Coulomb Effects in Pion Scattering by Protons under the Charge Independence Hypothesis*

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The differential cross sections for scattering of charged π-mesons by protons, observed so far at energies and angles for which Coulomb effects are negligible, were found compatible with the assumption of charge independence for the nuclear interaction between pion and nucleon. For the future discussion of scattering experiments now being performed at lower energies, the Coulomb effects are calculated, retaining the charge independence hypothesis for the nuclear part of the interaction and describing it by the phase shifts used in the higher energy region. For 35-Mev pions, an estimate based on the known magnitude of the total π^+-p cross section shows the Coulomb effects to reach 50 percent for a scattering angle of the order of 50° in the center-of-mass system.

ALL the experimental data obtained so far on the scattering of π-mesons by protons were found compatible with the assumption of charge independence of the (nonelectromagnetic) pion-nucleon interaction, and could be analyzed in terms of six phase shifts corresponding to the states of total isotopic spins $T=\frac{1}{2}, \frac{3}{2}$ and angular momenta $s_{\frac{1}{2}}, p_{\frac{1}{2}}, p_{\frac{3}{2}}$.[1] These data refer to pion energies of the order of 100 Mev, for which the Coulomb forces between proton and charged pion have negligible effects at the observed scattering angles.

Since scattering experiments are now being performed at Rochester with pion energies of the order of 35 Mev, for which Coulomb effects are likely to be observable, these effects have been investigated, retaining the charge independence hypothesis for the nuclear part of the interaction and trying to describe it by the same six phase shifts mentioned above.

This description is possible under the reasonable assumption that the Coulomb energy may be neglected for any proton-pion separation r smaller than the range r_0 of the nuclear interaction (r_0 is of the order of the Compton wavelength of the pion). The wave function in the center-of-mass system, which is a Coulomb wave function for $r \geqq r_0$, has then to be continuously matched at $r=r_0$ to the wave function determined by the nuclear interaction for $r \leqq r_0$. At $r=r_0$, the latter wave function is completely expressible in terms of the phase shifts which would be produced by the nuclear interaction in the absence of Coulomb forces, i.e., the exact analogs of the phase shifts used in the analysis of the higher energy scattering experiments. We shall use six such phase shifts, corresponding to the isotopic spin and angular momentum states mentioned above, so that the matching of Coulomb and nuclear wave functions has to be performed for s- and p-waves only, whereas for higher l-values the nuclear interaction will be neglected and regular Coulomb wave functions can be used.

For each of the three types of scattering,

$$\pi^+ + p \to \pi^+ + p, \quad \pi^- + p \to \pi^- + p, \quad \pi^- + p \to \pi^0 + n,$$

the differential cross sections per steradian in the center-of-mass system, for scattering without and with nucleon spin flip, respectively, are found to have the form

$$\frac{d\sigma^{(nf)}}{d\Omega} = \frac{1}{4k^2}\left| -\frac{i\epsilon\alpha}{\sin^2(\theta/2)} \exp\left(-i\epsilon\alpha \log \sin^2\frac{\theta}{2}\right) + P + Q\cos\theta \right|^2, \quad (1)$$

$$\frac{d\sigma^{(f)}}{d\Omega} = \frac{1}{4k^2}|R|^2 \sin^2\theta. \quad (2)$$

θ is the scattering angle and $\hbar k$ the momentum in the center-of-mass system; $\alpha = me^2/\hbar^2 k$, with e the charge and m the reduced mass of pion and nucleon; ϵ is $+1$ for $\pi^+ + p \to \pi^+ + p$ scattering, -1 for $\pi^- + p \to \pi^- + p$ and 0 for $\pi^- + p \to \pi^0 + n$. P, Q, R are rather complicated expressions involving α and the six phase shifts which would be produced by the nuclear interaction in the absence of Coulomb forces. They are obtained as follows. Denote the latter phase shifts, for $T=\frac{1}{2}, \frac{3}{2}$, respectively, by η_1, η_3 for the $s_{\frac{1}{2}}$ wave, by η_{11}, η_{31} for the $p_{\frac{1}{2}}$-wave and by η_{13}, η_{33} for the $p_{\frac{3}{2}}$ wave. Consider the radial Schrödinger equation,

$$\frac{d^2u}{d\rho^2} + \left(1 - \frac{l(l+1)}{\rho^2} - \frac{2\epsilon\alpha}{\rho}\right)u = 0, \quad (\epsilon = \pm 1, 0; \rho = kr),$$

and denote by $u_\epsilon^l(\rho, \eta)$ its solution with asymptotic behavior ($\rho \gg 1$):[2]

$$u_\epsilon^l(\rho, \eta) \sim \sin\left(\rho - l\frac{\pi}{2} + \arg\Gamma(l+1+i\epsilon\alpha) + \eta - \epsilon\alpha \log 2\rho\right).$$

Starting from η_1 we define two phase shifts η_1^+, η_1^-

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[1] Anderson, Fermi, Long, and Nagle, Phys. Rev. 85, 936 (1952); Anderson, Fermi, Nagle, and Yodh, Phys. Rev. 86, 793 (1952); Fermi, Anderson, and Nagle (to be published) and private communication of E. Fermi to R. E. Marshak.

[2] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, London, 1949), p. 53.

and two constants C_1^+, C_1^- by the equations

$$\left[\frac{d}{d\rho}\log u_{\pm 1}{}^0(\rho,\,\eta_1{}^{\pm})\right]_{\rho=kr_0}=\left[\frac{d}{d\rho}\log u_0{}^0(\rho,\,\eta_1)\right]_{\rho=kr_0};$$

$$C_1{}^{\pm}=\left[\frac{u_0{}^0(\rho,\,\eta_1)}{u_{\pm 1}{}^0(\rho,\,\eta_1{}^{\pm})}\right]_{\rho=kr_0}. \quad (3)$$

With η_3 we define similarly $\eta_3{}^{\pm}$ and $C_3{}^{\pm}$. Using $u_\epsilon{}^1(\rho,\,\eta)$ instead of $u_\epsilon{}^0(\rho,\,\eta)$, each p-phase shift $\eta_{\gamma\delta}$, $(\gamma,\,\delta=1,\,3)$ gives rise to $\eta_{\gamma\delta}{}^{\pm}$ and $C_{\gamma\delta}{}^{\pm}$. The quantities P, Q, R have then the following expressions:

For $\pi^++p\rightarrow\pi^++p$

$$P=\exp(2i\eta_3{}^+)-1,$$

$$Q=\frac{1+i\alpha}{1-i\alpha}[\exp(2i\eta_{31}{}^+)+2\exp(2i\eta_{33}{}^+)-3],$$

$$R=\exp(2i\eta_{33}{}^+)-\exp(2i\eta_{31}{}^+).$$

For $\pi^-+p\rightarrow\pi^-+p$

$$P=\zeta-1,\quad Q=\frac{1-i\alpha}{1+i\alpha}[\zeta_1+2\zeta_3-3],\quad R=\zeta_3-\zeta_1,$$

where

$$\zeta=\frac{2C_1{}^-\exp[i(\eta_1+\eta_1{}^-)]+C_3{}^-\exp[i(\eta_3+\eta_3{}^-)]}{2C_1{}^-\exp[i(\eta_1-\eta_1{}^-)]+C_3{}^-\exp[i(\eta_3-\eta_3{}^-)]}.$$

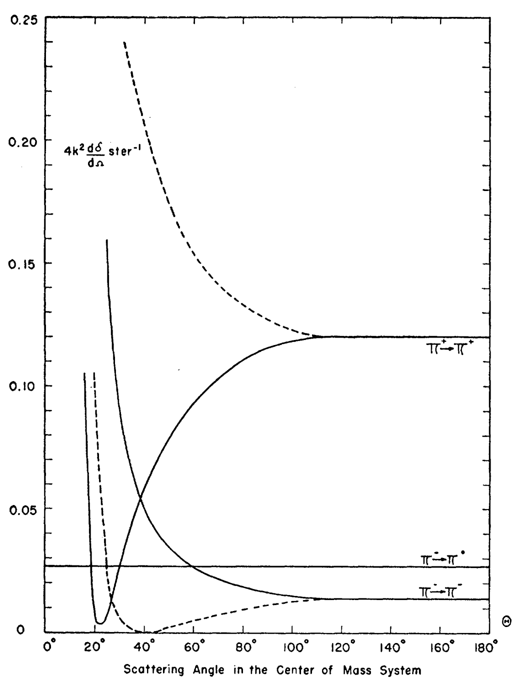

FIG. 1. Differential cross sections for scattering of 35-Mev pions by protons, assuming the scattering to take place in the $s_{\frac{1}{2}}$ state of isotopic spin $\frac{3}{2}$, with a phase shift of 10° (solid line) or −10° (dotted line).

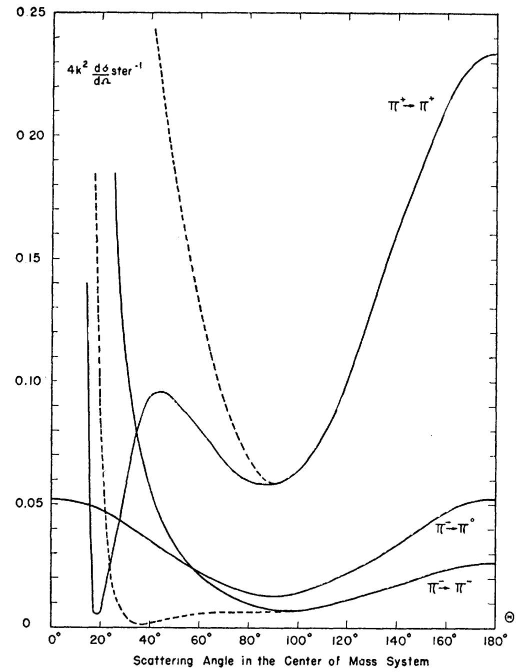

FIG. 2. Differential cross sections for scattering of 35-Mev pions by protons, assuming the scattering to take place in the $p_{\frac{3}{2}}$ state of isotopic spin $\frac{3}{2}$, with a phase shift of 7° (solid line) or −7° (dotted line).

ζ_1, ζ_3 are similar expressions with a second subscript 1,3, respectively, added in all C's and η's.

For $\pi^-+p\rightarrow\pi^0+n$,

$$P=\chi,\quad Q=\left(\frac{1-i\alpha}{1+i\alpha}\right)^{\frac{1}{2}}[\chi_1+2\chi_3],\quad R=\chi_3-\chi_1,$$

with

$$\chi=\sqrt{2}\frac{\exp(2i\eta_3)-\exp(2i\eta_1)}{2C_1{}^-\exp[i(\eta_1-\eta_1{}^-)]+C_3{}^-\exp[i(\eta_3-\eta_3{}^-)]}.$$

χ_1 and χ_3 are again obtained by adding a second subscript, 1 and 3, respectively, in all C's and η's.

All foregoing formulas are nonrelativistic. If the proton were infinitely heavy compared to the pion, consideration of the Klein-Gordon equation shows that, apart from a negligible term containing the square of the Coulomb potential, the only relativistic corrections required would be to take for $\hbar k$ the relativistic momentum of the pion and to divide α by $(1-\beta^2)^{\frac{1}{2}}$, where βc is the relative velocity of proton and pion. In the absence of a conclusive treatment of relativistic two-body problems, it seems reasonable to apply similar corrections to our case: $\hbar k$ will be taken as the relativistic momentum of pion and nucleon in the center-of-mass system, and α will be divided by $(1-\beta^2)^{\frac{1}{2}}$, with βc the relative velocity of pion and proton.

Since in all practical cases one has $\alpha\ll 1$, a convenient way to treat Eq. (3) is by power expansion in α.[3] One

[3] The more conventional W. K. B. method is not well justified for the pion-proton separations and the energies here considered.

finds to first order in α,

$$\eta_1^{\pm}=\eta_1\pm\alpha[C+\log(2kr_0)-\mathrm{Ci}(2kr_0)\cos(2\eta_1)+\mathrm{si}(2kr_0)\sin(2\eta_1)], \quad (4)$$

$$C_1^{\pm}=1\mp\alpha[\mathrm{Ci}(2kr_0)\sin(2\eta_1)+\mathrm{si}(2kr_0)\cos(2\eta_1)] \quad (5)$$

with

$$\mathrm{Ci}(x)=-\int_x^\infty\frac{\cos x}{x}dx,\quad \mathrm{si}(x)=-\int_x^\infty\frac{\sin x}{x}dx,$$

$$C=\text{Euler's constant}=0.5772.$$

$\eta_3^{\pm}$ and $C_3^{\pm}$ have an identical expression in η_3. Similar expressions could be worked out for the η's and C's belonging to the p-waves.

For α of the order of 10^{-2} or smaller ($\alpha=1.25\times10^{-2}$ for 35 Mev pions) and not too small η_1, the α-terms in (4) and (5) are quite small and a good approximation is obtained by putting $\alpha=0$ in all expressions P, Q, R; the Coulomb effects reduce then to the α-term in Eq. (1).

To show the order of magnitude of the Coulomb effects for 35 Mev pions, the differential cross sections have been plotted for two choices of phase shifts: $\eta_3=\pm10°$, all other η's$=0$ (Fig. 1), and $\eta_{33}=\pm7°$, all other η's$=0$ (Fig. 2). These phase shifts correspond in each case to a total π^+ scattering cross section of the order of 18×10^{-27} cm^2, (in agreement with the data available so far).[4] The solid lines correspond to positive phase shifts, the dotted lines to negative ones. The difference between positive and negative phase shifts for $\pi^-\rightarrow\pi^0$ scattering at all angles and $\pi^-\rightarrow\pi^-$ or $\pi^+\rightarrow\pi^+$ scattering for $\theta>\pi/2$ is small and has been neglected. It is interesting to note that the effects are as large as 50 percent for θ below 50°, where θ is the scattering angle in the center-of-mass system.

A more detailed account of this work is available as a University of Rochester report (NYO-3223). The author wishes to thank Professor R. E. Marshak for suggesting this investigation and for many valuable discussions. He is indebted to Mr. R. Grover for computational help.

[4] Barnes, Clark, Perry, and Angell, Phys. Rev. 87, 669 (1952).

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Disintegration Scheme of Fe^{59}†

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The radiations accompanying the disintegration of Fe^{59} have been investigated using lens spectrometer and scintillation counter techniques. Three beta-ray spectra of end points 1560±8 kev (0.3 percent), 462±3 kev (53.9 percent), and 271±3 kev (45.8 percent) lead to the ground state, to 1098-kev and 1289-kev excited states of Co^{59}. The 1560-kev spectrum has a forbidden shape; the shapes of the other partial spectra are allowed. Gamma-rays of 191±2 kev (2.8 percent), 1098±6 kev (56.7 percent) and 1289±6 kev (43 percent) represent all the transitions possible between the three Co^{59}-states. The conversion coefficients characterize the 191-kev and the 1098-kev transitions as magnetic dipoles, the 1289-kev transition as electric quadrupole.

A disintegration scheme is presented in which spins and parities are assigned to all the levels involved. The single-particle orbitals corresponding to the assigned spins and parities agree with those of the spin-orbit coupling shell model. Accordingly, the first excited state (1098-kev) of Co^{59} is an $f_{5/2}$ state, and its 1098-kev energy difference from the $f_{7/2}$ gound state represents the spin-orbit splitting of the f-levels, which is responsible for the existence of a closed shell at N or $Z=28$.

The angular correlation of the 191 kev–1098 kev gamma-gamma cascade was measured and found to support the proposed spin assignment.

INTRODUCTION

THE radiations of 46-day Fe^{59} were studied by Deutsch and co-workers in 1942.[1] They analyzed the beta-ray spectrum into two components of approximately equal intensities with end points of 257 and 460 kev. They showed that the low energy beta-ray group is followed by a 1.30-Mev gamma-ray, the high energy group by a 1.10-Mev gamma-ray.

As the two high energy gamma-rays are not in coincidence with one another, Fe^{59} proved to be a very useful source with which to test gamma-gamma angular correlation equipment and similar arrangements for spurious coincidences due to scattering. When such a test was run without the usual lead absorbers in front of the counters, a small number of gamma-gamma coincidences was observed and traced back to a weak gamma-gamma cascade in the Fe^{59} decay, i.e., in Co^{59}.[2]

A subsequent investigation of the angular correlation in this gamma-gamma cascade did not allow an unambiguous assignment of spins to the levels involved. However, it was felt that most of the ambiguities could

† Supported in part by the joint program of the ONR and AEC.
* Now with the Bartol Research Foundation, Swarthmore, Pennsylvania.
[1] Deutsch, Downing, Elliott, Irvine, and Roberts, Phys. Rev. 62, 3 (1942).
[2] F. R. Metzger, Phys. Rev. 85, 727 (1952).