indicate qualitative agreement with the expected distributions predicted by the statistical models of the nucleus. The various statistical treatments for the heavier elements (A > 50) predict a level spacing above a few Mev excitation energy which is very small compared to the natural widths of the levels.

On broad general lines it can be seen that the level densities of the residual nuclei, vanadium, chromium, and manganese, increase with increasing excitation of the residual nucleus. The general tendency of the intensity distributions of these three elements is a gradual decrease in the intensities of the groups as a function of increasing group energy. This is in qualitative agreement with the distributions predicted by Weisskopf and Ewing.9

The authors are indebted to Mr. L. M. Diana, Mr.

⁹ V. F. Weisskopf and D. H. Ewing, Phys. Rev. 57, 472 (1940).

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TABLE I. Energy levels (probable error ± 0.02 Mev).

Mg^{24}	Mg^{25}	Mg ²⁶	Mg	V ⁵¹	Cr ⁵²	Cr	Mn ⁵⁵
1.38	0.61	1.83	3.54	0.33	1.45	0.48	0.13
4.13	1.62	2.96	4.71	0.48	2.43	0.81	1.00
4.24	1.98		5.03	1.16	2.99	2.69	1.30
	2.56			1.84		2.79	1.56
	2.76			2.22		3.20	1.91
	3.41			2.43		3.46	2.27
	3.91			2.65		3.51	2.42
				3.11		3.65	2.59
				3.41		3.80	2.77
				3.58		3.99	2.85
				3.83		4 07	3.05
				3.96		4 78	3 21
				4 00		1.70	3 31
				4.07			2 12
				4.97			3.44
							3.04

K. B. Rhodes, Mr. R. F. Weise, Mr. E. M. Perkins, and Mr. R. A. Barjon for their collaboration in this project.

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Nuclear Energy Levels in the Region of the 28-Neutron Shell^{*,†}

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Energy levels of the titanium nuclei of mass numbers 47, 48, 49, 50, and 51 have been obtained by measuring the ranges of proton groups from the reactions Ti(d, p)Ti. With the use of enriched isotopes, it has been possible to assign thirty observed O values to their proper reactions. Good agreement was found in most cases in which results of this study could be compared with the results of others.

The data have been analyzed in terms of the strong spin-orbit coupling shell model of nuclear structure. A study of the binding energy of each residual nucleus and the energy of its first excited state revealed definite evidence for the existence of a closed shell at 28 neutrons.

I. INTRODUCTION

F the well-known magic numbers, the evidence for particular stability is poorest in the cases of 20 and 28. Mayer¹ did not expect the shells at 28 neutrons or protons to be strongly marked. Harvey² found a change in neutron binding energy of about 1 Mev, but the decrease apparently occurred between 29 and 30 neutrons, rather than between 28 and 29. Collins, Nier, and Johnson^{3,4} have reported discontinuities in the binding energy surface at 20 and 28 neutrons and protons.

The known excited states of nuclei have been studied by Pollard⁵ and by Scharff-Goldhaber⁶ in attempts to

- [‡] AEC Predoctoral Fellow.
 ¹ M. G. Mayer, Phys. Rev. **78**, 16 (1950).
 ² J. A. Harvey, Phys. Rev. **81**, 353 (1951).
 ³ Collins, Nier, and Johnson, Phys. Rev. **87**, 236 (1952).
 ⁴ Collins, Nier, and Johnson, Phys. Rev. **86**, 408 (1952).
 ⁵ E. C. Pollard, Nucleonics **2**, No. 4, 1 (1948); Phys. Rev. **82**, (1951).
 ⁶ (1951): and ONE Progress Report January 30, 1951 (up.

326 (1951); and ONR Progress Report, January 30, 1951 (unpublished).

observe regularities which might be attributed to shell structure. The most noticeable regularity which has been found is that the energy of the first excited state as a function of the number of protons or neutrons in the nucleus reaches a series of maximum values at closed shells. Figures 1 and 2 are plots of the first excited states which are known for light elements as functions of proton and neutron number. These plots show definite evidence for shell effects at 8 and 20 nucleons. Data of this sort are scare in the regions of the higher magic numbers.

The present investigation is concerned with the expected shell at 28 neutrons; a companion investigation⁷ is concerned with the 28-proton shell. The element titanium is particularly suited for this problem since it has stable isotopes containing 24, 25, 26, 27, and 28 neutrons. Analysis of the proton groups from the series of reactions Ti(d,p)Ti allows the study of the binding energies and energy levels of a series of nuclei having a constant number of protons and a continuous series of neutron numbers, 25 through 29. Any effects on the

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[†] Assisted by the joint program of the ONR and AEC. ‡ AEC Predoctoral Fellow.

⁶ G. Scharff-Goldhaber, Phys. Rev. 87, 218 (1952).

⁷ D. C. Hoesterey, thesis, Yale University (1952) (unpublished).

binding energy or energy level pattern of the series of nuclei from the expected closing of the $1f_{7/2}$ shell at 28 neutrons would be expected to appear in this study, since the constancies of the mode of excitation and the proton number make it plausible to interpret any variation solely in terms of the neutron number.

Pertinent experimental information concerning the titanium nuclei is not plentiful. The masses of the stable nuclei have been measured by Collins, Nier, and Johnson.⁴ Harvey² has reported *Q* values for the ground state (d, p) reactions on Ti⁴⁶, Ti⁴⁷, and Ti⁴⁸; these were obtained by measuring the ranges in aluminum of protons from a natural titanium target observed at 30° to an incident beam of 14-Mev deuterons. The Ti⁴⁸ ground-state peak was identified by its intensity; the other Q values were assigned on the basis that lighter isotopes should have higher energy ground-state groups.

Energy levels are known from radioactivity studies for only two of the titanium nuclei. The excitation energies of Ti⁴⁸ were first studied by Pollard,⁸ using the reaction $Sc^{45}(\alpha, p)Ti^{48}$; levels were found at 1.1 ± 0.4 and 2.3 ± 0.4 Mev. The decay scheme of V⁴⁸ has been investigated by Peacock and Deutsch,9 by Robinson, Ter-Pogossian, and Cook,¹⁰ and by Green, Ticho, and Richardson.¹¹ All of these experiments have yielded evidence for gamma-rays of 1.32 and 0.99 Mev, indicating that the positron transition of V48 leads to the second excited state of Ti⁴⁸, 2.31 Mev above the ground state. The first excited state may lie at either 0.99 or 1.32 Mev; both results are consistent with Pollard's value of 1.1 ± 0.4 Mev. The decay scheme of Sc⁴⁸ has been studied by Peacock and Deutsch⁹ and by Hamermesh et al.¹² The results of the latter group show evidence for three gamma-rays, one of energy 1.33 Mev and two of energy about 0.98 Mev. These results are consistent with the above interpretation of the positions of the excited states of Ti⁴⁸ and indicate in addition that the third excited state lies in the region of 3.3 Mev.

The other titanium nucleus whose energy levels are are known is Ti⁴⁶. A number of investigations¹³ indicate that the first excited state may be at either 0.89 or 1.12



FIG. 1. First excited states in Mev vs proton number up to Z=20. Mass numbers of nuclei are given beside level positions.

⁸ E. C. Pollard, Phys. Rev. 54, 411 (1938).
⁹ W. C. Peacock and M. Deutsch, Phys. Rev. 69, 306 (1946).
¹⁰ Robinson, Ter-Pogossian, and Cook, Phys. Rev. 75, 1099 (1949).
¹¹ Green, Ticho, and Richardson, Phys. Rev. 87, 195 (1952) and Phys. Rev. 86, 422 (1952).
¹² Hamermesh, Hummel, Goodman, and Engelkemeir, Phys. Rev. 87, 528 (1952).
¹³ C. L. Peacock and R. G. Wilkinson, Phys. Rev. 74, 297 (1948); E. T. Jurney, Phys. Rev. 74, 1049 (1948); F. T. Porter and C. S. Cook, Phys. Rev. 81, 298 (1951).



FIG. 2. First excited states in Mev vs neutron number up to N=24.

Mev, but the second excited state is definitely established at 2.01 Mev.

II. EXPERIMENTAL METHOD

The experimental procedure used in this work has been employed before.¹⁴⁻¹⁶ The bombardments in which protons at 90° to the direction of the incident beam were measured were carried out in a chamber built by Martin.¹⁶ The arrangement is shown in Fig. 3. A different chamber, one with a gold window thick enough to stop the beam, was used to measure protons at 0° . The electronic circuits associated with the proportional counter have been described elsewhere.¹⁵⁻¹⁷

The technique used was the peaked proportional counter method. Pulses from the counter are amplified and then put through a biased diode discriminator and into a blocking oscillator for shaping before recording. If the discriminator is adjusted so that the pulse from every proton passing through the counter is recorded, integral curves will be obtained when the proton yield for a fixed number of deuterons is plotted as a function of range.¹⁸ If an appropriate discriminator adjustment is made so that only those pulses with amplitudes greater than a certain fixed value are recorded, the peak in the Bragg ionization curve may be used to restrict observation to a small interval of range; in this way nearly differential or "peaked" group spectra are obtained. High peaking corresponds to a small range interval and vice versa.



FIG. 3. Arrangement of apparatus for counting of protons at 90° to the direction of the incident beam.

¹⁴ W. O. Bateson, Phys. Rev. 80, 982 (1950); P. W. Davison, ¹⁵ B. Benson, Phys. Rev. **73**, 8 (1948).
 ¹⁶ A. B. Martin, Phys. Rev. **71**, 127 (1947); **72**, 378 (1947).
 ¹⁷ H. A. Schultz and E. C. Pollard, Rev. Sci. Instr. **19**, 617

^{(1948).} ¹⁸ M. S. Livingston and H. A. Bethe, Revs. Modern Phys. 9, 245 (1937).

TABLE I. Composition of titanium samples.^a

Per- cent	Natural Ti	46	47	Enriched 48	49	50
46	7.95	82.68±0.09	1.83 ± 0.03	0.16 ± 0.01	1.09 ± 0.03	1.25 ± 0.01
47	7.75	5.24 ± 0.04	82.05 ± 0.09	0.32 ± 0.01	1.21 ± 0.03	1.23 ± 0.04
48	73.45	10.39 ± 0.04	14.62 ± 0.06	98.90 ± 0.02	15.71 ± 0.05	10.99 ± 0.07
49	5.51	0.86 ± 0.02	0.81 ± 0.01	0.48 ± 0.01	77.27 ± 0.07	1.84 ± 0.03
50	5.34	$0.84{\pm}0.02$	0.69 ± 0.01	0.14 ± 0.01	4.71 ± 0.08	84.69 ± 0.04

^a The following impurities were also detected but in no case was the amount of the impurity greater than 0.08%: Al, Na, Si, Mg, Cu, Ca, Ag, Fe, Cd, Cr, Mn, and Sn.

In this work, integral curves were used to determine the mean ranges of the few groups sufficiently well isolated to allow such observation. In most cases the increased resolution obtainable from peaked data was required. The approximate Q values determined from peaked data (extrapolated range-extrapolated beam approximation) were corrected by means of an empirical correction curve based upon integral data for each reaction and upon the accurate O value¹⁹ for the ground-state transition in the reaction $O^{16}(d,p)O^{17}$, oxygen having been the chief contaminant in each target material.

The five stable isotopes of titanium occur in nature with the abundances²⁰ given in Table I. Through the United States Atomic Energy Commission, it was possible to obtain samples of titanium dioxide highly enriched in each isotope. The composition of these enriched samples²¹ is also given in Table I. The samples were in the form of finely ground TiO₂ powder. Thin targets were constructed by painting on thin gold foils small amounts from suspensions of the TiO₂ samples in ethyl alcohol.

III. RESULTS

In view of the large amount of data obtained in this work, only a few curves of results can be shown. Figure 4 shows the proton groups observed at 90° in the reac-



FIG. 4. Proton groups from $\text{Ti}^{48}(d, p)$ Ti⁴⁹ observed at 90° at medium peaking. Each group is labeled by the mass number of the target nucleus and the number of the excited state to which it is attributed; lettered groups are unassigned. The dashed curves show the resolution of multiple groups into their components.

tion $Ti^{48}(d,p)Ti^{49}$ in a single cyclotron run at medium peaking. In addition to the groups from the 48-49 reaction, groups from the ground state and first excited state transitions in the reaction $O^{16}(d, p)O^{17}$ are evident. The group labeled B may be due to the ground state of the $C^{12}(d,p)C^{13}$ reaction. Carbon was probably present in all targets, since ethyl alcohol was used in their construction. The *Q* value as computed for the carbon reaction is 2.85 ± 0.15 MeV, which is in fair agreement with the result of Buechner et al.,²² 2.729 ± 0.009 Mev. The assignment has been left uncertain since the group did not appear consistently in all the data.

Ti⁴⁸ was chosen as the target for the first investigation of the series because of the high purity of the sample. It was advantageous to locate the groups from the 48-49 reaction first, since each of the other enriched samples contained Ti⁴⁸ as its chief impurity. It was convenient to be able to recognize these groups in later work. In some of the other reactions they interfered with the group spectrum under study to some extent; it was then necessary to subtract the yield from the 48-49



FIG. 5. Proton groups from $Ti^{46}(d,p)Ti^{47}$ observed at 90°. Dashed curve represents yield from competing reaction $Ti^{48}(d,p)$ -Ti⁴⁹. The labeling system is explained in Fig. 4.

reaction to determine what was left from the desired reaction.

The proton groups observed at 90° from a target of enriched Ti⁴⁶ are shown in Fig. 5, the result of a single cyclotron run at medium peaking. Just inside the group of longest range, there is a group of small yield whose extrapolated range agrees with that of 48-0, the ground-state group from the 48-49 reaction. It has been assumed that this group is 48-0. On the basis of this assignment, the dashed curve has been drawn in Fig. 5 to represent the yield from the 48-49 reaction. This curve was calculated from a run on Ti⁴⁸ made under similar peaking conditions as were used for the 46-47 data; the calculation was based on equalizing the yields of group 48-0 in the two cases. It is noteworthy that the largest maximum of the dashed curve corresponds quite well with one of the maxima of the solid curve. In view of the uncertainties involved in matching yields, due in particular to the critical de-

 ¹⁹ Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. 22, 291 (1950).
 ²⁰ A. O. Nier, Phys. Rev. 53, 282 (1938).

²¹ These samples were prepared by the Stable Isotopes Research and Production Division of the Oak Ridge National Laboratory by the method of electromagnetic separation and were analyzed in their laboratory.

²² Buechner, Strait, Sperduto, and Malm, Phys. Rev. 76, 1543 (1949).

pendence of yield on peaking, the agreement has been accepted, and the peak so labeled has been assigned to group 48–1. Extrapolated ranges of the inner groups were obtained from a curve of subtracted results, determined by taking the difference between the solid and dashed curves of Fig. 5. In all cases these ranges were in reasonably good agreement with those obtained from Fig. 5, neglecting the effects of the Ti⁴⁸ contamination. This result is fortuitous; it means that the Q values for the inner groups are not strongly dependent on the accuracy with which the relative yields from Ti⁴⁶ and Ti⁴⁸ are known.

The same procedure of subtracting the 48–49 yield was used where necessary in the cases of the other three reactions. The only case in which almost no interference was encountered was that of $Ti^{50}(d,p)Ti^{51}$. Figure 6 shows the proton groups observed in this reaction at 90°. It is noteworthy that the first excited state group 50–1 lies very near the ground-state group 50–0; with no other isotope was this the case. The



FIG. 6. Proton groups from $Ti^{50}(d, p)Ti^{51}$ observed at 90°. The dashed curves show the resolution of a multiple group into its components. The labeling system is explained in Fig. 4.

group 50-2 is too wide to be a single group and is almost certainly double. The indication is also strong that there is a large group from the 50-51 reaction in the region of the oxygen groups. Some indication of group 50-1 appeared in each of several runs made under various peaking conditions, but better resolution than that shown in Fig. 6 was impossible. Nor could the probable double structure of 50-2 be resolved. Data taken at low peaking showed the presence of a very weak group whose Q values agreed with that of 48-0; the subtraction of the yield from the 48-49 reaction from the yield from the Ti⁵⁰ sample made no difference in the Q values of any of the Ti⁵⁰ groups.

The results of the entire study are summarized in Table II. In cases of severe interference the Q values are only estimates or have large probable errors.

IV. COMPARISONS

The ground-state Q values for the series of reactions Ti(d,p)Ti can be compared to those calculated from the masses of Collins, Nier, and Johnson⁴ (CNJ) and to

Reaction	Q-Mev	Ene	rgy level—Mev	Relative yield at 90°
${ m Ti}^{46}(d,p){ m Ti}^{47}$	$\begin{array}{c} 6.45 {\pm} 0.05 \\ 5.05 {\pm} 0.08 \\ 4.06 {\pm} 0.09 \\ 3.81 {\pm} 0.09 \\ 3.36 {\pm} 0.09 \\ 2.75 {\pm} 0.09 \\ 2.27 {\pm} 0.09 \end{array}$	Ti ⁴⁷	$\begin{array}{c} 0\\ 1.40\pm0.08\\ 2.39\pm0.09\\ 2.64\pm0.09\\ 3.09\pm0.09\\ 3.70\pm0.09\\ 4.18\pm0.09\end{array}$	1 3.4 1.3 3.2 1.7 5 5
${ m Ti}^{47}(d, p){ m Ti}^{48}$	$\begin{array}{c} 8.14 \pm 0.05 \\ 6.81 \pm 0.09 \\ 5.83 \pm 0.16 \\ 4.83 \pm 0.09 \\ 3.64 \pm 0.12 \\ 3.2 \\ 2.9 \end{array}$	Ti48	$\begin{array}{c} 0\\ 1.33 \pm 0.09\\ 2.31 \pm 0.16\\ 3.31 \pm 0.09\\ 4.50 \pm 0.12\\ 5.0\\ 5.3\end{array}$	1 1 <5 10 15 18 25
$\mathrm{Ti}^{48}(d,p)\mathrm{Ti}^{49}$	5.81 ± 0.04 4.46 ± 0.05 4.11 ± 0.07 3.40 ± 0.07 2.70 ± 0.05	Ti ⁴⁹	$\begin{array}{c} 0\\ 1.35{\pm}0.05\\ 1.70{\pm}0.07\\ 2.41{\pm}0.07\\ 3.11{\pm}0.05\end{array}$	1 9 6 4 12
Ti ⁴⁹ (<i>d</i> , <i>p</i>)Ti ⁵⁰	$\begin{array}{c} 8.62 \pm 0.05 \\ 7.04 \pm 0.06 \\ 5.6 \\ 4.48 \pm 0.06 \\ 3.74 \pm 0.09 \\ 3.23 \pm 0.10 \\ 2.63 \pm 0.10 \end{array}$	Ti20	$\begin{array}{c} 0\\ 1.58 \pm 0.06\\ 3.0\\ 4.14 \pm 0.06\\ 4.88 \pm 0.09\\ 5.39 \pm 0.10\\ 5.99 \pm 0.10\end{array}$	1 <1 24 24 19 33
Ti ⁵⁰ (<i>d</i> , <i>p</i>)Ti ⁵¹	4.11 ± 0.07 3.50 ± 0.12 2.96 ± 0.08 2.5	Ti ⁵¹	$\begin{array}{c} 0 \\ 0.61 \pm 0.12 \\ 1.15 \pm 0.08 \\ 1.6 \end{array} \right\}$	1.0 0.3 0.8

TABLE II. Energy levels of titanium nuclei.

those reported by Harvey.² This comparison is made in Table III. The agreements in all cases are very good, except for the reaction $Ti^{47}(d,p)Ti^{48}$.

In the study of this one discrepancy, the following considerations are pertinent: (1) If the discrepancy is due to an error in the CNJ masses, then there must be at least two errors of the same magnitude since the Qvalues are computed from mass differences and the differences on either side of the one in question lead to Qvalues in good agreement with the measured ones. The possibility of a systematic error in the mass measurements would appear to be very slight, especially in view of the use of different substances for the doublet comparison in each case. (2) Since Harvey's experiments were carried out using natural titanium, it is possible that the Q value he attributed to the 47–48 reaction should in fact be assigned to the 49–50 reaction. It

TABLE III. Comparison of ground-state Q values.

	Grou	Ground-state Q value—Mev					
Reaction	Collins, Nier, and Johnson	Harvey	Pieper				
$Ti^{46}(d, p)Ti^{47}$ $Ti^{47}(d, p)Ti^{48}$ $Ti^{48}(d, p)Ti^{49}$ $Ti^{49}(d, p)Ti^{50}$ $Ti^{50}(d, p)Ti^{51}$	6.41 ± 0.10 9.40 ± 0.11 5.75 ± 0.07 8.66 ± 0.06	6.51 ± 0.10 8.82 ± 0.4 5.92 ± 0.05	$\begin{array}{r} 6.45 \pm 0.05 \\ 8.14 \pm 0.05 \\ 5.81 \pm 0.04 \\ 8.62 \pm 0.05 \\ 4.11 \pm 0.07 \end{array}$				

TABLE IV. Binding energies of titanium nuclei in Mev.

Mass number	Neutron number	Binding energy of last neutron	Total binding energy	Binding energy per particle
46	24		397.80 ± 0.09	8.648 ± 0.002
47	25	8.68 ± 0.05	406.48 ± 0.07	8.649 ± 0.001
48	26	10.37 ± 0.05	416.85 ± 0.05	8.684 ± 0.001
49	27	8.04 ± 0.04	424.89 ± 0.07	8.671 ± 0.001
50	28	10.85 ± 0.05	435.74 ± 0.08	8.715 ± 0.002
51	29	6.34 ± 0.07	442.08 ± 0.10	8.668 ± 0.002

would be in good agreement with the other results, considering its rather large probable error. (3) Recently Harvey²³ has observed the inelastic scattering of 7.7-Mev protons from a number of targets, including natural titanium. The first excited level in the case of titanium occurred at 1.00 ± 0.05 Mev and the second level at approximately 2.4 Mev. (4) In the present work the low yield made the detection of the group of Q value 8.14 Mev quite difficult. Although a search made at both 90° and 0° for a longer range proton group produced no result, the only conclusion which can be drawn is that any such longer range group must have a yield less than about 15 percent of the yield of the group of O value 8.14 Mev.

The most reasonable conclusion from these considerations is that the true ground-state group in the reaction $Ti^{47}(d, p)Ti^{48}$ was not observed in the present study; the group assigned as 47-0 would then correspond to the first excited state transition. If this is the case, the energy level scheme would agree with that determined from the inelastic proton scattering work of Harvey; the ground-state Q value would be 9.13 Mev. Since this latter value is not in particularly good agreement with CNJ's value of 9.40 Mev, even this conclusion cannot be accepted without reservation. The radioactivity studies discussed earlier throw no light on this matter, since all the gamma-rays involved can be fitted with equal ease into the two schemes in which the group of Q value 8.14 Mev corresponds to the ground state or to the first excited state.

V. BINDING ENERGIES

From the well-established value for the binding energy of the deuteron,²⁴ 2.226 ± 0.003 Mev, and the Q values measured in this work, the binding energy of the last neutron in each of the nuclei Ti⁴⁷ through Ti⁵¹ can be calculated. In addition, if the mass value of one of the isotopes is assumed (e.g., that of Ti⁴⁸, which is the best known^{4,25}), it is also possible to calculate from the present results the total binding energy and binding energy per particle of each of the nuclei Ti⁴⁶ through

Ti⁵¹. It is of interest to examine all these values to determine whether there is any significant change in these binding energies as a result of the completion of the $1f_{7/2}$ shell at 28 neutrons. The information is given in Table IV.

The values of the binding energy of the last neutron show readily the odd-even effect, that nuclei containing even numbers of neutrons have higher neutron binding energies than those with odd neutron numbers. It is noteworthy also that the binding energy of the 29th neutron is 1.7 Mev less than that of the 27th neutron, while the difference between the binding energies of the 27th and 25th neutrons is only 0.6 Mev. The weak binding of the 29th neutron is a definite indication of the closing of the 28-neutron shell. The same feature is shown in the values of the binding energy per particle, although the odd-even effect is smoothed here. The difference in the two cases is essentially that the effect is attributed entirely to the last neutron in the one case and is spread among all the nucleons in the other. In either approach the change in binding with the addition of the 29th neutron is noticeable, although it is much smaller than the discontinuities observed in the regions of the higher magic numbers.² No use has been made of the semiempirical mass formula for comparison purposes, in view of its lack of accuracy in this region of the nuclide chart.26

VI. ENERGY LEVELS

A complete summary of the energy levels found in the present work is shown in Fig. 7. The most striking feature of this plot is the position of the first excited state as a function of neutron number; it rises slightly with the addition of the 27th and 28th neutrons and then drops sharply at 29. This behavior is in agreement with that previously noted at the lower magic numbers, although here the effect is not so marked.

Another way of plotting the results of these experiments is shown in Fig. 8, in which the neutron binding energy (Q plus 2.23 Mev) for each state is shown. It

	6 -					- <u>-</u>		
Energy	5 -					÷		
in	4 -		-2-	-2-		-8-		
Mev	3 ~			-9-		()		
	2 -			÷				
	1 -	(***** 4 (*****)	-5-	-3-		- <u>11</u> -	- - -	
	0 -							
		46	47	48	49	50	51	Mass number of Tinucleus
		24	25	26	27	28	29	Neutron number

FIG. 7. Energy levels of titanium nuclei. The long lines represent actual values of the energy levels; the short lines above and below are the probable limits of error. Solid lines in parentheses indicate that the existence of a level is known, but its accurate measure-ment was impossible. Excited states of Ti⁴⁶, as determined from radioactivity studies, are given as dotted lines; the parentheses in this case indicate that the first excited state may lie at either of the two values shown.

26 E. Fermi, Nuclear Physics (University of Chicago Press, Chicago, Illinois, 1950), p. 8.

²³ J. A. Harvey, Phys. Rev. 88, 162 (1952) and private communication. ²⁴ R. C. Mobley and R. A. Laubenstein, Phys. Rev. 80, 309

^{(1950).} ²⁵ H. E. Duckworth and R. S. Preston, Phys. Rev. 82, 468 Nuclear Theory (John Wiley and (1951); H. A. Bethe, Elementary Nuclear Theory (John Wiley and Sons, Inc., New York, 1947).

is noteworthy that the level pattern seems to show a series of regularities, in all states of binding energy less than about 7 Mev, in the states of even A nuclei, and in those of odd A nuclei. The ground state of Ti⁵¹ seems to have corresponding states in the other nuclei. Unfortunately, not much significance can be attached to these regularities at this stage.

The results of a recent study by Kinsey and Bartholomew²⁷ of the neutron capture gamma-rays from natural titanium (Ti⁴⁸ contributes 95 percent of the total capture cross section) can be combined with the present results concerning excited states of Ti⁴⁹. Gamma-rays were observed corresponding to ten transitions; four of these were in good agreement with the four excited levels of Ti⁴⁹ found in the present study. No gamma-ray was found corresponding to the groundstate transition, nor was any such ray expected since it would presumably involve electric octupole radiation. Figure 9 summarizes the situation.

Kinsey and Bartholomew's results indicate extremely strong transitions to the first two excited states of Ti⁴⁹. The relative intensities of the gamma-rays to the first and second excited states were 53 and 32, respectively; no other ray had a relative intensity greater than 5. The possibility that the first excited state is a $2p_{\frac{3}{2}}$ level is in accord with the strength of the transition, for the capture, gamma-ray would then be electric dipole radiation. Breit²⁸ has suggested that the approximate ratio of 2 to 1 in the intensities of these two transitions



FIG. 8. Neutron binding energies of all observed states.

²⁷ B. B. Kinsey (private communication).

²⁸ G. Breit (private communication).



FIG. 9. Energy levels of Ti⁴⁹. The figures at the left are the energy levels in Mev and relative yields at 90° of proton groups determined by Pieper using the reaction $Ti^{48}(d, p)Ti^{49}$; those next to the arrows are the energies in Mev and relative intensities of the gamma-rays determined by Kinsey and Bartholomew from the reaction $Ti^{48}(n,\gamma)Ti^{49}$. Tentative level assignments are also given for some states.

might be interpreted as an argument that the second excited state is a $2p_{\frac{1}{2}}$ level. He points out that for dipole radiation and pure s_i to p transitions, neglecting frequency differences, the theoretical ratio of 2 to 1 applies; since the two strongest lines have intensities in approximately this ratio, with the higher energy ray the stronger, and since the energy difference is nearly the same as for the ground-state doublet of Li7, there appears to be reason for speculatively considering the first two excited states of Ti^{49} as forming a *p*-neutron doublet. Should these level assignments be correct, the splitting of the p doublet in Ti⁴⁹ is 0.35 ± 0.09 Mev from the (d,p) work and 0.345 ± 0.008 Mev from Kinsey and Bartholomew's data. A situation which might be interpreted in a similar manner exists in the case of Ca⁴¹, which has been studied by Sailor²⁹ and by Kinsey, Bartholomew, and Walker.³⁰ Here the splitting (if such exists) is 0.53 ± 0.05 Mev, while the intensity ratio is 83 to 11.

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²⁹ V. L. Sailor, Phys. Rev. 75, 1836 (1949)

²⁰ Kinsey, Bartholomew, and Walker, Phys. Rev. 85, 1012 (1952).