena. It is true that one cannot feel very optimistic about the quantization of a nonlinear classical field theory. But one hopes that this difficulty may be overcome, partly, by starting the quantization procedure with a Lagrangian[1] formulation of the quantum field theory.

In this paper we propose a new version of Einstein's latest unified field theory.[1a] The reasons for this modification will be made clear in the following. The same formalism and notation of Einstein's theory are used. The total field is described by a Hermitian tensor $g_{\alpha\beta}$ given as

$$g_{\alpha\beta}=a_{\alpha\beta}+i\varphi_{\alpha\beta}, \tag{1.1}$$

where

$$a_{\alpha\beta}=g_{\underline{\alpha\beta}} \quad\text{and}\quad \varphi_{\alpha\beta}=g_{\underset{\smile}{\alpha\beta}}, \quad i=(-1)^{\frac{1}{2}},$$

so that we have

$$(g_{\alpha\beta})^{\dagger}=(g_{\alpha\beta}). \tag{1.2}$$

The dagger (†) stands for Hermitian conjugate operation. We also have the general affine connection $\Gamma^{\gamma}_{\alpha\beta}$ given by

$$\Gamma^{\gamma}_{\alpha\beta}=\Gamma^{\gamma}_{\underline{\alpha\beta}}+i\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}}. \tag{1.3}$$

The Hermitian property of $\Gamma^{\gamma}_{\alpha\beta}$ in the covariant indices α and β is obvious.

Now, if we define $a^{\alpha\beta}$ as the normalized minors of $\mathrm{Det}\,a_{\alpha\beta}=a$, then, we have

$$a_{\alpha\mu}a^{\gamma\mu}=\delta_{\alpha}{}^{\gamma}.$$

The determinant of $g_{\alpha\beta}$, because of (1.2), is real and can be expressed as

$$g=a(1-\Omega-\Lambda^2), \tag{1.4}$$

where

$$\Omega=\tfrac{1}{2}\varphi_{\mu\nu}\varphi^{\mu\nu} \quad\text{(is an invariant)},$$
$$\Lambda=\tfrac{1}{4}f^{\mu\nu}\varphi_{\mu\nu} \quad\text{(is a pseudoscalar)},$$

and

$$f^{\alpha\beta}=\frac{1}{2(-a)^{\frac{1}{2}}}\epsilon^{\alpha\beta\mu\nu}\varphi_{\mu\nu}, \tag{1.5}$$

where $\epsilon^{\alpha\beta\mu\nu}$ is zero whenever any two indices are equal and is ±1 for even and odd permutations. All indices are raised by $a^{\alpha\beta}$.

We also have the contravariant tensor $g^{\alpha\beta}$ given by

$$g_{\alpha\mu}g^{\beta\mu}=\delta_{\alpha}{}^{\beta}.$$

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[1] J. Schwinger, Phys. Rev. 82, 914 (1951).
[1a] A. Einstein, *The Meaning of Relativity* (Methuen, London, 1951).

It can easily be shown that $\mathfrak{g}^{\alpha\beta}[=(-g)^{\frac{1}{2}}g^{\alpha\beta}]$ can be expressed in terms of $g_{\alpha\beta}$ as

$$\mathfrak{g}^{\underline{\alpha\beta}}=(-g)^{\frac{1}{2}}a^{\alpha\beta}+\mathfrak{g}^{\underset{\smile}{\alpha\mu}}\varphi^{\beta}{}_{.\mu}, \tag{1.6}$$

$$\mathfrak{g}^{\underset{\smile}{\alpha\beta}}=(-a)^{\frac{1}{2}}(\varphi^{\alpha\beta}+\Lambda f^{\alpha\beta})/(1-\Omega-\Lambda^2)^{\frac{1}{2}}. \tag{1.7}$$

In (1.6) we used the expression

$$\delta_\beta{}^\alpha\Lambda=f^{\alpha\mu}\varphi_{\beta\mu}.$$

The components of the affine connection $\Gamma^\gamma{}_{\alpha\beta}$ are defined by solving the equations

$$g_{\underset{+-}{\alpha\beta};\gamma}=g_{\alpha\beta,\gamma}-g_{\mu\beta}\Gamma^{\mu}_{\alpha\gamma}-g_{\alpha\mu}\Gamma^{\mu}_{\gamma\beta}=0. \tag{1.8}$$

By splitting up symmetric and antisymmetric parts (or by taking real and imaginary parts) of (1.8) it is easy to show that

$$\Gamma^{\gamma}_{\underline{\alpha\beta}}=\left\{\begin{matrix}\gamma\\ \alpha\beta\end{matrix}\right\}_a+a^{\gamma\mu}(\varphi_{\beta\nu}\Gamma^{\nu}_{\underset{\smile}{\alpha\mu}}+\varphi_{\nu\alpha}\Gamma^{\nu}_{\underset{\smile}{\mu\beta}}), \tag{1.9}$$

where $\left\{\begin{matrix}\gamma\\ \alpha\beta\end{matrix}\right\}_a$ are the Christoffel symbols formed from $a_{\alpha\beta}$.

Equations (1.8), when solved with respect to $\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}}$, give[2]

$$\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}}=-\tfrac{1}{2}I^{\gamma}_{\alpha\beta}+a^{\gamma\mu}\varphi_{\alpha\beta o\mu}, \tag{1.10}$$

where the sign (o) stands for covariant differentiation with respect to $\Gamma^{\gamma}_{\underline{\alpha\beta}}$, so that (1.10) is only an implicit solution of (1.8), and $I^{\gamma}_{\alpha\beta}=a^{\gamma\mu}I_{\alpha\beta\mu}$ is defined by

$$I_{\alpha\beta\gamma}=\varphi_{\alpha\beta,\gamma}+\varphi_{\beta\gamma,\alpha}+\varphi_{\gamma\alpha,\beta}. \tag{1.11}$$

Equation (1.11) represents the 4-current density, the dual of which is

$$\mathfrak{I}^\alpha=(1/3\,!)\,\epsilon^{\mu\nu\rho\alpha}I_{\mu\nu\rho}. \tag{1.12}$$

We add the four field conditions,

$$\Gamma_\alpha=\Gamma^{\gamma}_{\underset{\smile}{\alpha\gamma}}=0, \tag{1.13}$$

to Eqs. (1.8).

Equations (1.13) imply, because of (1.8), the four equations

$$\mathfrak{g}^{\underset{\smile}{\alpha\beta}}{}_{,\beta}=0. \tag{1.14}$$

2. THE EQUATIONS OF THE TOTAL FIELD

The existence of Bianchi identities have been of great use in the mathematical formulation of the ideas of general relativity. The right-hand sides of $G_{\alpha\beta}-\frac{1}{2}a_{\alpha\beta}G=-8\pi\bar{\gamma}T_{\alpha\beta}$ are the phenomenological descriptions of the sources of the gravitational field, but the fact that covariant divergence of the left-hand side vanishes identically has induced one to stick-in $T_{\alpha\beta}$ in an *ad hoc* manner. In unified field theory we can start, formally, in the same way and derive the sources of the field direct from the elements of the theory itself.

We first note that if we multiply both sides of (1.11) by $\mathfrak{g}^{\underset{\smile}{\alpha\beta}}$ and use Eqs. (1.14), the identities

$$\mathfrak{g}^{\underline{\alpha\mu}}{}_{|\mu}=-\tfrac{1}{2}\mathfrak{g}^{\underset{\smile}{\mu\nu}}I^{\alpha}_{\mu\nu} \tag{2.1}$$

follow immediately. The stroke (|) in (2.1) stands for covariant differentiation with respect to $\left\{\begin{matrix}\gamma\\ \alpha\beta\end{matrix}\right\}_a$.

If (1.8) and (1.13) are granted, it can be shown that the Hermitian tensor $R_{\alpha\beta}$ satisfies Bianchi-Einstein identities for the nonsymmetric field derived by Einstein[3] as

$$\mathfrak{g}^{\alpha\beta}[R_{\underset{+-}{\alpha\beta};\gamma}-R_{\underset{++}{\alpha\gamma};\beta}-R_{\underset{--}{\gamma\beta};\alpha}]=0. \tag{2.2}$$

These identities hold for all fields satisfying (1.8) and (1.13). It is now easy to verify the identity, with proviso (1.8) and (1.13),

$$\mathfrak{g}^{\alpha\beta}[g_{\underset{+-}{\alpha\beta};\gamma}-g_{\underset{++}{\alpha\gamma};\beta}-g_{\underset{--}{\gamma\beta};\alpha}]=0, \tag{2.3}$$

which is equivalent to (2.1).

After performing the semi-colon covariant differentiations and using

$$\mathfrak{g}^{\underline{\alpha\beta}}{}_{,\beta}=-\mathfrak{g}^{\mu\nu}\Gamma^{\alpha}_{\mu\nu}$$

(which follows from $\mathfrak{g}^{\overset{\alpha\beta}{+-}}{}_{;\gamma}=0$), the identity (2.2) can be written in a suggestive form as

$$(\mathfrak{g}^{\underline{\alpha\mu}}R_{\underline{\beta\mu}}-\tfrac{1}{2}\delta_\beta^\alpha\mathfrak{g}^{\underline{\mu\nu}}R_{\underline{\mu\nu}})_{||\alpha}=-\tfrac{1}{2}\mathfrak{g}^{\underset{\smile}{\mu\nu}}(R_{\underset{\smile}{\mu\nu},\beta}+R_{\underset{\smile}{\nu\beta},\mu}+R_{\underset{\smile}{\beta\mu},\nu}), \tag{2.4}$$

and (2.3) or (2.1) are equivalent to

$$(\mathfrak{g}^{\underline{\alpha\mu}}a_{\beta\mu}-\tfrac{1}{2}\delta_\beta^\alpha\mathfrak{g}^{\underline{\mu\nu}}a_{\mu\nu})_{||\alpha}=-\tfrac{1}{2}\mathfrak{g}^{\underset{\smile}{\mu\nu}}(\varphi_{\mu\nu,\beta}+\varphi_{\nu\beta,\mu}+\varphi_{\beta\mu,\nu}), \tag{2.5}$$

where (||) stands for covariant differentiation with respect to Christoffel symbols formed from $b_{\alpha\beta}$ which is defined by

$$b^{\alpha\beta}=\mathfrak{g}^{\underline{\alpha\beta}}/(-\mathrm{Det}\,\mathfrak{g}^{\underline{\alpha\beta}})^{\frac{1}{2}}.$$

Hence,

$$\mathfrak{g}^{\underline{\alpha\beta}}=(-b)^{\frac{1}{2}}b^{\alpha\beta},\quad b=\mathrm{Det}\,b_{\alpha\beta}=\mathrm{Det}\,\mathfrak{g}^{\underline{\alpha\beta}}, \tag{2.6}$$

and

$$b_{\alpha\mu}b^{\gamma\mu}=\delta^{\gamma}_{\alpha}. \tag{2.7}$$

The form of (2.4) does not change with respect to the transformation

$$R_{\underset{\smile}{\alpha\beta}}\rightarrow R_{\underset{\smile}{\alpha\beta}}+(\partial_\alpha B_\beta-\partial_\beta B_\alpha).$$

[2] B. Kurşunoğlu, Phys. Rev. **82**, 289 (1951).

[3] A. Einstein, Can. J. Math. **2**, 120 (1950).

This fact will be made use of in the Hamiltonian principle of the theory.

When $\varphi_{\alpha\beta}=0$, (2.4) reduces to

$$(\mathfrak{a}^{\alpha\mu}G_{\beta\mu}-\tfrac{1}{2}\delta^{\alpha}_{\beta}\mathfrak{a}^{\mu\nu}G_{\mu\nu})_{|\alpha}=0,$$

where $\mathfrak{a}^{\alpha\beta}=(-a)^{\frac{1}{2}}a^{\alpha\beta}$; hence the symmetric tensor $b_{\alpha\beta}$ may be taken as the generalized "metric" of the space-time. From the conservation laws of general relativity one might jump to the conclusion that the right-hand sides of (2.4) ought to vanish so that they would constitute a set of four field equations, but this is *non-sequitur*. Instead, we shall follow the analogy with the electromagnetic theory. In (2.4) the right-hand side being a covariant divergence of a symmetric tensor density [left-hand side of (2.4)] has the appearance of a generalized electromagnetic force density, that this is so will be shown more explicitly later on.

The identity (2.4) can be satisfied by taking $R_{\alpha\beta}=0$ (Einstein's theory), or we may reconcile the two identities by putting $R_{\alpha\beta}=\lambda g_{\alpha\beta}$ (Schrödinger's theory).[4] In both cases the field equations are over-determined (since we also have Eqs. (1.14) the total number of the field equations are 20; for consistency 8 identities are required, but there are only 4+1 identities), and also the forms of the identities are destroyed. It is very important to preserve the form of the identities, i.e., the reconciliation of (2.4) and (2.5) can be carried through by preserving the forms of "matter tensor" and "force-density," so that only in this way a consistent number of field equations can be secured. The expression in the bracket on the left-hand side of (2.4) can be regarded as a tensor describing the energy and momentum of the total field. We can think the same for (2.5) so that the required unification of the physical fields will be achieved, if we introduce a "fundamental constant" p of the dimension of $(\text{length})^{-1}$ by writing

$$\mathfrak{g}^{\overset{\alpha\mu}{-}}R_{\underline{\beta\mu}}-\tfrac{1}{2}\delta^{\alpha}_{\beta}\,\mathfrak{g}^{\overset{\mu\nu}{-}}R_{\mu\nu}+f\delta^{\alpha}_{\beta}(-b)^{\frac{1}{2}}$$
$$=-p^2(\mathfrak{g}^{\alpha\mu}a_{\beta\mu}-\tfrac{1}{2}\delta^{\alpha}_{\beta}\mathfrak{g}^{\overset{\mu\nu}{-}}a_{\mu\nu}),\quad\text{(I)}$$

and

$$R_{\underset{\smile}{\alpha\beta},\gamma}+R_{\underset{\smile}{\beta\gamma},\alpha}+R_{\underset{\smile}{\gamma\alpha},\beta}=-p^2(\varphi_{\alpha\beta,\gamma}+\varphi_{\beta\gamma,\alpha}+\varphi_{\gamma\alpha,\beta}),\quad\text{(II)}$$

where, in I, the third term on the left-hand side is a consequence of the covariant differentiation with respect to $\left\{\begin{matrix}\gamma\\ \alpha\beta\end{matrix}\right\}_b$, and f is a constant.

The minus sign before p^2 is very important and will explain itself when we consider the solutions of the field equations.

By contracting (I) with respect to β and α and arranging the terms, we obtain I as

$$R_{\underline{\alpha\beta}}=-p^2a_{\alpha\beta}+fb_{\alpha\beta}.$$

The constant f can be defined by imposing the condition that our field equations must reduce, in the absence of $\varphi_{\alpha\beta}$, to the free field equations of general relativity. Thus, we finally obtain

$$R_{\underline{\alpha\beta}}=-p^2(a_{\alpha\beta}-b_{\alpha\beta}),\tag{2.8}$$

$$R_{\underset{\smile}{\alpha\beta},\gamma}+R_{\underset{\smile}{\beta\gamma},\alpha}+R_{\underset{\smile}{\gamma\alpha},\beta}=-p^2I_{\alpha\beta\gamma},\tag{2.9}$$

$$\mathfrak{g}^{\underset{\smile}{\alpha\beta}}{}_{,\beta}=0.\tag{2.10}$$

In this way we have obtained 18 field equations for 16 field variables plus two trivial identities that follow by differentiation of (2.9) and (2.10). Hence, we have a consistent number of field equations. Note that p^2 is not a cosmological constant. The same field equations can also be obtained from an action principle,

$$\delta\int\mathfrak{L}d^4x=\delta\int[\mathfrak{g}^{\alpha\beta}R_{\alpha\beta}-2p^2\{(-b)^{\frac{1}{2}}-(-g)^{\frac{1}{2}}\}]d^4x=0.\tag{2.11}$$

The term in curly brackets can also be written as

$$\{[-\mathrm{Det}\,\mathfrak{g}^{\alpha\beta}]^{\frac{1}{2}}-[-\mathrm{Det}\,\mathfrak{g}^{\alpha\beta}]^{\frac{1}{2}}\}=(\sqrt{-b})[1-(1-M-L^2)^{\frac{1}{2}}],$$

where

$$M=\tfrac{1}{2}\chi_{\mu\nu}\chi^{\mu\nu},\quad L=\tfrac{1}{4}\chi^{*\mu\nu}\chi_{\mu\nu},$$

$$\chi_{\mu\nu}=\frac{b_{\alpha\mu}b_{\beta\gamma}}{\sqrt{-b}}\mathfrak{g}^{\alpha\beta},\quad \chi^{*\mu\nu}=\frac{1}{\sqrt{-b}}\epsilon^{\mu\nu\alpha\beta}\chi_{\alpha\beta}.$$

In order to impose the conditions (1.14), we add the term $\mathfrak{g}^{\underset{\smile}{\alpha\beta}}B_{\alpha\beta}$ and regard B_α as auxiliary field variable which can be eliminated from the field equations. Lagrangians of Einstein and Schrödinger theories are

$$\mathfrak{L}_E=\mathfrak{g}^{\alpha\beta}R_{\alpha\beta}$$

and

$$\mathfrak{L}'_S=\mathrm{Det}(-R_{\alpha\beta})^{\frac{1}{2}}\ (\text{or } \mathfrak{g}^{\alpha\beta}R_{\alpha\beta}-2\lambda(-g)^{\frac{1}{2}}).$$

The variation of $I=\int\mathfrak{L}d^4x$ consists of adding the independent effects of changing the field components at each point by $\delta_0\mathfrak{g}^{\alpha\beta}$, $\delta_0\Gamma^{\gamma}_{\alpha\beta}$, δ_0B_α and of altering the region of integration by a displacement δx^μ of the points on the boundary surfaces. The first kind of variations lead to the field equations and the second one to the Bianchi-Einstein identities for the nonsymmetric field. The variation of the first term in (2.11) is given in Einstein's theory, and that of the second term can easily be obtained.

3. CONSERVATION LAWS

Now, let us introduce the pseudo quantity

$$\mathfrak{B}=\mathfrak{g}^{\alpha\beta}\mathfrak{B}_{\alpha\beta},\tag{3.1}$$

where

$$\mathfrak{B}_{\alpha\beta}=\Gamma^{\gamma}_{\alpha\beta}\Gamma^{\nu}_{\underline{\gamma\nu}}-\Gamma^{\gamma}_{\alpha\mu}\Gamma^{\mu}_{\gamma\beta},\tag{3.2}$$

[4] E. Schrödinger, Proc. Roy. Irish Acad. LI. **A213** (1948).

which is the nonlinear part of

$$R_{\alpha\beta}=\Gamma^{\gamma}_{\alpha\beta,\gamma}-\Gamma^{\gamma}_{\underline{\alpha\gamma},\beta}+\Gamma^{\gamma}_{\alpha\beta}\Gamma^{\mu}_{\underline{\gamma\mu}}-\Gamma^{\gamma}_{\alpha\mu}\Gamma^{\mu}_{\gamma\beta}=R_{\underline{\alpha\beta}}+iR_{\underset{\smile}{\alpha\beta}}, \quad (3.3)$$

where

$$\Gamma^{\gamma}_{\underline{\alpha\gamma}}=\partial\log(-g)^{\frac{1}{2}}/\partial x^{\alpha}.$$

The variation of (3.1), using $\mathfrak{g}^{\overset{\alpha\beta}{+-}}{}_{;\gamma}=0$, gives

$$\delta\mathfrak{B}=-\mathfrak{B}_{\alpha\beta}\delta\mathfrak{g}^{\alpha\beta}+\mathfrak{B}^{\gamma}_{\alpha\beta}\delta\mathfrak{g}^{\alpha\beta}{}_{,\gamma}, \quad (3.4)$$

where

$$\mathfrak{B}^{\gamma}_{\alpha\beta}=\Gamma^{\gamma}_{\alpha\beta}-\delta^{\gamma}_{\beta}\Gamma^{\nu}_{\underline{\alpha\nu}}. \quad (3.5)$$

The Hamiltonian derivatives of $\mathfrak{B}$ with respect to $\mathfrak{g}^{\alpha\beta}$ results in

$$\frac{\partial\mathfrak{B}}{\partial\mathfrak{g}^{\alpha\beta}}-\frac{\partial}{\partial x^{\gamma}}\frac{\partial\mathfrak{g}\mathfrak{B}}{\partial\mathfrak{g}^{\alpha\beta}{}_{,\gamma}}=-R_{\alpha\beta}. \quad (3.6)$$

Multiplying (3.6) by $\mathfrak{g}^{\alpha\beta}{}_{,\mu}$, we get

$$-\mathfrak{g}^{\alpha\beta}{}_{,\gamma}R_{\alpha\beta}=\frac{\partial}{\partial x^{\mu}}(\delta^{\mu}_{\gamma}\mathfrak{B}-\mathfrak{g}^{\alpha\beta}{}_{,\gamma}\mathfrak{B}^{\mu}_{\alpha\beta}), \quad (3.7)$$

or we can write

$$-\frac{1}{2}\mathfrak{g}^{\underline{\alpha\beta}}{}_{,\gamma}R_{\underline{\alpha\beta}}=-\frac{1}{2}\mathfrak{g}^{\underset{\smile}{\alpha\beta}}{}_{,\gamma}R_{\underset{\smile}{\alpha\beta}}+\frac{1}{2}\frac{\partial}{\partial x_{\mu}}(\delta^{\mu}_{\gamma}\mathfrak{B}-\mathfrak{g}^{\alpha\beta}{}_{,\gamma}\mathfrak{B}^{\mu}_{\alpha\beta}). \quad (3.8)$$

We can cancel the left-hand side of (3.8) by writing the identities (2.2) as

$$\tfrac{1}{2}\mathfrak{g}^{\underline{\alpha\beta}}{}_{,\gamma}R_{\underline{\alpha\beta}}=-\tfrac{1}{2}\mathfrak{g}^{\underset{\smile}{\alpha\beta}}(R_{\underset{\smile}{\alpha\beta,\gamma}}+R_{\underset{\smile}{\beta\gamma,\alpha}}+R_{\underset{\smile}{\gamma\alpha,\beta}})$$
$$-(\mathfrak{g}^{\underline{\alpha\beta}}R_{\underline{\alpha\gamma}}-\tfrac{1}{2}\delta^{\beta}_{\gamma}\mathfrak{g}^{\underline{\mu\nu}}R_{\underline{\mu\nu}})_{,\beta}. \quad (3.9)$$

On adding up (3.8) and (3.9) and using the field equations (1.14), i.e., we are again using the 4-hermiticity conditions $\Gamma_{\alpha}=0$, it follows that

$$\mathfrak{T}^{\beta}_{\gamma,\beta}=0, \quad (3.10)$$

where

$$4\pi p^{2}\mathfrak{T}^{\beta}_{\gamma}=(\mathfrak{g}^{\underline{\alpha\beta}}R_{\underline{\alpha\gamma}}-\tfrac{1}{2}\delta^{\beta}_{\gamma}\mathfrak{g}^{\underline{\mu\nu}}R_{\underline{\mu\nu}})$$
$$-(\mathfrak{g}^{\underset{\smile}{\alpha\beta}}R_{\underset{\smile}{\alpha\gamma}}-\tfrac{1}{2}\delta^{\beta}_{\gamma}\mathfrak{g}^{\underset{\smile}{\mu\nu}}R_{\underset{\smile}{\mu\nu}})+\tfrac{1}{2}(\mathfrak{g}^{\mu\nu}{}_{,\gamma}\mathfrak{B}^{\beta}_{\mu\nu}-\delta^{\beta}_{\gamma}\mathfrak{B}) \quad (3.11)$$

is the pseudo stress-energy-momentum tensor of the total field.

If we set $\varphi_{\alpha\beta}=0$, (3.11) reduces to

$$\mathfrak{a}^{\alpha\beta}G_{\alpha\gamma}-\tfrac{1}{2}\delta^{\beta}_{\gamma}\mathfrak{a}^{\mu\nu}G_{\mu\nu}+\tfrac{1}{2}(\mathfrak{a}^{\mu\nu}{}_{,\gamma}\mathfrak{L}^{\beta}_{\mu\nu}-\delta^{\beta}_{\gamma}\mathfrak{L}), \quad (3.12)$$

where $\mathfrak{L}^{\beta}_{\mu\nu}$ and $\mathfrak{L}$ are the corresponding expressions of $\mathfrak{B}^{\beta}_{\mu\nu}$ and $\mathfrak{B}$ in general relativity. The expression (3.12) is the total energy momentum tensor of general relativity, and it is conserved. In the absence of matter, when the field equations $G_{\alpha\beta}=0$ are used, (3.12) consist merely of the last term representing gravitational field energy density.

A similar situation appears to be the case in Einstein's theory. When the field equations $R_{\alpha\beta}=0$ are used in (3.11) the energy momentum tensor of Einstein's field consists again of the last pseudo term of (3.11), and there is nothing to take the place of the matter in the ordinary sense, and Einstein's field equations are not complete.

There is one important point to be noticed: Because of the existence of the electromagnetic field the last term of (3.11) differs considerably from its counterpart $\mathfrak{L}$ in general relativity; e.g., it cannot be made to vanish at a point in any special coordinate system. Despite this the pseudo term cannot represent the total field energy density.[5]

The above arguments, as far as a nonsymmetric generalization of general relativity is concerned, make it clear beyond any shadow of doubt that the expression (3.11) is the genuine energy momentum tensor of the total field, and its form is most suggestive and provides another argument in favor of the fact that the field equations are neither $R_{\alpha\beta}=0$ nor $R_{\alpha\beta}=\lambda g_{\alpha\beta}$ both of which cause the vanishing of the field energy density.

If in (3.11) we use the field equations (2.8), we get

$$4\pi p^{2}\mathfrak{T}^{\beta}_{\gamma}=-p^{2}[\delta^{\beta}_{\gamma}((-b)^{\frac{1}{2}}-(-g)^{\frac{1}{2}}-\tfrac{1}{2}\mathfrak{g}^{\underset{\smile}{\mu\nu}}\varphi_{\mu\nu})+\mathfrak{g}^{\underset{\smile}{\beta\mu}}\varphi_{\gamma\mu}]$$
$$-(\mathfrak{g}^{\underset{\smile}{\alpha\beta}}R_{\underset{\smile}{\alpha\gamma}}-\tfrac{1}{2}\delta^{\beta}_{\gamma}\mathfrak{g}^{\underset{\smile}{\mu\nu}}R_{\mu\nu})+\tfrac{1}{2}(\mathfrak{g}^{\mu\nu}{}_{,\gamma}\mathfrak{B}^{\beta}_{\mu\nu}-\delta^{\beta}_{\gamma}\mathfrak{B}). \quad (3.13)$$

The expression in square brackets in (3.13), when the cubes and higher orders of φ's are neglected, reduces to

$$-(-a)^{\frac{1}{2}}(\tfrac{1}{4}\delta_{\gamma}{}^{\beta}\varphi_{\mu\nu}\varphi^{\mu\nu}-\varphi^{\beta\mu}\varphi_{\gamma\mu}).$$

The reason for not having a minus sign before $\mathfrak{T}^{\alpha}_{\beta}$ in (3.11) is due to the fact that the tensor $R_{\alpha\beta}$ defined by (3.3) has an opposite sign to that of conventional form of $R_{\alpha\beta}$.

We may also define an energy-momentum 4-vector by writing

$$\mathfrak{P}_{\alpha}=\int_{\sigma}\mathfrak{T}^{\beta}_{\alpha}d\sigma_{\beta}, \quad (3.14)$$

where $\mathfrak{P}_{\alpha}$ is a pseudo vector, and $d\sigma_{\beta}$ is a four-dimensional surface element.

Finally it can be shown that, because of $\Gamma_{\alpha}=0$, the field equations are gauge-invariant.

4. LINEAR APPROXIMATIONS TO THE FIELD EQUATIONS

The two spherically symmetric static solutions of Einstein-Schrödinger theories were obtained by Papapetrou.[6] It would be of great interest if one could obtain

[5] B. Kurşunoğlu, Proc. Phys. Soc. (London) **A65**, 81 (1952).
[6] A. Papapetrou, Proc. Roy. Irish Acad. **A52**, 69 (1948).

the same for the present theory. But this is found to be more difficult in this case. In order to see more explicitly the form of the right-hand sides of the field equations (2.8) we first introduce a constant q (with the dimensions of an electric field strength) and write the physical tensor $g_{\alpha\beta}$ as

$$g_{\alpha\beta}=a_{\alpha\beta}+iq^{-1}\varphi_{\alpha\beta}. \qquad (4.1)$$

The numerical value of q will be calculated in later sections. For the static spherically symmetric case the right-hand sides of (2.8) have the forms

$$-p^2(a_{11}-b_{11})=-p^2a_{11}\left(1-\frac{1}{(1-E^2/q^2)^{\frac{1}{2}}}\right),$$

$$-p^2(a_{22}-b_{22})=-p^2a_{22}(1-(1-E^2/q^2)^{\frac{1}{2}}),$$

$$-p^2(a_{33}-b_{33})=-p^2a_{33}(1-(1-E^2/q^2)^{\frac{1}{2}})\sin^2\theta,$$

$$-p^2(a_{44}-b_{44})=-p^2a_{44}\left(1-\frac{1}{(1-E^2/q^2)^{\frac{1}{2}}}\right).$$

The comparison of these with the kinetic energy $-m_0c^2[1-(1-v^2/c^2)^{-\frac{1}{2}}]$ is most suggestive;[7] while the latter puts a limit to the velocity of light, the former puts a limit to the electric field strength E. Thus for the consistency of the above expressions we have to impose the condition

$$|E|<q \qquad (4.2)$$

upon the electric field E.

Now the symmetric and the antisymmetric parts of $R_{\alpha\beta}$ are

$$R_{\underline{\alpha\beta}}=(\Gamma^{\gamma}_{\underline{\alpha\beta},\gamma}-\Gamma^{\gamma}_{\underline{\alpha\gamma},\beta}+\Gamma^{\gamma}_{\underline{\alpha\beta}}\Gamma^{\nu}_{\underline{\gamma\nu}}-\Gamma^{\gamma}_{\underline{\alpha\nu}}\Gamma^{\nu}_{\underline{\gamma\beta}})+\Gamma^{\gamma}_{\underset{\smile}{\alpha\nu}}\Gamma^{\nu}_{\underset{\smile}{\gamma\beta}}, \qquad (4.3)$$

$$R_{\underset{\smile}{\alpha\beta}}=\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta},\gamma}+\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}}\Gamma^{\gamma}_{\underline{\gamma\nu}}-\Gamma^{\gamma}_{\underset{\smile}{\alpha\mu}}\Gamma^{\mu}_{\underline{\gamma\beta}}-\Gamma^{\gamma}_{\underset{\smile}{\mu\beta}}\Gamma^{\mu}_{\underline{\alpha\gamma}}=\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}o\gamma}. \qquad (4.4)$$

We split up the field variables in the form

$$g_{\alpha\beta}=-\delta_{\alpha\beta}+h_{\alpha\beta}+i\varphi_{\alpha\beta}, \qquad (4.5)$$

where $h_{\alpha\beta}$ and $\varphi_{\alpha\beta}$ represent weak gravitational and electromagnetic fields, respectively, where we use the convention $x_4=ict$.

Let us assume that we can neglect

(1) the squares of $h_{\alpha\beta}$;
(2) the cubes and higher orders of $\varphi_{\alpha\beta}$;
(3) gravitational and electromagnetic interaction terms.

Then, using (1.9) and (1.10), one can write

$$\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}}=\tfrac{1}{2}I_{\alpha\beta\gamma}-\partial_\gamma\varphi_{\alpha\beta}, \qquad (4.6)$$

$$\Gamma^{\gamma}_{\underline{\alpha\beta}}=\tfrac{1}{2}(\partial_\gamma h_{\alpha\beta}-\partial_\beta h_{\alpha\gamma}-\partial_\alpha h_{\beta\gamma})$$
$$+\tfrac{1}{2}(\varphi_{\beta\nu}I_{\alpha\nu\gamma}+\varphi_{\nu\alpha}I_{\beta\gamma\nu})+\varphi_{\beta\nu}\partial_\nu\varphi_{\alpha\gamma}+\varphi_{\nu\alpha}\partial_\nu\varphi_{\gamma\beta}. \qquad (4.7)$$

[7] M. Born and L. Infeld, Proc. Roy. Soc. (London) CXLIV, **A425** (1934).

The remaining field variables are

$$g=-h+(1-\Omega), \qquad (4.8)$$

$$b=1-h=a, \qquad (4.9)$$

$$b_{\alpha\beta}=-\delta_{\alpha\beta}+h_{\alpha\beta}-T'_{\alpha\beta}, \qquad (4.10)$$

where

$$T_{\alpha\beta}=\tfrac{1}{4}\delta_{\alpha\beta}\varphi_{\mu\nu}\varphi_{\mu\nu}-\varphi_{\alpha\mu}\varphi_{\beta\mu}. \qquad (4.11)$$

From (1.11) and (1.12) we can write

$$I_{\alpha\beta\gamma}=\epsilon_{\alpha\beta\gamma\mu}J_\mu, \qquad (4.12)$$

$$\partial_\beta f_{\alpha\beta}=J_\alpha, \qquad (4.13)$$

where

$$f_{\alpha\beta}=-\tfrac{1}{2}\epsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}=\partial_\alpha A_\beta-\partial_\beta A_\alpha, \qquad (4.14)$$

and A_α are the potentials of the electromagnetic field. Using (4.14), $T'_{\alpha\beta}$ becomes

$$T'_{\alpha\beta}=\tfrac{1}{4}\delta_{\alpha\beta}\varphi_{\mu\nu}\varphi_{\mu\nu}-\varphi_{\alpha\mu}\varphi_{\beta\mu}=-(\tfrac{1}{4}\delta_{\alpha\beta}f_{\mu\nu}f_{\mu\nu}-f_{\alpha\mu}f_{\beta\mu})=-T_{\alpha\beta}.$$

With these results the field equations (2.8) reduce to

$$\tfrac{1}{2}\Box h_{\alpha\beta}+\tfrac{1}{2}(\delta_{\alpha\beta}J_\mu J_\mu-J_\alpha J_\beta)+\partial_\mu\varphi_{\beta\nu}\partial_\nu\varphi_{\alpha\mu}$$
$$+\partial_\alpha\partial_\beta(\tfrac{1}{4}f_{\mu\nu}f_{\mu\nu})+\tfrac{1}{2}(f_{\alpha\mu}J_{\beta\mu}+f_{\beta\mu}J_{\alpha\mu}$$
$$-\delta_{\alpha\beta}f_{\mu\nu}J_{\mu\nu})=p^2T_{\alpha\beta}, \qquad (4.15)$$

where

$$J_{\alpha\beta}=\partial_\alpha J_\beta-\partial_\beta J_\alpha,\ \text{and}\ \Box=\partial_\mu\partial_\mu,\ \text{and}\ \partial_\alpha=\partial/\partial x^\alpha, \qquad (4.16)$$

and the coordinate conditions

$$\partial_\mu h_{\alpha\mu}=\tfrac{1}{2}\partial_\alpha h, \quad (h=h_{\mu\mu}) \qquad (4.17)$$

are used. Because of the Bianchi-Einstein identities for the nonsymmetric field the conditions (4.17) are consistent with the field equations.

In the absence of charges we have

$$\tfrac{1}{2}\Box h_{\alpha\beta}+\partial_\mu\varphi_{\beta\nu}\partial_\nu\varphi_{\alpha\mu}+\partial_\alpha\partial_\beta(\tfrac{1}{4}f_{\mu\nu}f_{\mu\nu})=p^2T_{\alpha\beta}. \qquad (4.18)$$

A general solution of the wave equation (4.18) for the gravitational potentials $h_{\alpha\beta}$ can be obtained by expressing $\varphi_{\alpha\beta}$ as a superposition of plain waves by means of Fourier integral representation of $\varphi_{\alpha\beta}$. It follows, by differentiation, from (1.11) that $\varphi_{\alpha\beta}$ satisfy

$$\Box\varphi_{\alpha\beta}=0, \qquad (4.19)$$

which is solved by

$$\tfrac{1}{2}\varphi_{\alpha\beta}(x)=\int[\varphi_{\alpha\beta}(k)+\overset{*}{\varphi}_{\alpha\beta}(k)]$$
$$\times\exp(ik_\mu x_\mu)\delta(k_\mu{}^2)d^4k, \qquad (4.20)$$

where the coefficients $\varphi_{\alpha\beta}$ are undefined, and $\delta(k_\mu{}^2)$ is Dirac's δ-function. The vector k_μ is the wave-number four vector. For the functions $h_{\alpha\beta}$ we write

$$\tfrac{1}{2}h_{\alpha\beta}(x)=\int[h_{\alpha\beta}(k)+\overset{*}{h}_{\alpha\beta}(k)]\exp(ik_\mu x_\mu)d^4k. \qquad (4.21)$$

When these are substituted in (4.18), one obtains the gravitational potentials as functions of Maxwell's radiation field. To these solutions one adds the solutions for free gravitational fields.

We identify $\varphi_{\alpha\beta}$ and its dual $f_{\alpha\beta}$ in accordance with the Maxwell's equations (1.11) and (1.14) as

$$\begin{aligned}(\varphi_{23}, \varphi_{31}, \varphi_{12}; \varphi_{41}, \varphi_{42}, \varphi_{43}) &= (iE_1, iE_2, iE_3; H_1, H_2, H_3),\\ (f_{23}, f_{31}, f_{12}; f_{41}, f_{42}, f_{43}) &= (H_1, H_2, H_3; iE_1, iE_2, iE_3).\end{aligned} \tag{4.22}$$

By using (4.1) and comparing (4.15) with $G_{\alpha\beta}=-2\bar{\gamma}T_{\alpha\beta}$ of general relativity, we get

$$p^2q^{-2}=2\bar{\gamma}/c^4, \tag{4.23}$$

where $\bar{\gamma}$ = gravitational constant.

5. THE STRUCTURE OF THE ELECTRON

The most interesting feature of the unified field theory is its restriction of the charge-current distribution. This is the essential deviation from Maxwell-Lorentz electrodynamics and is expressed by the field equations (2.9). With the above approximation it works out as

$$(\Box-\kappa^2)J_\alpha=0, \tag{5.1}$$

where

$$\kappa^2=2p^2. \tag{5.2}$$

For an electron at rest the charge density, as follows from (5.1), is

$$\rho=-iJ_4=(e\kappa^2/4\pi)e^{-\kappa r}/r,$$

so that

$$\int\rho dV=4\pi\int_0^\infty \rho r^2dr=e. \tag{5.3}$$

For sufficiently large κ the function ρ behaves as a delta-function multiplied by $4\pi/\kappa^2$ regardless of the order of limits ($r\to0$, $\kappa\to\infty$) or otherwise ($\kappa\to\infty$, $r\to0$). It is easy to see that when the origin $r=0$ is included the function ρ is a solution of

$$(\nabla^2-\kappa^2)\rho=-e\kappa^2\delta(\mathbf{r}), \tag{5.4}$$

where

$$\delta(\mathbf{r})=\delta(x)\delta(y)\delta(z).$$

When rationalized units are used Eqs. (4.13) can be written as

$$\Box A_\alpha=-4\pi J_\alpha \quad\text{and}\quad \partial_\mu A_\mu=0,$$

so that in a static spherically symmetric case the potential $\varphi=-iA_4$ is given by

$$\varphi=(e/r)(1-e^{-\kappa r}), \tag{5.5}$$

where we use the fact that, in case $J_\alpha=0$, the electrodynamics is the same as classical theory.

Our charge distribution has the range of nuclear forces (κ will follow in the next section). It may be objected that the charge of an electron is an indivisible unit; therefore a shape factor κ is out of place, but this is no more an unreasable description of the electron than is the point electron model.[8]

The second term $-(e/r)r^{-\kappa r}$ of φ may be interpreted as causing a force holding the electron to itself.

A further approximation to the field equations (2.9) can be obtained by retaining gravitational and interaction terms in the equations. On multiplying both sides of (2.9) by $\epsilon^{\alpha\beta\gamma\rho}$ we obtain

$$-\kappa^2J_\rho=\epsilon^{\alpha\beta\gamma\rho}\Gamma^{\sigma}_{\underset{\smile}{\alpha\beta}|\sigma,\gamma}. \tag{5.6}$$

Hence, using the expressions for the Γ's, after some lengthy calculations we get

$$\begin{aligned}(\Box-\kappa^2)J_\alpha=&-\kappa^2\gamma J_\alpha-\tfrac{1}{2}(\partial_\mu\gamma)J_{\alpha\mu}+(\partial_\gamma J_\mu)\partial_\mu\gamma_{\alpha\gamma}\\ &-(2\partial_\gamma J_\mu)\partial_\alpha\gamma_{\mu\gamma}+(2\partial_\gamma f_{\mu\rho})\partial_\rho\partial_\alpha\gamma_{\gamma\mu}+(\tfrac{1}{2}\partial_\rho\partial_\mu\gamma)\partial_\rho f_{\alpha\mu}\\ &-(\tfrac{1}{2}\partial_\mu\gamma)\partial_\alpha J_\mu-(\partial_\beta\partial_\gamma J_\alpha)\gamma_{\beta\gamma}+4(\partial_\beta\partial_\gamma f_{\alpha\mu})\partial_\beta\gamma_{\mu\gamma}\\ &\times(\partial_\rho f_{\gamma\alpha})\partial_\rho\partial_\gamma\gamma-(\partial_\alpha\partial_\gamma\gamma)J_\gamma+(2\partial_\rho J_\alpha)\partial_\rho\gamma,\end{aligned} \tag{5.7}$$

where

$$\gamma_{\alpha\beta}=h_{\alpha\beta}-\tfrac{1}{2}\delta_{\alpha\beta}h,\quad \gamma=-h,\quad \partial_\beta\gamma_{\alpha\beta}=0.$$

The effect of the terms on the right-hand side of (5.7) may be of significance only for heavy particles or for an assembly of particles. The right-hand side of (5.7) is a 4-vector and it is of course conserved. In general, we are going to assume that the effect of those terms are equivalent to a delta-singularity and replace (5.7) by

$$(\Box-\kappa^2)\mathbf{J}_\alpha=-\kappa^2 j_\alpha, \tag{5.8}$$

where

$$\begin{aligned}j_\alpha(x)&=\int_{-\infty}^{s} e\delta(x-\xi)V_\alpha(\xi)ds,\\ \delta(x)&=\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4),\end{aligned} \tag{5.9}$$

and ξ_α represent the center of the electron and V_α is a four-vector of the second order.

The 4-vector $\mathbf{j}_\alpha$ acts as the source of the 4-current J_α. This may also be regarded as a boundary condition imposed on the current density. We infer from (5.8) that the charge distribution of the electron has a range κ^{-1}.

The electrostatic field due to an electron at rest is

$$E=-d\varphi/dr=(e/r^2)[1-e^{-\kappa r}-\kappa re^{-\kappa r}], \tag{5.10}$$

from which the field at $r=0$ comes out as

$$E(0)=e\kappa^2/2, \tag{5.11}$$

and the potential has its maximum at $r=0$ as

$$\varphi(0)=e\kappa. \tag{5.12}$$

At $r=0$, φ and E have a finite discontinuity.

[8] A point electron may be regarded as an infinitely compressed form of the actual (extended) electron, and for this process of localization into a point an infinite amount of energy has to be used. This may also be seen quite easily by expanding the delta-function in terms of the eigenfunctions of a suitable energy operator. I , of course, leads to the so-called infinite self-energy of the electron in classical and quantum field theories.

Unified field theory describes the charge density of an elementary particle as a short range field. It is not possible to measure the effects of an electron "radius" κ^{-1} by having two electrons collide with an energy of the order mc^2. Quantum theoretically the wavelength corresponding to this energy is $\hbar/mc$, which is much larger than κ^{-1}. Thus it is impossible to locate the electron at that energy better than within $\hbar/mc$. During a collision their average distance will never be within a distance comparable with κ^{-1}.

A solution of (5.8) can be obtained by means of Fourier integral representation as

$$J_\alpha^{\text{ret}}(x)=\kappa^2\int\bar{\Delta}^{\text{ret}}(x-x')j_\alpha(x')d^4x', \tag{5.13}$$

where

$$\bar{\Delta}^{\text{ret}}(x)=\bar{\Delta}(x)-\tfrac{1}{2}\Delta(x)$$

are Lorentz invariant scalar functions,[9] and they vanish outside the light cone and are singular on the light cone so that the solutions (5.13) are causally correct. The 4-current $J_\alpha{}^{\text{ret}}$ given by (5.13) is conserved.

It can easily be seen, by writing (5.13) in terms of Fourier components, that

$$\int_{-\infty}^{\infty}J_4(x)dt=\frac{e\kappa^2}{4\pi}\frac{e^{-\kappa r}}{r}.$$

The equations,

$$\Box A_\alpha=-4\pi J_\alpha,\quad (\Box-\kappa^2)J_\alpha=-\kappa^2 j_\alpha, \tag{5.14}$$

in quantum electrodynamics, imply a two-particle picture (photon+meson). Their combined form, i.e., a fourth-order equation in A_α, have been discussed extensively,[10] and it is found that a fourth-order partial differential equation is not, because of occurrence of negative probabilities, consistent with the physical reality. In the present case this objection does not arise, since Eqs. (5.14) can be replaced by

$$\Box A_\alpha=-4\pi\kappa^2\int\bar{\Delta}^{\text{ret}}(x-x')j_\alpha(x')d^4x', \tag{5.15}$$

which means that electromagnetic field can be split up as a "short-range" and "long-range" parts. Thus one can regard a neutral vector meson field as part of the electromagnetic field. Equations (5.15) are solved by

$$A_\alpha{}^{\text{ret}}(x)=4\pi\kappa^2\int D^{\text{ret}}(x-x')\bar{\Delta}^{\text{ret}}(x'-x'')\times j_\alpha(x'')d^4x'd^4x'', \tag{5.16}$$

where $D^{\text{ret}}(x)$ is obtained by putting $\kappa=0$ in the expression of $\bar{\Delta}^{\text{ret}}(x)$. In actual case there was no need for the definition of the current density J_α given by (1.11).

[9] J. Schwinger, Phys. Rev. **75**, 677 (1949).
[10] A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950).

Equations (2.9) approximate to

$$(\Box-\kappa^2)\partial_\mu f_{\alpha\mu}=-\kappa^2 j_\alpha, \tag{5.17}$$

so that the second set of Maxwell's equations are to be obtained by solving (5.17). In this sense, we can say that the sources of the electromagnetic field are contained in our field equations.

In the present theory, if one attempted to construct an S-matrix of quantum electrodynamics in interaction representation, then the interaction Hamiltonian would be $\mathfrak{H}=-(1/c)J_\mu{}^{\text{ret}}A_\mu$ instead of $(1/c)j_\mu A_\mu$. The former contains the invariant function $\bar{\Delta}^{\text{ret}}(x-x')$. Strictly speaking the S-matrix,

$$P\exp\left[-\frac{i}{\hbar c}\int_{-\infty}^{\infty}\mathfrak{H}(x)d^4x\right],$$

when expanded will have, with each term, associated various powers of $\bar{\Delta}^{\text{ret}}(x)$ so that one expects the results for any physical process to be convergent. It, of course, is not possible to say without going into details, that this proposition will be free of objections.

6. THE NATURE OF THE ELECTROMAGNETIC MASS

The energy-momentum tensor (3.13), when the approximation procedure of Sec. 4 is used, can easily be put into the form

$$\begin{aligned}4\pi p^2\mathfrak{T}_{\alpha\beta}=&-p^2T_{\alpha\beta}+\tfrac{1}{2}f_{\beta\mu}J_{\alpha\mu}+\tfrac{1}{4}\delta_{\alpha\beta}J_\mu J_\mu\\&-\tfrac{1}{8}\delta_{\alpha\beta}\Box(f_{\mu\nu}f_{\mu\nu})+\tfrac{1}{2}J_\mu\partial_\beta f_{\alpha\mu}\\&+\tfrac{1}{2}\partial_\alpha f_{\mu\nu}\partial_\beta f_{\mu\nu}-\tfrac{1}{4}\delta_{\alpha\beta}J_{\mu\nu}f_{\mu\nu}.\end{aligned} \tag{6.1}$$

The tensor $\mathfrak{T}_{\alpha\beta}$ is nonsymmetric; it can be symmetrized by using a standard method[11] so as to secure the conservation of the angular momenta, but for the following purpose its symmetrization is only of an academic interest and, therefore, no attempt will be made for it.

For the static spherically symmetric case, in spherically polar coordinates diagonal components of $\mathfrak{T}_{\alpha\beta}$ are given by

$$\begin{aligned}4\pi p^2\mathfrak{T}_{11}=&\tfrac{1}{2}p^2E^2-4\pi^2\rho^2+\tfrac{1}{4}\nabla^2E^2\\&-2\pi E\frac{d\rho}{dr}+\frac{1}{4}\frac{1}{r}\frac{dE^2}{dr}+\frac{1}{2}\sin^2\theta\cos^2\varphi\\&\times\left[\frac{1}{2r}\frac{dE^2}{dr}-\left(\frac{dE}{dr}\right)^2+4\pi E\frac{d\rho}{dr}-\kappa^2E^2\right],\\4\pi p^2\mathfrak{T}_{22}=&\tfrac{1}{2}p^2E^2-4\pi^2\rho^2+\tfrac{1}{4}\nabla^2E^2\\&-2\pi E\frac{d\rho}{dr}+\frac{1}{4}\frac{1}{r}\frac{dE^2}{dr}+\tfrac{1}{2}\sin^2\theta\sin^2\varphi\\&\times\left[\frac{1}{2r}\frac{dE^2}{dr}-\left(\frac{dE}{dr}\right)^2+4\pi E\frac{d\rho}{dr}-\kappa^2E^2\right],\end{aligned} \tag{6.2}$$

[11] G. Wentzel, *Quantum Theory of Fields* (Interscience Publications, New York, 1949).

$$4\pi p^2\mathfrak{T}_{33}=\tfrac{1}{2}p^2E^2-4\pi^2\rho^2+\tfrac{1}{4}\nabla^2E^2$$
$$-2\pi E\frac{d\rho}{dr}+\frac{1}{4}\frac{1}{r}\frac{dE^2}{dr}+\tfrac{1}{2}\cos^2\theta$$
$$\times\left[\frac{1}{2r}\frac{dE^2}{dr}-\left(\frac{dE}{dr}\right)^2+4\pi E\frac{d\rho}{dr}-\kappa^2E^2\right],$$
$$4\pi p^2\mathfrak{T}_{44}=-\tfrac{1}{2}p^2E^2-4\pi^2\rho^2+\tfrac{1}{4}\nabla^2E^2, \tag{6.3}$$

where the electrostatic field E is given by

$$-if_{4s}=E_s=(x_s/r)\,|E|,\quad (s=1,2,3) \tag{6.4}$$

and $|E^2|=E_1^2+E_2^2+E_3^2$ is a function of r only.

By integrating $\mathfrak{T}_{44}$ through the whole of space we can calculate the constant κ in terms of the mass and charge of the electron. The following integrals will be used throughout the following discussions:

$$\int E^2dV=4\pi\int_0^\infty\frac{d\varphi}{dr}\frac{d\varphi}{dr}r^2dr=2\pi e^2\kappa,$$
$$\int\rho^2dV=4\pi\int^\infty r^2\left(\frac{e\kappa^2}{4\pi}\frac{\bar{e}^{\kappa r}}{r}\right)^2dr=\frac{1}{8\pi}e^2\kappa^3, \tag{6.5}$$
$$\int\nabla^2E^2dV=4\pi\left[r^2\frac{dE^2}{dr}\right]_0^\infty=0,\quad\int E\frac{d\rho}{dr}dV=-\tfrac{1}{2}e^2\kappa^3.$$

Now

$$-\mathfrak{T}_{44}=\text{energy density}=\sigma c^2,$$

where σ = mass density, so that

$$-4\pi p^2\int\mathfrak{T}_{44}dV=\tfrac{1}{2}p^2\int E^2dV+4\pi^2\int\rho^2dV.$$

Hence,

$$m_0c^2=\tfrac{1}{2}\kappa e^2,$$

and

$$\kappa=2m_0c^2/e^2. \tag{6.6}$$

Note that each term in the above contributes half of the rest mass of the electron. The appearance of the factor 2 in (6.6) is a most important feature of the entire theory. This we shall explain a little later. The constant p follows as

$$p=(1/\sqrt{2})\kappa=\sqrt{2}m_0c^2/e^2.$$

The constant q, using (4.23), is

$$q=m_0c^4/e^2\sqrt{\bar{\gamma}}=1.2\times10^{37}\text{ esu.}^{11a} \tag{6.7}$$

Thus the inequality (4.2) finds its most convincing explanation. It follows also from (5.11) that

$$E^2(0)/q^2=4\cdot\bar{\gamma}m_0^2/e^2$$
$$=4\cdot\text{gravitational force/electrostatic force.}$$

[11a] From (6.7), (5.11), and (4.2) it follows that the rest mass of any elementary charged particle satisfies the inequality:

$$m_0<\tfrac{1}{2}|e|/\sqrt{\bar{\gamma}}\approx10^{-6}\text{ g.}$$

For the radial and transverse stress components we have

$$\int\mathfrak{T}_{11}dV=\int\mathfrak{T}_{22}dV=\int\mathfrak{T}_{33}dV=0, \tag{6.8}$$

which is the so-called Laue theorem. Thus, our electron is a stable structure.

The integrals of the rest of $\mathfrak{T}_{\alpha\beta}$ give

$$\int\mathfrak{T}_{mn}dV=0,\quad\int\mathfrak{T}_{4m}dV=\int\mathfrak{T}_{m4}dV=0,\ m\neq n.$$

Now, a few words on the electromagnetic mass are necessary. The potential $\varphi(r)$ reaches its maximum at $r=0$, and it is $\varphi(0)=e\kappa=2m_0c^2/e$. Since the entire mass is contributed by an electromagnetic field with a "short-range" and a "long-range" parts, then, from a classical point of view in bringing a positron and electron charge together all external fields are canceled out, and an energy of $2m_0c^2$ is released which is the rest mass of these particles in virtue of their fields. This may also mean that the rest mass of the two particles is equivalent to the work done in separating them against their mutual attraction after they "are created."[12] Similar arguments may be applied for the origin of the neutral matter.

7. EQUATIONS OF MOTION

It has been shown[13] that in the presence of an electromagnetic field the equations $G_{\alpha\beta}=-2\bar{\gamma}T_{\alpha\beta}$ gave a correct law of motion for a charged particle. It has also been shown[14] that Einstein's theory gives no interaction between a charged particle and electromagnetic field: a result that could be deduced immediately from our discussion on the energy-momentum tensor of the field.

In the present theory a particle is to be represented as a concentration of the field energy density into a very small space-time region where, contrary to the assumption of a singularity, the laws of field are known and the magnitude of the field strength can be expressed in terms of finite but large numbers. Thus the application of the methods of general relativity to the present case is not suitable, and it has to be modified. The surface integral conditions[15] of general relativity are empty in this case.

It is quite easy to show from the assumption that mass is entirely of an electromagnetic origin and from the field equations that the equations of motion of an electron in an external field have the correct form.

Now let

$$f_{\alpha\beta}=\overset{(s)}{f}_{\alpha\beta}+\overset{(e)}{f}_{\alpha\beta}, \tag{7.1}$$

[12] R. P. Feynman, Phys. Rev. **74**, 939 (1948).
[13] L. Infeld and P. R. Wallace, Phys. Rev. **57**, 797 (1940).
[14] L. Infeld, Acta Polonica **X**, 284 (1951).
[15] A. Einstein and L. Infeld, Can. J. Math. **1** (1949).

where

$$\overset{(s)}{f}_{\alpha\beta}=\text{proper field of the electron},$$

$$\overset{(e)}{f}_{\alpha\beta}=\text{external field}.$$

We assume that the acceleration of the particle is not too large, so that we can make use of a Lorentz transformation.

When the field equations (1.8) are granted, then, as in general relativity, the field equations can also be obtained from an action principle[4]

$$\delta\int \mathfrak{H}d^4x=0,$$

where

$$-8\pi p^2\mathfrak{H}=\mathfrak{B}+2p^2(-b)^{\frac{1}{2}}-(-g)^{\frac{1}{2}}. \tag{7.2}$$

In this case the field variables $\mathfrak{g}^{\underset{\vee}{\alpha\beta}}=\mathfrak{g}^{\alpha\beta\gamma}{}_{,\gamma}$ and $\mathfrak{g}^{\underline{\alpha\beta}}$ are to be varied independently. The complete antisymmetric tensor density $\mathfrak{g}^{\alpha\beta\gamma}=\epsilon^{\alpha\beta\gamma\delta}A_\delta$ is the dual of A_α. With the approximation of Sec. 4 we have

$$\mathfrak{B}=\tfrac{1}{2}J_\mu J_\mu-(\varphi_{\rho\mu}\varphi_{\gamma\rho})_{,\gamma\mu},$$

the last term of which can be dropped from the action principle, and we get

$$-4\pi p^2\int\mathfrak{H}d^4x=\delta\int(\tfrac{1}{4}p^2f_{\mu\nu}f_{\mu\nu}+\tfrac{1}{4}J_\mu J_\mu)d^4x. \tag{7.3}$$

Now, using (7.1) we obtain

$$-4\pi\int\mathfrak{H}d^4x=\delta\int[\tfrac{1}{4}\overset{(s)}{f}_{\mu\nu}\overset{(s)}{f}_{\mu\nu}+\tfrac{1}{4}p^{-2}J_\mu J_\mu]d^4x$$
$$+\tfrac{1}{2}\delta\int\overset{(s)}{f}_{\mu\nu}\overset{(e)}{f}_{\mu\nu}d^4x+\tfrac{1}{4}\delta\int\overset{(e)}{f}_{\mu\nu}\overset{(e)}{f}_{\mu\nu}d^4x. \tag{7.4}$$

We define the quantity $\mathfrak{L}$ by

$$4\pi\mathfrak{L}=\int\overset{(s)}{\mathfrak{H}}d^3x+\int\tfrac{1}{2}\overset{(s)}{f}_{\mu\nu}\overset{(e)}{f}_{\mu\nu}d^3x. \tag{7.5}$$

Hence,

$$4\pi\delta\int\mathfrak{H}d^4x=4\pi\delta\int\mathfrak{L}dt,$$

where

$$\overset{(s)}{\mathfrak{H}}=\tfrac{1}{4}\overset{(s)}{f}_{\mu\nu}\overset{(s)}{f}_{\mu\nu}+\tfrac{1}{4}p^{-2}J_\mu J_\mu.$$

The last term in (7.4), because of $\partial_\nu f^{(e)}{}_{\mu\nu}=0$, drops out, also in the rest frame of the electron we have (writing $4\pi J_\alpha$ for J_α)

$$-4\pi\int\overset{(s)}{\mathfrak{H}}d^3x=\tfrac{1}{2}\int E^2dV+4\pi^2p^{-2}\int\rho^2dV=4\pi m_0c^2,$$

so that we can write

$$-\int\overset{(s)}{\mathfrak{H}}d^3x=m_0c^2(1-v^2/c^2)^{\frac{1}{2}},$$

where v = velocity of the center of the electron.

From these results it follows that (7.4) is

$$\delta\int m_0c^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}dt+\tfrac{1}{2}\delta\int\overset{(e)}{f}_{\mu\nu}\overset{(s)}{f}_{\mu\nu}d^4x=0. \tag{7.6}$$

Now, we have

$$\overset{(e)}{f}_{\mu\nu}=\partial_\mu\overset{(e)}{A}_\nu-\partial_\nu\overset{(e)}{A}_\mu,\quad \partial_\nu\overset{(s)}{f}_{\mu\nu}=4\pi J_\mu.$$

Using these equations in (7.6) and integrating by parts and dropping the divergence terms we get

$$\mathfrak{L}=m_0c^2(1-v^2/c^2)^{\frac{1}{2}}-\int\overset{(e)}{A}_\mu J_\mu d^3x. \tag{7.7}$$

Thus we have obtained the action function of Maxwell-Lorentz theory. An electron behaves as a mechanical system with the rest mass m_0 acted on by the external field $\overset{(e)}{f}_{\alpha\beta}$.

If we assume that the electron is moving in a constant external field $\overset{(e)}{A}_\mu$, then we obtain

$$\int\mathfrak{L}dt=\int m_0c^2(1-v^2/c^2)^{\frac{1}{2}}dt-e\int A_\mu V_\mu dtd^3x.$$

Thus in a slowly varying field the generally covariant force law can be stated as

$$\frac{dV^\alpha}{d\tau}+\left\{\begin{matrix}\alpha\\ \beta\gamma\end{matrix}\right\}_b V^\beta V^\gamma=\frac{e}{m_0c^2}\Psi^{\alpha\mu}V_\mu, \tag{7.8}$$

where $V_\mu=dx_\mu/d\tau$,

$$\Psi_{\alpha\beta}=-\tfrac{1}{2}\epsilon_{\alpha\beta\mu\nu}\mathfrak{g}^{\underset{\vee}{\mu\nu}}=(f_{\alpha\beta}-\Lambda\varphi_{\alpha\beta})/(1-\Omega-\Lambda^2)^{\frac{1}{2}}, \tag{7.9}$$

and

$$d\tau^2=b_{\alpha\beta}dx^\alpha dx^\beta. \tag{7.10}$$

We must mention the fact that these results could also be deduced, without making the above approximations, merely from the fact that the expression (7.2) is already contained in $\mathfrak{T}_4{}^4$ and contributes the total rest mass m_0 of the electron. In (7.8) $\Psi_{\alpha\beta}$ refers to the actual field, i.e., the external field plus the field produced by the particles themselves.

8. DEFLECTION OF LIGHT

For a static spherically symmetric field, Eqs. (4.15), in spherical polar coordinates, give

$$\nabla^2\tfrac{1}{2}h_{44}=\tfrac{1}{2}p^2q^{-2}E^2, \tag{8.1}$$

$$\nabla^2\tfrac{1}{2}h_{11}=-\tfrac{1}{2}p^2q^{-2}E^2+4\pi q^{-2}E\frac{d\rho}{dr}+8\pi^2\rho^2q^{-2}$$
$$-\frac{1}{2}q^{-2}\frac{1}{r}\frac{dE^2}{dr}+q^{-2}\sin^2\theta\cos^2\varphi$$
$$\times\left[\left(\frac{dE}{dr}\right)^2-\frac{1}{2}\frac{1}{r}\frac{dE^2}{dr}+p^2E^2\right],$$

$$\nabla^2 \tfrac{1}{2} h_{22} = -\tfrac{1}{2} p^2 q^{-2} E^2 + 4\pi q^{-2} E \frac{d\rho}{dr} + 8\pi^2 \rho^2 q^{-2} - \frac{1}{2} q^{-2} \frac{1}{r} \frac{dE^2}{dr} + q^{-2} \sin^2\theta \sin^2\varphi \times \left[\left(\frac{dE}{dr} \right)^2 - \frac{1}{2} \frac{1}{r} \frac{dE^2}{dr} + p^2 E^2 \right], \quad (8.2)$$

$$\nabla^2 \tfrac{1}{2} h_{33} = -\tfrac{1}{2} p^2 q^{-2} E^2 + 4\pi q^{-2} E \frac{d\rho}{dr} + 8\pi^2 \rho^2 q^{-2} - \frac{1}{2} q^{-2} \frac{1}{r} \frac{dE^2}{dr} + q^{-2} \cos^2\theta \times \left[\left(\frac{dE}{dr} \right)^2 - \frac{1}{2} \frac{1}{r} \frac{dE^2}{dr} + p^2 E^2 \right].$$

All other components vanish because of spherical symmetry.

Now if we consider the gravitational field of an electron at rest the above expressions, when integrated through the whole of space, give

$$\int -\nabla^2 h_{11} dV = \int -\nabla^2 h_{22} dV = \int -\nabla^2 h_{33} dV = \int \nabla^2 h_{44} dV = 2\pi e^2 \kappa p^2 q^{-2} = \frac{8\pi\bar{\gamma}}{c^2} m_0. \quad (8.3)$$

The contributions of the terms on the right-hand side of the equations (8.2) to the mass of the electron, except the first term, vanish.[16]

From (8.3) it follows that for a macroscopic distribution of mass density Eqs. (8.1)–(8.2) can be written as

$$\nabla^2 h_{44} = \kappa_0 \sigma, \quad (8.4)$$

$$\nabla^2 h_{11} = -\kappa_0 \sigma + \kappa_0 \sigma_1 + \kappa_0 \sigma_2 \sin^2\theta \cos^2\varphi, \quad (8.5)$$

$$\nabla^2 h_{22} = -\kappa_0 \sigma + k_0 \sigma_1 + \kappa_0 \sigma_2 \sin^2\theta \sin^2\varphi, \quad (8.6)$$

$$\nabla^2 h_{33} = -\kappa_0 \sigma + \kappa_0 \sigma_1 + \kappa_0 \sigma_2 \cos^2\theta, \quad (8.7)$$

where

$$\sigma = (1/4\pi c^2) \textstyle\sum_n E_n^2 = \text{density of mass per unit volume}, \quad (8.8)$$

and

$$\kappa_0 = 8\pi\bar{\gamma}/c^2.$$

The densities σ_1 and σ_2 represent the rest of the terms on the right-hand sides of the equations (8.2). Note that because of (8.3), we have

$$\int (\sigma_1 + \sigma_2 \sin^2\theta \cos^2\varphi) dV = \int (\sigma_1 + \sigma_2 \sin^2\theta \sin^2\varphi) dV = \int (\sigma_1 + \sigma_2 \cos^2\theta) dV = 0. \quad (8.9)$$

In (8.8) E_n can be replaced by its maximum value $\frac{1}{2} e_n \kappa^2$ where κ can take two values κ_e and κ_p for electron and proton, respectively, so that

$$E_n = 2e_n (m_{0n} c^2 / e_n^2)^2,$$

and

$$E_n^2 = 4 m_{0n} c^2 (m_{0n} c^2 / e_n^2)^3.$$[17]

If we put

$$V_0 = \pi (e^2 / m_0 c^2)^3 = \text{“volume of the particle,”}$$

then, we get

$$\sigma = \textstyle\sum_n (m_{0n} / V_n).$$

With the arguments given at the end of Sec. 6 we are now in a position to regard the system (8.4)–(8.7) as representing the equations of a gravitational field produced by the density σ of matter. The solutions of (8.4)–(8.7) can be written down at once, as

$$h_{44} = -\frac{\kappa_0}{4\pi} \int \frac{\sigma(r') dV'}{|r - r'|}, \quad (8.10)$$

$$h_{11} = h_{22} = h_{33} = \frac{\kappa_0}{4\pi} \int \frac{\sigma(r') dV'}{|r - r'|} - \frac{\kappa_0}{4\pi} \int \frac{\sigma_1(r')}{|r - r'|} dV' - \frac{\kappa_0}{4\pi} \frac{1}{3} \int \frac{\sigma_2(r') dV'}{|r - r'|}. \quad (8.11)$$

Hence, because of the second and third terms on the right-hand side of (8.11), the potentials h_{11}, h_{22}, h_{33} are different from that of general relativity. But since our main interest is, as will be seen in the following, in the partial derivatives of h_{11}, h_{22}, h_{33}, h_{44} with respect to the coordinate x and integration of the results between the limits $(-\infty, \infty)$, the contribution of the second and third terms on the right-hand side of (8.11) will, because of (8.9), vanish, *viz.*,

$$\int_{-\infty}^{\infty} \frac{\partial h_{11}}{\partial x} dz = \frac{\kappa_0}{4\pi} \int \int_{-\infty}^{\infty} \sigma(r') dV' \frac{\partial}{\partial x} \left(\frac{1}{|r - r'|} \right) dz - \frac{\kappa_0}{4\pi} \int [\sigma_1(r') + \tfrac{1}{3} \sigma_2(r')] dV' \times \int_{-\infty}^{\infty} \frac{\partial}{\partial x} \left(\frac{1}{|r - r'|} \right) dz = -\frac{\kappa_0}{2\pi} M R^{-1},$$

where

$$M = \int \sigma(r') dV' = \text{total mass}.$$

[16] It is clear that the value of κ given by (6.6) enables us to regard m_0, calculated as an electromagnetic mass, also as the gravitational mass of a particle. The same constant m_0 in Sec. 7 behaves as a mechanical mass. Thus electromagnetic mass = inertial mass = gravitational mass, which is the statement of the "principle of equivalence." The constant κ could also be calculated from (8.3).

[17] This gives a density of matter of the order 10^{13} g cm^{-3}, which is similar to the liquid drop model of the nucleus.

Now, if we neglect the term $q^{-2}T_{\alpha\beta}dx^\alpha dx^\beta$ in the expression of the metric of the space-time, we can write

$$d\tau^2=-\left(1+\frac{\kappa_0}{4\pi}\int\frac{\sigma(r')}{|r-r'|}dV'\right)(dx^2+dy^2+dz^2)+c^2\left(1-\frac{\kappa_0}{4\pi}\int\frac{\sigma(r')dV'}{|r-r'|}\right)dt^2. \quad (8.12)$$

This result is the same as the one obtained by Einstein in general relativity. The required deflection of light passing near a strong gravitational field follows from

$$d\tau^2=0.$$

Hence, if L is the velocity of light, we have approximately

$$L=c\left(1-\frac{\kappa_0}{4\pi}\int\frac{\sigma(r')}{|r-r'|}dV'\right), \quad (8.13)$$

so that the deflection of light is

$$\alpha=\int_{-\infty}^{\infty}\frac{1}{L}\frac{\partial L}{\partial x}dz=\frac{\kappa_0 M}{2\pi R}=1.75', \quad (8.14)$$

in complete agreement with the result of general relativity.[18]

The author is not aware of any other unified field theory comprising the results obtained in the foregoing. We think that these implications of the theory are important enough to warrant our confidence in its validity as a correct physical theory.

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[18] It can easily be seen that for $\kappa=0$ the above theory reduces to Einstein's version of the unified field theory.

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Mobility in High Electric Fields

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An extension of conductivity theory to high fields, subject to the usual simplifying assumptions, is carried out for the cases in which the change of energy of an electron in a collision can be neglected. This yields a relationship between mobility and relaxation time which is valid over a wide range of fields.

THE formal theory of conduction can be extended simply to cover the case of high fields when the scattering processes which give rise to the resistance are to a good approximation elastic. This condition is satisfied in monatomic solids for the range of temperature and field strength in which collisions are mainly with acoustical modes of lattice vibrations and impurities. Experiments indicate that in germanium at room temperature, for example, this is the case up to about 4000 volts per cm.[1] This condition is also satisfied for electrons in a gas when the collisions are nonionizing ones with atoms or ions.

The distribution function for the electrons in an electric field will be denoted by $f(\mathbf{k}, \mathbf{E})$, where $\mathbf{k}$ is the wave vector and $\mathbf{E}$ is the electric field intensity. If we neglect crystal anisotropy, since the scattering is elastic the distribution function in the presence of the field will be nearly isotropic in k space. It can be shown that it is a good approximation to take

$$f(\mathbf{k}, \mathbf{E})=f_0(k, E)-f_1(k, E)\cos\theta, \quad (1)$$

where θ is the angle between $\mathbf{k}$ and the field direction, chosen as the z axis, and f_1 is much smaller than f_0. In low fields, of course, f_0 will be the zero field equilibrium distribution. In the steady state, the rate of change of f due to the field must be balanced by the rate of change due to collisions, or

$$(eE/\hbar)(\partial f/\partial k_z)+(\partial f/\partial t)_c=0. \quad (2)$$

It has been shown that probabilities of scattering by lattice vibrations or imperfections are independent of electric field intensity up to fields of the order of 6×10^5 volts per cm.[2] In the approximation that the scattering is elastic, transitions will take place to states on the constant energy surface. Let the probability of transition per unit time from a state near $\mathbf{k}$ to one of a group of states in area dS' of the constant energy surface be denoted by $P(\mathbf{k}, \mathbf{k}')dS'$. Then

$$(\partial f/\partial t)_c=-\int_{S'}[f(\mathbf{k}, \mathbf{E})P(\mathbf{k}, \mathbf{k}')-f(\mathbf{k}', \mathbf{E})P(\mathbf{k}', \mathbf{k})]dS'. \quad (3)$$

To carry this further it is necessary to assume that $P(\mathbf{k}, \mathbf{k}')=P(\mathbf{k}', \mathbf{k})$ and depends only on the angle between $\mathbf{k}$ and $\mathbf{k}'$. Since collisions can only redistribute electrons around the constant energy surface, f_0 does not contribute to this term. The rate of change of f_1 may be found by considering an element of phase

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[2] J. Bardeen and W. Shockley, Phys. Rev. 80, 69 (1950).