Coulomb and Nuclear Interactions of Cosmic-Ray Mesons and Protons in Lead*

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A cloud-chamber experiment shows that positive and negative particles at sea level in the momentum range 0.3–3.1 Bev/c scatter in 5 cm of lead in the same manner. The experimental scattering distributions have a large-angle "tail" which agrees rather well with the theoretical distributions of Molière and of Snyder and Scott who assumed that the nucleus scatters like a point charge. These distributions should not be valid here because the charge distribution within the nucleus must be considered in detail for such energetic mesons. The recent calculation of Olbert specifically neglecting the scattering of particles which penetrate the nucleus predicts a much smaller probability for large-angle scattering than observed. A rough extrapolation of the calculations of Amaldi *et al.* to the case of lead shows that it is possible to explain perhaps 10 percent of the observed large-angle scattering as due to inelastic electric scattering within the nucleus, if the radius of the electric charge on a proton is taken to be 2×10^{-14} cm. Negative particles observed at 3.4-km altitude scatter in lead much the same as the sea-level particles. The scattering of positive particles observed at 3.4-km altitude indicates that protons in the momentum range 1–4.8 Bev/c suffer a small amount of nuclear scattering in lead corresponding to $(0.13\pm0.07)\sigma_g$, where σ_g is the geometrical cross section. All nuclear interactions combined give a total interaction cross section for lead of $(0.38\pm0.09)\sigma_g$. These surprisingly small cross sections are compared with other data from the literature.

I. INTRODUCTION

A CLOUD-CHAMBER experiment has been performed at sea level and at 3.4-km altitude to study the scattering of cosmic-ray mesons and protons in lead. The results of this investigation are presented in two parts. The first part, including Secs. III and IV, deals with the electrical scattering of mu-mesons. The second part (Secs. V to VII) deals with the interactions of protons in lead, including star production and meson production in addition to the nuclear scattering of protons.

The multiple Coulomb scattering of charged particles has been investigated theoretically by several authors.[1–5] The various results are generally in agreement in the region of highly multiple scattering and predict a distribution which is approximately Gaussian. However, some of the results differ for the region of plural and single scattering. The single scattering depends upon the assumptions made concerning the charge distribution within the nucleus. Molière[3] and Snyder and Scott[4] assumed a point charge which gives a differential distribution in $(p\beta\theta)$ for single scattering varying for large angles ($p\beta\theta > \sim 3$ degree-Bev/c for the present investigation) about as $1/(p\beta\theta)^3$, where p is the momentum, $\beta = v/c$ (v and c are, respectively, the velocity of the incident particle and the velocity of light), and θ is the angle of scattering projected on a plane. On the other hand, Olbert[5] has assumed that the probability of scattering of fast particles through angles larger than $\theta_{\max} \approx ƛ/R$ is identically zero, where $ƛ = \hbar/p$ is the wavelength of the incident particle and $R = 1.4\times10^{-13}A^{\frac{1}{3}}$ is the nuclear radius. A stands for the atomic mass number. The resulting distribution indicates a much reduced probability for large-angle scattering of particles of high energy. For the approximations used in all of the calculations as well as for the present observations, only angles $\theta < 1$ radian are to be considered. Thus, for $ƛ > R$, Olbert assumes no cutoff within the angular range considered, and the different theories become identical.

The modification of the scattering due to the charge distribution inside the nucleus must be taken into account when $ƛ < R$. A calculation by Amaldi *et al.*[6] shows that the amount of elastic Coulomb scattering (coherent scattering) inside the nucleus is very small. In this respect, Olbert's assumption is valid. On the other hand, the calculation of Amaldi *et al.* for the effect of incoherent scattering within the nucleus indicates that Olbert's assumption may have to be modified, as will be shown later, for those particles which penetrate the nucleus.

The work of Hanson *et al.*[7] on the multiple scattering of 15.7-Mev electrons has shown that electrons scatter in gold according to the predictions of Molière's theory within the experimental accuracy of 2 percent, with the possible exception that Molière's theory predicts slightly fewer scatterings in the region of plural scattering. Since $ƛ > R$ for 15.7-Mev electrons Molière's distribution should here be valid for all angles. Very few data are available for the scattering of energetic mu-mesons capable of probing the nucleus. Previous experiments[8–10] on mesons, which gave simultaneous information on the

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[1] W. Bothe, *Handbuch der Physik* (Julius Springer, Berlin, 1933), Vol. 22, II, p. 1.
[2] E. J. Williams, Proc. Roy. Soc. (London) **169**, 531 (1939); S. Goudsmit and J. L. Saunderson, Phys. Rev. **57**, 24 (1940); **58**, 36 (1940).
[3] G. Molière, Z. Naturforsch. **3a**, 78 (1948).
[4] H. S. Snyder and W. T. Scott, Phys. Rev. **76**, 220 (1949); W. T. Scott, Phys. Rev. **85**, 245 (1952).
[5] S. Olbert, Phys. Rev. **87**, 319 (1952).
[6] Amaldi, Fidecaro, and Mariani, Nuovo cimento **VII**, 553 (1950).
[7] Hanson, Lanzl, Lyman, and Scott, Phys. Rev. **84**, 634 (1951).
[8] F. L. Code, Phys. Rev. **59**, 229 (1941).
[9] J. G. Wilson, Proc. Roy. Soc. (London) **A174**, 73 (1940).
[10] J. A. Vargus, Phys. Rev. **56**, 480 (1939).

momentum and scattering angle, have not provided sufficient information at the large angles to decide between the various theories. Amaldi and Fidecaro[11] have obtained experimental results for very large-angle scattering ($p\beta\theta > \sim 15$ degree Bev/c) revealing a small cross section (of the order of 2×10^{-30} cm^2/nucleon) for mesons with energies greater than 300 Mev, but since they dealt only with very large angles, their experiment did not provide details of the distribution at intermediate scattering angles. The purpose of the first part of the present experiment was to obtain sufficient data on the electrical scattering of mu-mesons in the region of plural and relatively small-angle single scattering to check the various theories.

The second part of this experiment concerns the interaction of protons with lead nuclei. The cosmic-ray flux at 3.4-km altitude is rather rich in protons, as previous results have indicated.[12,13] At this altitude protons form about 20 percent of the total penetrating component in the momentum range from 0.3 to 9.6 Bev/c. Thus protons constitute a large fraction of the positive penetrating component alone. Therefore, a cloud-chamber investigation of the interactions of particles with positive charge only should lead to measureable results.

Until quite recently only meager information was available on the interaction in lead of protons with momenta in the range 1–5 Bev/c. Anderson and Neddermeyer[14] performed an experiment at 4.3-km altitude with a cloud chamber in a magnetic field. The arrangement was such as to provide in principle the desired information. However, this experiment was performed before the various components of cosmic rays were known, and the results cannot be applied here. Other experiments[15–17] were performed later at mountain altitudes with cloud chambers containing various arrangements of metal plates, but these experiments did not include a magnetic field to measure the momentum or the sign of charge of the particles involved. Therefore, the effects due to protons alone could not be isolated except in the cases where multiple scattering measurements and relative ionization could be used to identify the protons. In these cases the energy was still largely undetermined. Thus it seemed of interest to make a study of the proton-interaction cross section, where the momentum of the proton would be measured directly. It is the purpose of the second part of this paper to give the results of the present experiment on the interaction of protons in lead and to discuss these in view of the recent results of others.

[11] E. Amaldi and G. Fidecaro, Nuovo cimento **VII**, 535 (1950).
[12] Miller, Henderson, Potter, and Todd, Phys. Rev. **84**, 981 (1951).
[13] W. L. Whittemore and R. P. Shutt, Phys. Rev. **86**, 940 (1952).
[14] C. D. Anderson and S. Neddermeyer, Phys. Rev. **50**, 263 (1936).
[15] W. E. Hazen, Phys. Rev. **63**, 213 (1943); **65**, 67 (1944).
[16] M. J. Dandin, Compt. rend. **218**, 830 (1944).
[17] W. M. Powell, Phys. Rev. **69**, 385 (1946).

II. APPARATUS

The experimental arrangement, designed specifically to measure simultaneously the momentum and scattering of cosmic-ray particles has been described briefly.[13] A few additional details will now be given. The apparatus is similar in principle to that used by Brode for mass measurements[18] and by Glazier *et al.*[19] for momentum measurements. Two cloud chambers were mounted one above and one below the air gap of a permanent magnet[20] providing a field of about 9000 gauss. Through the center of the lower chamber a lead plate, 5 cm thick, was mounted horizontally. A 1-cm plate was mounted below the 5-cm plate. An arrangement of Geiger counters connected in coincidence selected particles which traversed the air gap of the magnet as well as the two cloud chambers. It was possible, although very unlikely, that a meson not originally in the solid angle determined by the counter system could be scattered into it by the iron pole pieces. Fiducial wires were located in the cloud chambers in order to enable one to recognize events of this type. A system of mirrors allowed one camera to view both cloud chambers simultaneously along the axis of each. A second camera viewed both chambers in a similar fashion with a stereoscopic angle of 20°.

The circular expansion cloud chambers were of conventional design, 6 inches deep and 19 inches in diameter. The cylindrical walls of the chamber were made of $\frac{3}{32}$-inch Plexiglas coated on the inside with a thin solution of polystyrene dissolved in toluene. This thin coating prevented the alcohol vapor from destroying the surface of the walls. The pressure of argon and condensable vapor (a 50–50 mixture of water and ethyl alcohol) in the chamber when fully compressed was 1.4 atmospheres at sea level and 1.1 atmospheres at 3.4-km altitude. Fixed-volume expansions were made in the usual way by releasing air from behind a Neoprene diaphragm through a fast-acting valve.[20] The time of 0.008 sec between passage of a particle and completion of the expansion was sufficiently short to give tracks of less than one-millimeter width. After a 5-second delay following a fast expansion caused by an incident particle the chamber was recompressed. Thereafter three slow expansions taking about one second each followed at 15-second intervals. Finally, the chamber was allowed to recover in the compressed state for an additional 70 seconds, making the minimum total time between fast expansions two minutes.

Illumination for each chamber was provided by 2 fourteen-inch quartz flash tubes, 5-mm inside diameter, mounted in parabolic reflectors and placed on both sides of the chambers. Each flash tube was energized by a condenser bank of 120 microfarads charged to 2000

[18] R. B. Brode, Revs. Modern Phys. **21**, 37 (1949).
[19] Glazier, Hammermesh, and Safanov, Phys. Rev. **80**, 625 (1950).
[20] R. P. Shutt and W. L. Whittemore, Rev. Sci. Instr. **21**, 643 (1950); **22**, 73 (1951).

volts. The lead plates in the bottom cloud chamber were covered on both sides by polished ferrotype plates to improve the illumination in the region around the plates. The light delay after expansion was 0.05–0.10 sec. The photographs were made on 35-mm Linagraph Ortho perforated film, using a demagnification of about 15. The cameras located at the front and at the 20° position had apertures of $f/9$ and $f/11$, respectively.

The momentum p is given by $p=5.3/\theta_p$, when p is measured in units of Bev/c, and θ_p is the angle in degrees between the track in the top chamber and in the top half of the bottom chamber. This approximate expression for p is no longer valid when $\theta_p>20°$; but for the mesons under consideration $\theta_p<15°$. As described,[13] the scattering in the walls of the cloud chambers and interposed Geiger counters caused an error of less than one percent in the momentum measurement and was negligible compared to the distortions due to turbulence. The momentum p could be measured in the range 0.300 ± 0.006 to 11 ± 4 Bev/c. The lead plates in the bottom chamber were used for the study of the projected angle θ of scattering for particles whose momenta were measured. For each singly occurring particle the quantity $p\theta$ could be evaluated. The scattering theories show that the proper variable to study is $p\beta\theta$, not $p\theta$, which is determined by the above measurements. For mesons of the momenta under consideration, however, the value of β is very nearly unity, and therefore, the quantity $p\theta$ can actually be used to characterize the scattering. On the other hand, protons of the same momentum as the mesons under consideration have a value of β substantially less than unity, ranging down to 0.31 for protons with $p=0.30$ Bev/c. Therefore, a correction to the measured quantity $p\theta$ must be applied for protons.

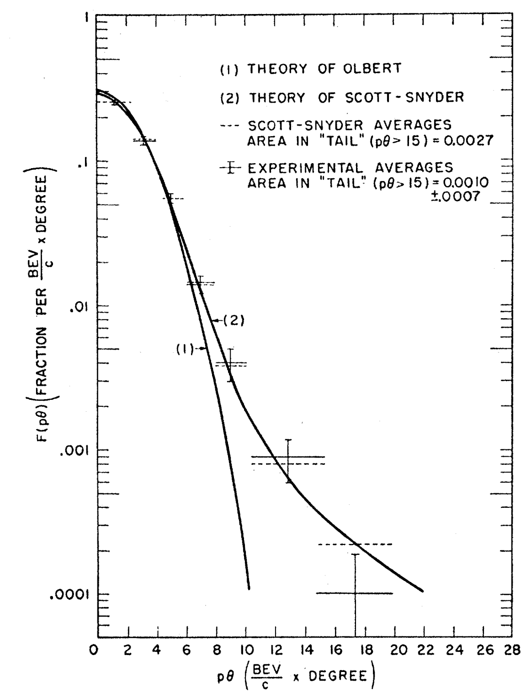

FIG. 1. The differential distribution $F(p\theta)$ in projected scattering variable $p\theta$ below 5 cm of lead for *negative* mu-mesons with $0.30<p<3.1$ Bev/c observed at sea level. In all distributions only those particles were included which ionized at the minimum rate above and below the 5-cm plate.

III. RESULTS OF EXPERIMENT ON SCATTERING OF MU-MESONS IN LEAD

The differential distributions $F(p\theta)$ of scattering in 5 cm of lead of negative and positive mesons observed *at sea level* are given in Figs. 1 and 2, respectively. Mesons in the momentum range from 0.300 to 3.1 Bev/c have been included with the exception of positive particles in the range 0.70 to 1.0 Bev/c. The omission of the latter group from the data as well as the inclusion of only those particles which ionized at the minimum rate both above and below the 5-cm plate aids in the elimination of a contribution due to nuclear interactions of protons in a momentum range where protons may still be numerous.[21] Protons with $p<0.7$ Bev/c were stopped in the 5-cm plate by ionization losses. Protons with $p>1.0$ Bev/c could not have been stopped by ionization losses alone. It should be noted that except for somewhat larger statistical fluctuations the same distributions are exhibited individually for positive as well as for negative particles if the data are subdivided into groups with momenta in the ranges 0.3–1.0, 1.0–1.9, and 1.9–3.1 Bev/c. The distributions in both Fig. 1 and Fig. 2 are normalized so that the total area under each curve is unity.

Some of the scattering data were affected by errors in measuring θ_p and θ which tended to distort the distributions. The data included in Fig. 1 and Fig. 2 were affected only to a negligible extent by these errors; however, data for momenta >3.1 Bev/c which were not included were affected to a very important extent in the following manner. The scattering distribution can be represented approximately by a Gaussian curve except for a "tail," starting at $p\theta=7$ degree-Bev/c, which is small compared to the bulk of the scattering. This "tail" contains the large angles, compared to which the measuring errors are negligible. The measuring errors can also be represented by Gaussian curves having a root-mean-square width representing an average magnitude of the error. If σ_t is the root-mean-square width of the true scattering distribution and σ_e is the root-mean-square deviation of a large number of measurements of $p\theta$, caused by the various measuring errors, then the root-mean-square width σ_0 of the observed scattering distribution is given by $\sigma_0^2=\sigma_t^2+\sigma_e^2$. Thus the Gaussian curve with width σ_t broadens to a Gaussian curve with width σ_0 because of the measuring errors. The average measuring error for θ_p was taken to be about 0.3°. The error in the measurement of the projected angle of scattering below the 5-cm plate was taken to be 0.5°.

[21] M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).

Therefore, σ_e increases with p, since $p \propto 1/\theta_p$, but is constant with θ. For mesons with $p<3.1$ Bev/c, the scattering data for the 5-cm plate were not appreciably affected by these errors, as a computation shows. For $p>3.1$ Bev/c, however, the effects of these errors became important, resulting in an observed scattering distribution about twice as broad as the true distribution for particles in the range $4.8<p<11.0$ Bev/c. For the 1-cm lead plate, the situation was much worse because the error in measuring the scattering angle below the 1-cm plate was at least 1.0°, due to the added turbulence and poor lighting in the bottom section as well as to the splitting of the tracks caused by electrostatic charges on the Plexiglas walls of the cloud chamber. Thus all of the scattering data from the 1-cm plate were affected in a significant and nonuniform manner by the various errors. Since it was preferred to use only those data for which the computations show that corrections were unimportant, only the data in Fig. 1 and Fig. 2 were used to exhibit the Coulomb scattering in lead.

A different type of error might affect the "tail" of the distributions given in Fig. 1 and Fig. 2. As mentioned above, the "tail" contains particles scattered into angles large compared to the measuring errors. Moreover, the momenta involved could also be measured with good accuracy. The only serious source of error for these events lay in the possibility that an incident particle not in the solid angle selected by the counter trays was scattered into the bottom counter and cloud chamber by the pole tips of the magnet in such a manner that the apparent momentum was considerably increased. This effect would have made the observed values of $p\theta$ too large and contributed falsely to a "tail." The observed "tail" was larger than the curve of Olbert (approximately Gaussian) by a number of events corresponding to 2.5 percent of the incident particles. A calculation shows that the number of mesons scattered in the pole tips could not have amounted to more than 0.15 percent of the incident flux. Furthermore, a detailed examination of the individual events forming the "tail," making use of the fiducial wires mentioned above, showed no case where a meson hit a magnet pole face. Therefore, the experimental distributions in Fig. 1 and Fig. 2 were not influenced significantly by measuring errors or instrumental uncertainty.

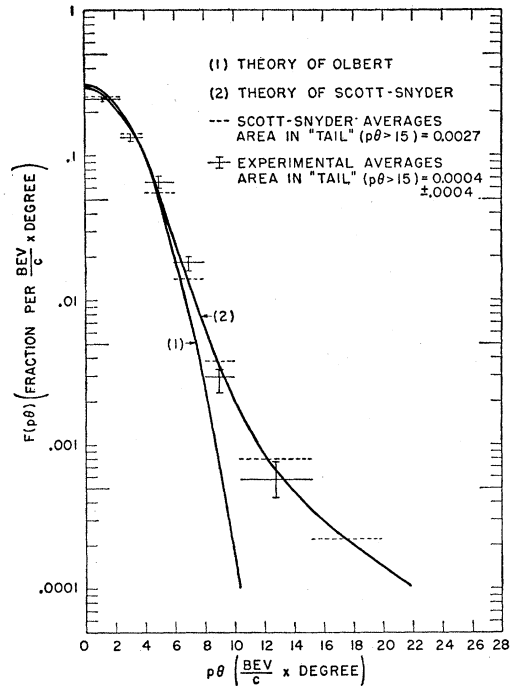

FIG. 2. The differential distribution $F(p\theta)$ below 5 cm of lead for *positive* mu-mesons with $0.30<p<3.1$ Bev/c observed at sea level, omitting $0.70<p<1.0$ Bev/c.

IV. DISCUSSION OF RESULTS ON MU-MESON SCATTERING

The theoretical distributions of Olbert (curve 1) and Snyder and Scott or Molière (curve 2) are shown also in Fig. 1 and Fig. 2, together with the experimental data. The theories of Molière and of Snyder and Scott are represented by the same distribution. The results of Olbert, however, are quite different. As discussed in Sec. I, the difference between the theories lies in the treatment of the electric charge distribution within the nucleus. Since $\lambda\!\!\!^-$ is about 2×10^{-14} cm ($\ll R=8\times10^{-13}$ cm) for a meson with $p=1$ Bev/c, one sees that the theories of Molière and of Snyder and Scott cannot be applied here, since the nucleus can no longer be considered as a single point charge. Surprisingly enough, their theories agree rather well with the experimental results up to much larger values of $p\theta$ than expected. As pointed out in Sec. III, mesons with momenta in smaller intervals (e.g., 0.3–1.0, 1.0–1.9, 1.9–3.1 Bev/c) separately gave the same experimental distributions.

The scattering distribution predicted by Olbert does not agree with the experimental results. In fact, even for values as small as $p\theta=7$, the distribution of Olbert, assuming no scattering within the nucleus, predicts a probability for scattering which is smaller than observed by a factor of two, and it becomes progressively smaller than observed for larger values of $p\theta$, reaching a value 10^3 times too small at $p\theta=15$, and 10^6 times too small at $p\theta=20$. The theory of Olbert could be brought into substantial agreement with the present results only by requiring that the electric charge of the whole lead nucleus is concentrated in a volume with radius of less than one sixth of the nuclear radius of 8×10^{-13} cm. Unless one attributes the observed excess scattering to the nuclear interaction of mu-mesons, one must conclude that the theory of Olbert has to be modified to include the effect of the distribution of charge within the nucleus.

The difference between Olbert's and the experimental distributions represents a "tail" containing about 2.5 percent of the total area under the curves. It must be emphasized that neither protons nor electrons nor pi-mesons could cause the "tails" observed in Fig. 1 and Fig. 2. In the first place, since the scattering distribution of negative particles showed the same "tail" as that observed for the positive particles, protons could not be the cause. Moreover, protons with $p>1$ Bev/c form not more than 1 percent of the penetrating component at sea level.[21] Since the mean free path for nuclear scattering in lead for protons will be shown in Sec. VI to be about 1200 g/cm^2, the 5-cm lead plate caused only about 5 percent of the protons to suffer nuclear scattering. Therefore protons gave rise to less than $0.05\times 1=0.05$ percent of all the scattering. Furthermore, electrons were not sufficiently numerous at sea level to cause this "tail." For the momentum region above 0.300 Bev/c, only about one percent of the ionizing tracks were due to electrons.[22] Cascade theory[23] indicates that of those electrons which existed in this momentum range only 5–10 percent should have produced a shower containing just *one* particle below 5 cm of lead. This indicates that not more than $0.1\times 1=0.1$ percent of the incident particles were electrons which could have given a falsely identified large angle event. Furthermore, this expected number of one-particle electron showers agrees also with that which one can predict on the basis of the observed electron showers with two or more particles. The number of pi-mesons incident on the apparatus must be very small (<0.1 percent), since it is known that very few mu-mesons (and hence, pi-mesons) are produced below 3.4-km altitude[13] and also that few protons (~ 1 percent)[21] are incident at sea level with sufficient energy to produce energetic pi-mesons. Thus the effects of protons, electrons, and pi-mesons on the production of the "tail" are negligible at sea level.

It might be noted that, if the electrical scattering theory of Olbert were assumed to be strictly correct, the "tail" observed for mu-mesons would indicate a non-electrical interaction. The cross section one obtains by attributing to nuclear effects all large-angle scattering beyond that predicted by Olbert is about 10^{-25} cm^2 per lead nucleus. This is a value very similar to that obtained previously by many workers, who attributed to nuclear scattering the difference between the observed scattering and that predicted by the theory of Williams. In this regard, see Code,[8] Wilson,[9] Vargus,[10] Shutt,[24] Sinha,[25] and Sahiar.[26] The cross section obtained from the present experiment would be essentially the same whether Williams' or Olbert's theory is used. The present results are compatible with those of Amaldi *et al.*,[11] obtained by a different method for very large scattering angles only, if one attributes to the electron component the few particles we observed to be scattered through $p\theta>15$.

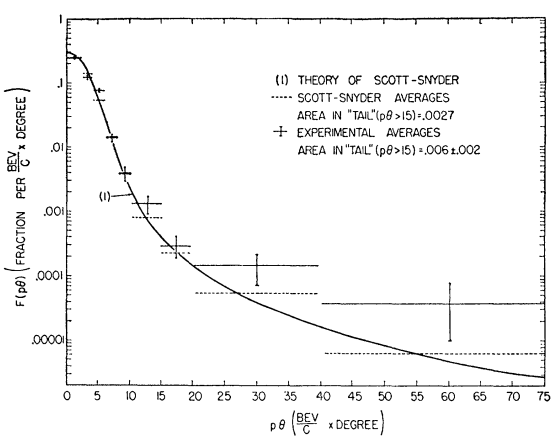

FIG. 3. The differential distribution $F(p\theta)$ below 5 cm of lead for *negative* particles observed at 3.4-km altitude with $1.0<p<4.8$ Bev/c.

To investigate whether the observed scattering for the larger values of $p\theta$ could be wholly due to electrical interactions within the nucleus, the theory by Amaldi *et al.*[6] has been used to estimate the effect of incoherent scattering resulting in excitation of the nucleus. While Amaldi's calculations apply only to light nuclei such as lithium and carbon, these calculations, nevertheless, should give an indication of the order of magnitude to be expected for this type of scattering in a heavy nucleus like lead, if a correction factor is introduced to take account of the Z dependence of the incoherent Coulomb scattering. A computation shows that incoherent scattering within the nucleus might account for 5 to 10 percent of the observed large-angle scattering if the radius of the charge distribution in the individual protons is taken as 2×10^{-14} cm, and for only about one percent if the radius is taken as 1.4×10^{-13} cm. This theory as applied to lead is too crude to allow any detailed conclusions. However, it seems unlikely that a detailed calculation carried out for lead specifically would yield a cross section larger by an order of magnitude. Hence, the large observed cross section may indicate some non-Coulomb interaction in addition to an electric charge distribution for single protons (as well as mu-mesons) much smaller than the size corresponding to the proton radius of 1.4×10^{-13} cm. We conclude that the present experiment is sufficient to indicate the need for a treatment of the contribution to scattering of an incident mu-meson by the individual protons within a lead nucleus.

V. NUCLEAR SCATTERING OF PROTONS OBSERVED AT 3.4-km ALTITUDE

The equipment described in Sec. II was operated at an altitude of 3.4 km in the same manner as at sea level. Since protons form a significant part of the penetrating component at 3.4 km, one would expect to observe not only the electrical scattering of the incident particles,

[22] E. J. Williams, Proc. Roy. Soc. (London) **A172**, 194 (1939).
[23] D. J. X. Montgomery, *Cosmic Ray Physics* (Princeton University Press, Princeton, 1949), p. 341.
[24] R. P. Shutt, Phys. Rev. **61**, 6 (1942).
[25] M. S. Sinha, Phys. Rev. **68**, 153 (1945).
[26] A. Sahiar, Proc. Indian Acad. Sci. **34**, 201 (1951).

but also nuclear scattering of protons and other events such as stars and meson production.

Figure 3 shows the differential distribution observed at 3.4-km altitude for negative particles with momenta in the range 1.0–4.8 Bev/c. It is in very good agreement with the sea-level data exhibited in Fig. 1 and Fig. 2, except that, for the larger angles where $p\theta>15$, about 6.0 ± 4.2 times as many particles were observed at altitude as at sea level. This excess number forms about 0.5 percent of all the scattered particles. Although this small excess for large angles might be taken to indicate the nuclear scattering of pi-mesons in the natural cosmic-ray beam, it probably only indicates the presence of a considerable number of electrons in the incident beam at 3.4-km altitude,[27] some of which triggered the apparatus even though anticoincidence counters were used at the sides. As shown in Sec. IV, an energetic electron ($E>1$ Bev) has less than a 10-percent chance to produce a one-particle shower below 5 cm of lead. This would imply that electrons may have formed 5 percent of the incident flux, if the additional large-angle scattering is due to electrons. Additional evidence of a rather large component of electrons occurring singly in the cloud chamber is provided by the fact that the intensity of electron showers with two or more particles observed at 3.4-km altitude was about 2 times the intensity observed at sea level in spite of the use of anticoincidence counters. An auxiliary experiment, making use of carbon placed above the apparatus in order to increase the number of pi-mesons observed, showed that pi-mesons scattered by nuclear interactions contributed less than 0.1 percent to the scattering distribution. Furthermore, a computation of the intensity of pi-mesons expected at 3.4-km altitude on the basis of the meson production spectrum of Sands[28] gives a value of about 0.25 percent of the penetrating component. The rather good agreement in the range $0<p\theta<15$ with the results obtained at sea level is taken as evidence that the negative particles observed here are mainly mu-mesons.

The positive particles at 3.4-km altitude scattered in quite a different manner from the negative particles at the same altitude. Figure 4 shows the scattering results obtained for positive particles alone in the momentum range from 1.0 to 4.8 Bev/c. The lower limit of 1.0 Bev/c was chosen to lie considerably above the upper limit of 0.7 Bev/c for absorption of protons by ionization losses in the 5-cm lead plate, so that both protons and mesons had to ionize at the minimum rate below the plate. Also protons with less momentum could have suffered sufficient Coulomb scattering in the lead to scatter out of the illuminated volume. The upper limit of 4.8 Bev/c was chosen because protons of larger momenta formed a very small part of the incident beam. The large range of momenta (1.0–4.8 Bev/c) was chosen to include enough protons to make the results significant.

[27] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).

[28] M. Sands, Phys. Rev. **77**, 180 (1950).

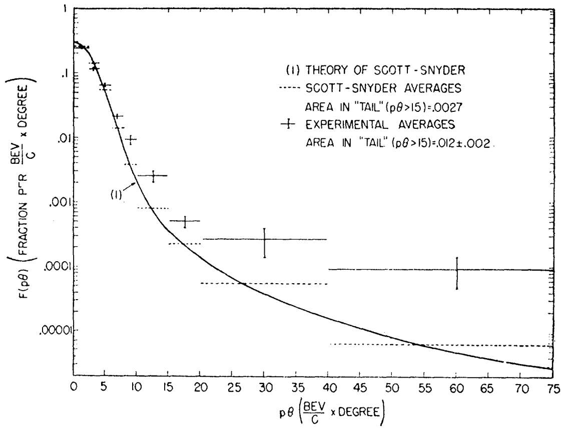

FIG. 4. The differential distribution $F_t(p\theta)$ below 5 cm of lead for *positive* particles observed at 3.4-km altitude with $1.0<p<4.8$ Bev/c. Approximately 27 percent of the incident positive particles were protons; the rest were mainly mu-mesons.

The scattering distributions for positive (Fig. 4) and negative (Fig. 3) particles differ in the following manner. In the first place, the area under the scattering distribution for $p\theta>15$ for positive particles is 2.0 ± 0.7 times that for the negative particles. This fact alone might indicate only a fluctuation in the number of electrons. However, one should note that the distribution for negative particles agrees with the sea-level distributions everywhere except for $p\theta>15$, whereas the distribution for positive particles deviates for values below $p\theta=15$ as well as for larger values, as should be the case if protons were included. The deviation for values of $p\theta<15$ nearly disappears when one makes a correction for the fact that the scattering distribution for protons included in Fig. 4 should be plotted as a function of $p\beta\theta$ instead of $p\theta$, since β for many of the protons included is considerably less than unity.

Using the data already presented in a previous publication,[13] one finds that 590 ± 60 protons[29] were incident on the apparatus accompanied in the same time interval by 1589 ± 40 positive and 1392 ± 37 negative particles, which penetrated the 5-cm lead plate. These particles were mainly mu-mesons except for a few electrons, as noted above. This information is included in Table I, where N_p stands for the number of protons; N_μ^+, positive mu-mesons; N_μ^-, negative mu-mesons.

The total experimental scattering distribution $F_t(p\theta)$ for positive particles can be expressed as the sum of several individual distributions:

$$F_t(p\theta)=n_\mu^+F_{el}(p\theta)+n_p(1-w)F_{el}'(p\theta)+n_pw\int_{-\infty}^{+\infty}F_p(p\theta')F_{el}'[p(\theta-\theta')]d(p\theta'),\quad (1)$$

[29] This number is about 10 percent less than the number calculated from the data of reference 13 because, of those particles whose momenta could be measured, 10 percent fell too near the edges of the scattering plate to provide reliable scattering data.

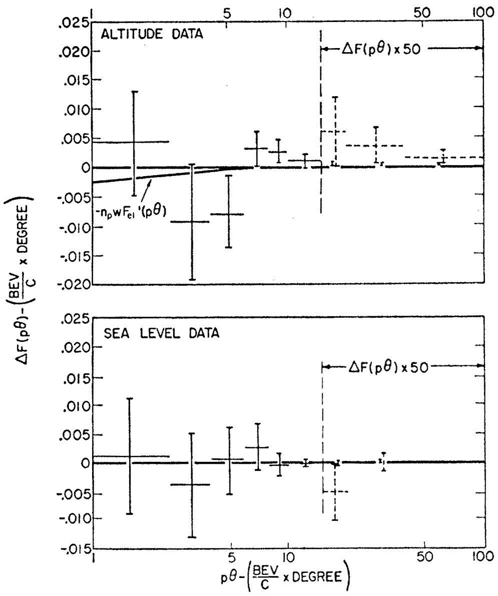

FIG. 5. The differential distribution of the difference $\Delta F(p\theta)$ between the total experimental scattering distribution for positive particles and that for negative particles. It is shown in the text that $\Delta F(p\theta)$ for $p\theta > 10$ represents the nuclear scattering of protons. The data obtained at 3.4-km altitude show evidence for nuclear scattering, whereas the data obtained at sea level for mu-mesons do not.

where

$$\int_{-\infty}^{+\infty} F_t(p\theta)d(p\theta)=1;\quad n_\mu{}^+=N_\mu{}^+/(N_\mu{}^++N_p),$$

$$n_p=N_p/(N_\mu{}^++N_p).$$

$F_{el}(p\theta)$ is the differential electric scattering distribution of mu-mesons where β is essentially unity, $F_{el}'(p\theta)$ is the electrical scattering distribution of protons and depends on β, because many of the protons included are relatively slow, w is the fraction of the N_p protons which suffers nuclear scattering with a distribution given by $F_p(p\theta)$. w and $F_p(p\theta)$ are to be determined. Since the momentum distribution of protons is given,[13] one can determine the distribution $F_{el}'(p\theta)$ from the distribution $F_{el}(p\theta)$ by transforming from the variable $p\theta$ to $p\beta\theta$ and remembering that $F_{el}(p\beta\theta)$ is a universal function in all electrical scattering theories. In order to eliminate the effect of electrons from the results, the *experimental* distribution for negative particles (Fig. 3) has been used for $F_{el}(p\theta)$. The last term in the integral Eq. (1) gives the distribution of nuclear scattering modified by the ever-present scattering due to electrical interaction. It will be shown that the distribution of nuclear scattering in the laboratory frame of reference is much broader than the distribution for electrical scattering. Therefore, one can write

$$\int_{-\infty}^{+\infty} F_p(p\theta')F_{el}'[p(\theta-\theta')]d(p\theta')$$

$$\approx F_p(p\theta)\int_{-\infty}^{+\infty} F_{el}'[p(\theta-\theta')]d(p\theta')=F_p(p\theta),$$

since

$$\int_{-\infty}^{+\infty} F_{el}'[p(\theta-\theta')]d(p\theta')=1.$$

With this simplification one obtains from (1)

$$F_t(p\theta)-[n_\mu{}^+F_{el}(p\theta)+n_pF_{el}'(p\theta)]$$

$$=n_pw[F_p(p\theta)-F_{el}'(p\theta)]=\Delta F(p\theta).\quad (2)$$

Therefore, the difference between the total differential scattering distribution $F_t(p\theta)$ and the electrical scattering distribution $[n_\mu{}^+F_{el}(p\theta)+n_pF_{el}'(p\theta)]$ represented as $\Delta F(p\theta)$ does not give directly the nuclear scattering distribution for protons, but is decreased by the electrical scattering of the fractional n_pw protons that were scattered by nuclear interactions. However, the electrical scattering distribution decreases rapidly as $p\theta$ increases. For a sufficiently large value of $p\theta$ (10 for the present experiment) $F_{el}'(p\theta)$ can be neglected compared to $F_p(p\theta)$. For larger values of $p\theta$, $\Delta F(p\theta)$ gives the nuclear scattering directly. It should be mentioned that elastic diffraction scattering of protons is completely negligible for $p\theta > 10$ for two reasons. In the first place, a computation has shown that this type of scattering decreases to a very small fraction of its maximum when $p\theta > 6$. Furthermore, the cross section for this scattering, even for small angles, should be very small compared to electrical scattering because the nucleus is presumably not completely opaque (see discussion below).

Figure 5 gives the experimental values of $\Delta F(p\theta)$ for the data obtained at sea level and 3.4-km altitude. Since protons formed less than one percent of the penetrating component with $p > 1.0$ Bev/c at sea level, one would expect that here $\Delta F(p\theta)$ should be identically zero for all values of $p\theta$. The data of Fig. 5 seem to be consistent with this view for the data obtained at sea level, except for statistical fluctuations. In particular, for $20 < p\theta < 40$, $\Delta F(p\theta)$ is identically zero and for $p\theta > 40$ no particles of either sign of charge were observed. On the other hand, $\Delta F(p\theta)$ obtained at 3.4-km altitude has small but finite positive values for $10 < p < 80$. For the smaller values of $p\theta$ the effect of the electrical scattering of the mu-mesons and protons becomes very important and obscures the effect of nuclear scattering of the protons since the absolute error becomes relatively much larger. Also shown in Fig. 5 is the electrical distribution $-n_pwF_{el}'(p\theta)$ for protons. As pointed out above, for $p\theta > 10$ the electrical scattering is negligible compared to the observed scattering.

The final net nuclear scattering is obtained by subtracting $\Delta F(p\theta)$ obtained at sea level from $\Delta F(p\theta)$ obtained at 3.4-km altitude. Subtracting the values obtained at sea level will tend to eliminate any instrumental errors. The resulting values have statistical significance only for $p\theta > 10$. An extrapolation must be made to account for the scattering between $p\theta = 0$ and $p\theta = 10$. Two extrapolations were performed assuming for the scattering in the center-of-mass system an isotropic distribution and a $\cos^2\theta$ distribution, respectively. Each extrapolation was joined to the experimental value at $p\theta = 10$, where $\Delta F(p\theta)$ represents the true nuclear scattering. For the isotropic distribution the total area under the curve corresponds to the nuclear scattering of 25 ± 12 out of $N_p = 590\pm 60$ protons. For the $\cos^2\theta$ distribution, the area corresponds to 30 ± 15 out of 590 ± 60 protons. One sees that the precise nature of the scattering distribution is relatively unimportant with the present experimental error, since these two very different distributions yield similar results. We will assume that the average of these, 28 ± 14, is the best estimate of the number of protons which scatter by nuclear interaction in 5 cm of lead. The mean free path for nuclear scattering can be calculated in the usual manner from these data. In 5 cm of lead (57 g/cm^2), 590 ± 60 protons yield 28 ± 14 scattered protons. The mean free path for nuclear scattering in lead is $57\times(590\pm 60)/28\pm 14 = 1200\pm 600$ g/cm^2. Since the geometrical cross section σ_g for lead nuclei corresponds to 160 g/cm^2, we see that our scattering results imply a cross section for nuclear scattering of $\sigma_s = (0.13\pm 0.07)\sigma_g$.

A fractional error in N_p produces a larger fractional error in σ_s which is not only inversely proportional to N_p but also depends on ΔF which itself depends sensitively on N_p (Eq. (2)). As an example, if N_p were actually 40 percent smaller than indicated by the present experiment,[13] one would obtain $\sigma_s = 0.31\sigma_g$. However, such a large error in N_p, which would be 4 times the statistical error given, appears to be quite unlikely.

VI. OTHER NUCLEAR INTERACTIONS OF PROTONS

In Sec. V we have derived from the data obtained at 3.4-km altitude a cross section for nuclear scattering. By examining the data for other evidence of nuclear interactions we can obtain a total interaction cross section. Nuclear stars have been classified by the symbols $N=0$, $N=1$, and $N\geqq 2$, referring to the lower limit of the number of densely ionizing prongs observed below the 5-cm lead plate, since others may be absorbed in the lead. Protons with $p>1.0$ Bev/c could only be stopped ($N=0$) by some nuclear interaction in the 5-cm plate, but the class of events with $N=0$ could include electrons stopped by multiplication in the lead plate. However, if many more positive than negative particles stop, the excess should be attributed to protons since there must be equal numbers of positive and negative electrons. At 3.4-km altitude in the momentum interval 1–4.8 Bev/c, 17 positive particles stopped in 5 cm of lead, whereas only 3 negative particles did likewise. This indicates that approximately $17-3=14$ particles were not electrons and hence should be designated as protons. On the other hand, at sea level 5 positive and 4 negative particles with $1.0<p<3.1$ Bev/c stopped in the 5-cm lead plate. There was no indication in this for the presence of protons.

Using the above classification, we have included the available information in Table I. Also included in parentheses are the corresponding data for the 1-cm lead plate. In some instances, the 1-cm plate could not supply the data. For instance, the measuring errors were so large that no scattering data were obtained. Also, it was impossible to identify events in which mesons were definitely produced because the criterion for this event was that the produced particles pass through an additional 1-cm plate without deflection or multiplication. This criterion could obviously not be used for the bottom plate.

The data listed in Table I under "Possible meson production" consist of shower events with low multiplicity (2–5), hard to distinguish from electron showers, because they did not satisfy the criterion for meson production listed above. However, these events did have two distinguishing features. First, about one-third of these events gave secondaries which appeared to diverge from a point rather deep in the 5-cm lead block, while ordinary electron showers of 4 or 5 particles appear to diverge from a point very near the surface. Second, all of the incident particles were positive. At sea level, the present experiment gave shower events with similar numbers of positive and negative initiating particles. Therefore, the observed events may well represent the production of mesons by protons.

Before calculating the total interaction cross section, we will compare the data obtained from the 1-cm plate with those from the 5-cm plate. The number 41 ± 7 of events in the 5-cm plate (4th to 6th row, last column, of

TABLE I. Data from 5-cm lead plate used for computing the scattering and total interaction cross sections for protons. Parentheses indicate same data for 1-cm lead plate.

	Momentum range			
	1–1.9 Bev/c	1.9–3.1 Bev/c	3.1–4.8 Bev/c	1–4.8 Bev/c
N_p	378 ± 50	122 ± 25	90 ± 15	590 ± 60
$(N_\mu^+ + N_p)$	1043	694	442	2179
N_μ^-	596	479	317	1392
Stars $N=0$	11 (0)	3 (0)	0 (0)	14 (0)
Stars $N=1$	12 (1)	2 (0)	2 (0)	16 (1)
Stars $N\geqq 2$	5 (1)	4 (5)	2 (1)	11 (7)
Definite meson prod.	0	1	2	3
Possible meson prod.	4	4	0	8
Nuclear scattering	18 ± 11	10 ± 8		28 ± 14
Total No. of nuclear interactions				80 ± 16

Table I) can be compared directly with the corresponding number 8 ± 3 for the 1-cm plate if the latter number is increased by a factor of 4/3 to account for the difference in solid angle covered by the two plates. The resulting ratio of similar events becomes $(41\pm7)/(8\pm3)4/3=3.9\pm1.6$. This is somewhat smaller than the ratio of 5.0 for the thicknesses of the two plates, but it is not unreasonable for the small number of events observed. Therefore, this is additional evidence that all events occurring in the 5-cm plate were counted.

The mean free path for the total interaction of protons with $1.0<p<4.8$ Bev/c in lead will now be calculated on the basis of the data given in Table I. When 590 ± 60 protons are incident on 5 cm of lead, 80 ± 16 nuclear, interactions are produced. Calculating the mean free path for total interaction, one finds $(590\pm60)57/(80\pm16)=420\pm95$ g/cm^2. Since the geometrical cross section corresponds to about 160 g/cm^2 of lead, one finds that the above interaction length corresponds to $(0.38\pm0.09)\sigma_g$. This cross section is actually averaged over the momentum range from 1.0 to 4.8 Bev/c. Since a large part of the incident protons have momenta in the range 1.0 to 1.9 Bev/c, it is possible to calculate the cross section for this group alone. Table I gives all the necessary data. One calculates, therefore, a total interaction length of $(378\pm50)57/(50\pm13)=430\pm140$ g/cm^2. This corresponds to $(0.37\pm0.12)\sigma_g$.

VII, NUCLEAR INTERACTIONS OF PROTONS: DISCUSSION AND CONCLUSION

The present experiment has provided a cross section for the nuclear scattering of $(0.13\pm0.07)\sigma_g$ and for the total interaction of $(0.38\pm0.09)\sigma_g$ for protons in lead. The total cross section appears considerably smaller than one would expect on the basis of observed cross sections for neutrons and protons of lower energy. The experiments performed with 85- and 280-Mev neutrons indicate that lead exhibits nearly a geometrical cross section.[30] Presumably protons should interact with a lead nucleus in a similar manner. However, the present experiment deals with a more energetic beam of protons (0.43–3.5 Bev) which may exhibit a different interaction in lead nuclei. Recent experiments have given some information on the interaction of protons in photographic plates. Bernardini *et al.*[31] have performed an experiment in which the interaction of protons of 0.375 Bev was studied. They obtained a total interaction cross section of $(0.56\pm0.11)\sigma_g$. Camerini *et al.*[32] have investigated in photoplates nuclear interactions produced by very energetic cosmic-ray particles. Their results show that protons with energies <1.0 Bev suffer elastic interactions in silver and bromine and that other types of interaction become of comparable importance only for energies >2.0 Bev. Their results do not provide an absolute cross section for total interaction. In view of the results of Bernardini *et al.* and Camerini *et al.* obtained essentially for the interaction of protons with silver and bromine, one would expect that protons should interact with nearly geometrical cross section with a heavier nucleus like lead. Nevertheless, the results of the present experiment indicate a weaker interaction.

Experiments performed on the particles producing penetrating showers have shown that the more energetic particles (energy >10 Bev) interact strongly in matter. Boehmer and Bridge[33] have demonstrated that the neutral particles (presumably neutrons) producing events with the highest multiplicity have a mean free path in carbon of 85 ± 12 g/cm^2 and in lead of 143 ± 30 g/cm^2. Lead and carbon perhaps display a smaller cross section for neutrons of lower energy, but this is not certain because the efficiency of the detector of penetrating showers decreases in a complicated manner as the multiplicity and hence the energy of the incident particle decreases. Froehlich *et al.*[34] have shown that protons of about 10 Bev which produce penetrating showers have a mean free path in lead of about 160 to 190 g/cm^2.

The cross sections obtained in the present experiment indicate a weaker interaction of protons with lead nuclei in the energy range 0.43–3.5 Bev than is observed for protons of smaller and greater energy. The principal source of uncertainty for the result lies in the determination of the number of incident protons. This experiment has been used to determine the proton flux at 3.4-km altitude to an estimated accuracy of 10 percent. Similar measurements of Miller *et al.* gave a flux of protons which at an energy of 0.43 Bev is 25 percent smaller than that deduced from the present experiment. If one were to reduce our proton intensity by 25 percent, the scattering and total interaction cross sections become, respectively, $\sigma_s=(0.27\pm0.09)\sigma_G$, $\sigma_t=(0.60\pm0.08)\sigma_g$. If all the errors were to add in such a manner as to make the largest possible, although not likely, cross section, then $\sigma_t=0.68\sigma_g$. The total interaction cross section would be geometrical if the flux of protons were only 60 percent of that obtained from the present experiment, provided all the errors should add to give the largest possible cross section. Such a low value for the flux would not only be outside the limits of error of the present experiment by a factor of four but would also be considerably smaller than that determined by Miller. Therefore, one concludes that the cross section which protons (0.43–3.5 Bev) exhibit for interaction with lead is less than geometrical and is most probably $(0.38\pm0.09)\sigma_g$ with an upper limit that might be stretched to $0.68\sigma_g$.

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The ratio of the scattering cross section to the total interaction cross section is relatively unaffected by an uncertainty in the proton flux. The best value of this ratio r is $(0.13\pm0.07)/(0.38\pm0.09)=0.34\pm0.19$. If, on the other hand, the proton intensity should be 75 percent of that used in the present calculations, then $r=(0.27\pm0.09)/(0.60\pm0.08)=0.45\pm0.18$.

We have had the benefit of several discussions of mu-meson scattering with Professor B. Rossi, Professor W. Scott, and Mr. S. Olbert. Mr. Olbert kindly provided some of his results prior to publication. Professor Scott has also made available the results of the Snyder and Scott and the Molière scattering calculations in a form directly applicable to the present experiment.

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The Temperature Dependence of Electrical Resistance

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The electrical resistance of a simple metal can be computed over a wide range of temperature by considering the thermal vibrations and the scattering from individual ions. Observations by MacDonald and Mendelssohn make possible a comparison with experiment from room temperature down to very low values. Moderate agreement for lithium and sodium can be obtained by a suitable choice of parameters, but the values necessary are not in close agreement with those suggested by other phenomena.

An approximate method for taking into account all three normal modes of vibration with a given propagation vector also gives moderate agreement with the observations.

The current phase of the theory of electrical conductivity began in 1928 when Sommerfeld applied the Fermi-Dirac statistics to the Lorentz-Drude treatment of the conduction electrons in a metal.[1] It was then soon recognized that the electron scattering, to which electrical resistance is due, is similar in many respects to x-ray scattering. As long as the crystal ions are fixed in their lattice positions the electrons move without any obstructions, but the presence of irregularities due to strains, impurities, or thermal motions, gives rise to a random scattering and to resistance.

With this picture it is obvious why resistance decreases with decreasing temperature, but it was not at first clear why the zero-point motion of the ions does not produce, in all metals, a very large residual resistance. The explanation of this fact was given in 1929, when it was shown that the statistics of the electrons, combined with the conservation of energy in the scattering process, lead to a vanishing probability of scattering as the temperature approaches zero.[2]

The scattering of an electron wave occurs on a sound wave in the crystal. The scattering is subject to three restrictions:

1. There must be conservation of propagation vectors of the incident electron wave, the sound wave, and the scattered electron wave. The propagation vector of the scattered wave must be equal to the sum of the propagation vectors of the incident wave and the sound wave. This is analogous to conservation of momentum.
2. There must be conservation of energy in the scattering process. The sound wave can gain or lose $h\nu$ of energy and the electron must lose or gain the same amount. Since $h\nu$ for the sound wave is small compared with the electron energies, the electron will not have its energy changed very much. It can be scattered from a point near the surface of the Fermi distribution only to another point near the same surface. In particular, at very low temperatures, when the sound vibration in question has its zero-point energy and can lose no more, the electron cannot gain energy at all but can only lose it.
3. The statistical distribution must be taken into account because of the Pauli exclusion principle. At very low temperatures the electron cannot gain energy because the lattice vibration has no energy to lose. The electron can then only be scattered if it can lose energy to the lattice. But at such very low temperatures, the surface of the Fermi distribution is quite sharp. Practically all the states of energy lower than the one from which an electron is to be scattered are occupied. Hence, the electron is practically not scattered at all and the resistance reaches very low values.

The detailed analysis of these three restrictions, the conservation of propagation vector, the conservation of energy, and the Pauli exclusion principle, leads directly to a proportionality of the resistance to the fifth power of the temperature in the limit of very low temperatures.

Thus in the two temperature limits of high temperature and very low temperature, the behavior of electrical resistance of ordinary metals is rather well understood on the basis of general principles. For high enough temperatures the resistance is proportional to the temperature, and for low enough temperatures is proportional to the fifth power of the temperature. The intervening region shows the peculiarities characteristic of the individual metals, and it has long been recognized that no one-parameter curve can describe the resistances of all pure metals.

Until recently there have not been extensive measurements over a wide enough region to permit detailed

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