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Production of Charged π-Mesons in H, D, C, Cu, and Pb by 381-Mev Protons*

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The differential cross section for the production of charged π-mesons at 90° from a 381-Mev proton beam has been measured in H, D, C, Cu, and Pb. Nuclear emulsions, imbedded in a tapered copper absorber, placed 90° below the internal circulating proton beam of the Nevis cyclotron detected mesons of both signs, from 0 to 125 Mev. The beam was monitored by means of the C^{11} activity produced in the target to determine the absolute cross section. The hydrogen and deuterium results were obtained by subtracting the carbon cross section from polyethylene and deuterated paraffin, respectively. Assuming the reaction in H is $p+p\rightarrow\pi^{+}+d$ with a $\cos^2\theta$ angular distribution, the total cross section for H at 381 Mev is calculated from the 90° value to be $(7.3\pm2.3)\times10^{-28}$ cm². The spectra for heavy nuclei are calculated with the assumptions that meson production occurs through nucleon-nucleon collisions and that the meson receives the maximum available energy in the c.m. system. Good agreement with the experimental results is found for matrix elements proportional to meson momentum. The π^+ production cross section increases with a dependence slower than $A^{\frac{2}{3}}$, whereas the π^- increases approximately as $A^{\frac{2}{3}}$. This asymmetry is also found in the large π^+/π^- ratio of 10 in carbon and about 25 in deuterium.

I. INTRODUCTION

A FUNDAMENTAL experiment related to understanding the role of π-mesons in nuclear forces is the study of meson production in various targets bombarded by a beam of high energy protons. The Berkeley group,[1] using the 184-inch synchrocyclotron, has investigated the production process in hydrogen as a function of meson emission angle at 340-Mev proton energy. They have also measured the 90° production cross section for carbon[2] and lead[3] at the same proton energy. Using the 385-Mev Columbia Nevis cyclotron we have extended this study of meson production to higher energies and to include additional elements. The elements investigated were hydrogen, deuterium, carbon, copper, and lead.

The hydrogen reaction involves the elementary nucleon-nucleon collision $p+p\rightarrow\pi^{+}+(n+p)$ or (d). The next simplest nucleus, deuterium, is very loosely bound, and the two nucleons are expected to interact independently with the incident proton. The nucleon-nucleon reactions in deuterium leading to charged meson production are

$$(a)\quad p+p\rightarrow\pi^{+}+p+n \quad\text{or}\quad p+p\rightarrow\pi^{+}+d;$$
$$(b)\quad p+n\rightarrow\pi^{+}+n+n;$$
$$(c)\quad p+n\rightarrow\pi^{-}+p+p.$$

The deuterium production cross section compared to the hydrogen cross section gives a measure of the relative magnitudes of the $(p\text{-}n)$ and $(p\text{-}p)$ interactions entering into π^+ meson production (reactions a and b). A comparison of the π^- and the π^+ cross section in deuterium will yield information about the relative magnitudes of the $(p\text{-}n)$ interaction entering into reaction (c) and the $(p\text{-}n)$ and $(p\text{-}p)$ interactions or reactions (a) and (b). The use of carbon, copper, and lead targets allows an investigation of meson production as a function of target atomic weight.

II. EXPERIMENTAL ARRANGEMENT

The mesons were produced by bombarding the target material to be studied with the internal circulating proton beam of the Nevis cyclotron. Charged mesons produced in the downward direction, at 90° to the beam, were detected in a nuclear emulsion imbedded in a tapered copper absorber placed $2\frac{1}{2}$ in. below the target.

The geometry of the experimental arrangement in relation to the cyclotron is shown in Fig. 1. The target holder, target, absorber block, and photographic plate were all mounted on a probe, the details of which are shown in Fig. 2. This arrangement enabled the target and plate to be changed after each exposure by withdrawing the probe through a vacuum lock. The position of the probe was such that the target was at a radius of 73.5 in., corresponding to a beam energy of 381 Mev, as determined from the magnetic field strength.

Ilford C-2 200-micron nuclear emulsions were chosen as detectors to make possible positive identification of mesons in regions of high background. The characteristic small-angle scattering and rapid grain density variation of a slow meson in the C-2 emulsion make the identification relatively simple for a trained observer, even with an extremely high background. The C-2 sensitivity was chosen because it has a maximum cutoff of about 10 Mev for π-mesons and 65 Mev for protons, thus eliminating high energy proton tracks from the background.

The nuclear emulsion technique has the important advantage of permitting a determination of the sign of

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[1] F. Cartwright, Ph.D. thesis, University of California Radiation Laboratory Report UCRL-1278, 1951 (unpublished); V. Peterson, Phys. Rev. 80, 136 (1950); Peterson, Iloff, and Sherman, Phys. Rev. 81, 647 (1951).

[2] C. Richman and H. A. Wilcox, Phys. Rev. 78, 496 (1950).

[3] M. Weissbluth, University of California Radiation Laboratory Report UCRL-568, 1950 (unpublished).

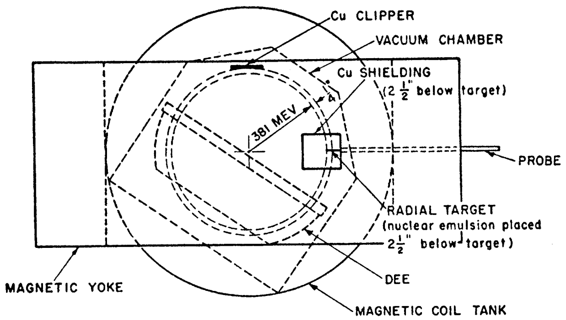

FIG. 1. Floor plan of Nevis cyclotron showing experimental arrangement for proton bombardment of radial target.

the charge and the type of meson detected, as well as the direction of incidence. Consequently, a separation of the π^+ and π^- mesons can be made.

The nuclear emulsions were imbedded inside a copper absorber block inclined at 10 degrees to the direction of the impinging mesons. Mesons of different energies traverse successively greater distances of absorber before they come to rest in the emulsion. Thus, the position of a meson ending in the plate is a measure of the distance it has traveled in the absorber, and hence of its energy. This method gives an energy detection interval from 0 to 125 Mev on one photographic plate, enabling a complete positive and negative spectrum to be obtained from a single exposure.

An experiment of this nature requires rectilinear trajectories from target to detector for mesons of all energies. To achieve this in a cyclotron, where the strong magnetic field causes mesons to travel in helical orbits whose radii of curvature vary with meson energy, the detector is placed directly below the target. In this position, mesons traveling straight down (90° from the proton beam) are moving parallel to the magnetic field lines and are unaffected by the field. To shield against unwanted helical trajectories, which correspond to other angles of emission from the target, a large shielding block of copper (14 in.×14 in.×5 in.) was placed around the absorber. The shielding block limits the meson trajectories arriving at the absorber plate to a bundle of mesons, containing all energies produced in the reaction, which travel in straight-line paths from the target to the plate.

The angular resolution of the apparatus was $\pm 5°$, so that the measurements included mesons emitted at $(90°\pm 5°)$ from the proton beam.

In the initial exposures, the target was in a vertical position and supported from above, as shown in Fig. 2. The vertical geometry has the advantage of utilizing the entire proton flux and giving relatively low background in the plates. The background tracks were almost exclusively limited to protons originating in the target or recoil protons due to neutrons emitted from the target.

The internal proton beam has a mean vertical amplitude between $\frac{1}{4}$ in. and $\frac{1}{2}$ in. Hence, mesons formed at the upper edge of the target had to traverse approximately $\frac{1}{4}$ in. to $\frac{1}{2}$ in. of target material before they could arrive at the detector. For light elements, this is not a serious energy resolution factor at high meson energies (above about 30 Mev), and does not appreciably affect the spectrum of an element like carbon. However, for hydrogen where the maximum energy of the mesons emitted at 90° in the laboratory system is 24 Mev, this effect considerably distorts the spectrum.

In order to improve the energy resolution, successive exposures were made with a target (thin in vertical dimensions) placed radially as shown in Fig. 1. In order to define the proton beam energy, a copper "clipper" was placed $\frac{1}{4}$ in. radially behind the target but displaced 90° in azimuth from the target, so that the neutrons emitted by the clipper could not strike the emulsion. However, this method introduced a severe, randomly oriented, proton background which obscured the low energy meson regions, making this technique unsuitable for hydrogen. The deuterium, copper, and lead spectra, which extend to much higher meson energies, were successfully obtained with this arrangement.

III. DETERMINATION OF CROSS SECTIONS

Let N be the number of mesons of energy E, detected in the corresponding region S of nuclear emulsion. Then, if $(d^2\sigma)/d\omega dE)$ is the differential cross section for meson production per target nucleus per unit solid angle and per unit energy interval at an energy E and an angle θ, it can readily be shown that

$$\frac{d^2\sigma}{d\omega dE}=\frac{A}{I\rho Ld}\times\frac{N}{S}\times\frac{r^2}{t[dE/dR]_E}, \tag{1}$$

where I = number of protons striking target, d = thickness of target, t = thickness of emulsion, L = Avogadro's number, ρ = density of target material, A = atomic weight of target, S = area of emulsion in which N mesons are found, $[dE/dR]_E$ = rate of energy loss in emulsion at energy E of creation, and r = distance from target to region S in emulsion.

Now, the yield Y of C^{11} (in targets in which carbon is present) via the reaction $C^{12}(p,pn)C^{11}$ is given by

$$Y=\sigma_c(I\rho Ld/A), \tag{2}$$

where σ_c, the cross section for this reaction, has been studied at Berkeley as a function of incident proton energy. Thus we may eliminate various parameters in Eq. (1), such as I, the number of protons incident upon the target, by a knowledge of Y, which may be obtained from an absolute monitoring of the positron decay of the C^{11}, as described below; thus, we obtain

$$\frac{d^2\sigma}{d\omega dE}=\frac{\sigma_c}{Y}\times\frac{N}{S}\times\frac{r^2}{t[dE/dR]_E}. \tag{3}$$

The evaluation of each of the terms involved in Eq. (3) is discussed below.

The C^{11} cross section σ_c at 381 Mev was found by extrapolating the known linear behavior at lower energies[4] to 381 Mev. The cross section value used was 0.036 ± 0.003 barn.

The number of C^{11} atoms formed in the target is inferred from the C^{11} positron annihilation radiation measured by inserting the target or monitoring foil (for the case of noncarbon containing targets) shortly after exposure, in a lead shield containing an end-windowed Geiger tube separated from the target by $\frac{1}{8}$ in. thick aluminum converter. Calibration of geometry and counter efficiency was made with the aid of a solution of Cu^{64} whose β-activity was compared to that of a RaD+E source calibrated by the Bureau of Standards. From the decay scheme of Cu^{64} and its branching ratio,[5] the number of positron decays per unit volume of solution was computed.

A known volume of this Cu^{64} solution was coated over a polyethylene target, approximating the activity distribution in a bombarded target; and from the number of positron annihilation counts observed, the absolute calibration of the monitoring system was obtained.

The monitored counting rate, which was always consistent with the known 20.5-minute half-life of C^{11}, was extrapolated back to the time the cyclotron was turned off and corrected for the short exposure time, leading to evaluation of the parameter Y.

The ratio N/S, the number of mesons per unit emulsion area ending in a region of the emulsion r cm from the target, was obtained by the usual meson scanning procedure. For this process a binocular microscope having a precision stage motion from which the position on the plate could be determined to within 0.01 cm was employed. High resolution oil immersion objectives were used.

The number of π^+ and π^- mesons ending in the C-2 emulsion is derived using the well-known phenomenological classification of meson endings into π, ρ, σ_1, and true σ-stars. The number of π^+ mesons is given directly by the number of π–μ events plus the number of σ_1 mesons, which are interpreted principally as π–μ events in which the μ-meson leaves the emulsion before traversing its full 600-μ range. The number of π^-, or star-producing mesons, must be corrected for the zero prong stars which would otherwise be classified as ρ-mesons. The number of π^- mesons is given by 1.37 times the number of observed σ-stars.[6]

The $\pi^\pm$ mesons were highly collimated, with less than 1 percent coming from directions other than that of the target.

In order to improve statistics and at the same time check on observer detection efficiency, plates were exposed in "sandwich." In all cases, results from separate plates scanned by different observers agreed within the statistical limits. As a further check, scanned plates were interchanged between two observers, and the areas were independently rescanned.

From the mean coordinates of the scanned region, the range of the π-mesons in the absorber was computed. The energy of the mesons was determined from available range-energy tables.[7]

The number of mesons observed in the plates in an energy interval dE and a solid angle interval $d\omega$ differs from the number of mesons produced in this energy and angular interval for several reasons. The largest correction is due to mesons being lost through interaction with the nuclei of the copper absorber block. Recent experimental evidence has shown that the $\pi^\pm$ meson interaction cross section is geometrical.[8] The interaction mean free path used for copper was 11.7 cm. This gives a correction of almost 50 percent for a 100-Mev meson.

Small angle Coulomb scattering in the absorber will cause mesons that would normally arrive at the plate to be scattered out. However, since the experiment is carried out with "poor geometry," to the first approximation, as many mesons scatter into the plate as would scatter out.

The third correction is due to π-mesons decaying in flight before they reach the emulsion. The correction is complicated by the fact that the mesons spend part of their time-of-flight in the copper, where they continuously lose energy and thus change their velocity. Since the absorber is only 2.5 in. from the target, the correction is very small ($\sim$2 percent) and can be neglected in view of the known meson mean lives.

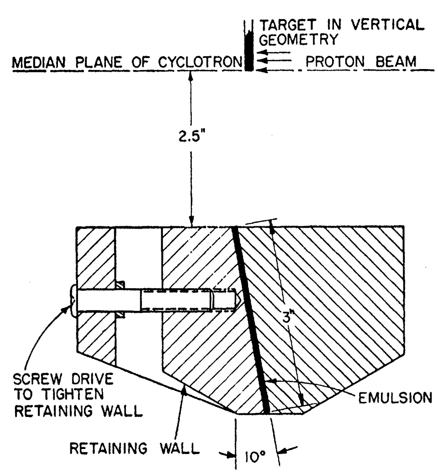

FIG. 2. Plan of probe head, containing nuclear emulsion and shielding material, which is inserted into the cyclotron.

[4] Aamodt, Peterson, and Phillips, University of California Radiation Laboratory Report UCRL-526, 1949).

[5] C. S. Cook and L. M. Langer, Phys. Rev. **73**, 601 (1948).

[6] F. L. Adelman and S. B. Jones, Phys. Rev. **75**, 1468 (1949).

[7] H. Bethe, Brookhaven National Laboratory Report T-7, 1949 (unpublished).

[8] Bernardini, Booth, Lederman, and Tinlot, Phys. Rev. **80**, 924 (1950); H. Bradner and B. Rankin, Phys. Rev. **80**, 916 (1950); Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951); L. Lederman (private communication).

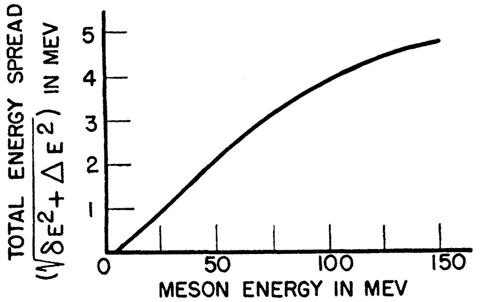

FIG. 3. Energy resolution of the apparatus *vs* meson energy.

The thickness of the emulsion after development was measured microscopically to within 1 micron. To correct for the shrinkage of the emulsion, a shrinkage factor was determined by measuring the emulsion thicknesses of test plates before development with a strain gauge and measuring them after development with a microscope. The thickness before development was measured to within 5 percent.

The rate of meson energy loss in emulsion, $[dE/dR]_E$, was calculated from an experimental range-energy curve for protons in Ilford C-2 emulsion,[9] and it was converted to mesons by using a ratio $(M_P)/(M_\pi)=6.65$. The resulting analytic range-energy relation used was $R=41.8E^{1.722}$, with E in Mev and R in microns.

IV. ENERGY RESOLUTION

The energy resolution in this experiment can be divided into the energy spread of the bombarding proton beam and the energy resolution of the detector.

Since the proton beam is an internal circulating beam, it is subject to energy degradation due to radial oscillations of the ions and multiple traversals of the targets.

The energy resolutions of the apparatus can be divided into two separate effects: (1) The finite vertical amplitude of the proton beam of the cyclotron which is between $\frac{1}{4}$ in. and $\frac{1}{2}$ in., so that mesons formed at the upper edge of the target had to traverse $\sim\frac{1}{4}$ in. of target material before they could arrive at the detector block. This gives a resolution that is energy dependent, being much poorer at lower meson energies. Above about 30 Mev this effect is negligible. (2) The detector itself has a finite resolution width due to range straggling and multiple scattering. The nuclear emulsion imbedded in the absorber measures the meson density as a function of depth of penetration of absorber. The projected path length of the meson (straight-line trajectory through the absorber) is taken to be the range of the meson; the energy is determined from a range-energy graph. Thus, in order to obtain the energy resolution, the distribution in range of absorber of a monoenergetic beam of mesons must be investigated.

Straggling introduces a range spread due to a statistical fluctuation of the number of collisions and energy loss per collision of particles traversing the absorber. The distribution is Gaussian, and its half-width is a function of meson energy.

Multiple scattering causes a particle's path to deviate from a straight line into a curved trajectory. This gives rise to a distribution in projected depth for a beam of monoenergetic particles. Since the effects of range straggling and multiple scattering are statistically independent, the two widths are added in quadrature to give the resultant resolution of the detector. A plot of resultant resolution *versus* meson energy is shown in Fig. 3. For meson energies lower than about 30 Mev, the meson energy spread due to the detector is seen to be negligible. Thus, a sensitive measure of the energy spread in the beam would be to find a reaction giving monoenergetic mesons of lower than 30 Mev for a monoenergetic proton beam and to measure the energy distribution of the detected mesons. In practice, the production of mesons in hydrogen provides such a line source of mesons of 24 Mev at 90 degrees to a 381-Mev proton. Since all energy spreads are due to factors which degrade the beam energy, one would expect to find a fairly sharp upper energy cutoff and a low energy smear due to the beam spread. However, in addition to the proton beam spread, the mesons also lose energy in the target. If we assume that the beam height is $\frac{1}{2}$ in., from the half-width of the hydrogen spectrum presented later (Fig. 6), the spread in beam energy resulting from multiple traversals and radial oscillation can be computed. This calculation gives a proton beam spread of the order of 10 Mev.

V. RESULTS

A. Carbon

A carbon target 0.3 g/cm² was placed in the vertical geometry described previously and exposed for 2.5 minutes in an average beam current of approximately 5×10^{-10} ampere. Figure 4 shows the 90° differential cross section $(d^2\sigma^+)/(d\omega dE)$ for the production of π^+ mesons, based on 550 $\pi-\mu$ decays.

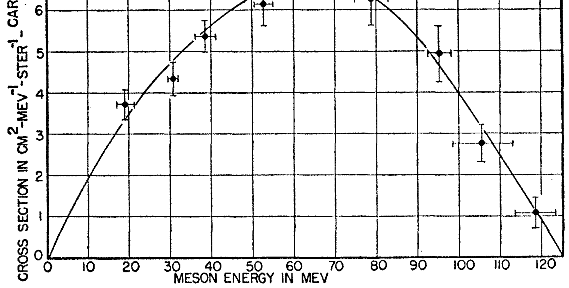

FIG. 4. Experimental results for the differential cross section for production of π^+ mesons in carbon at 90°±5° from a 381-Mev proton beam.

[9] Bradner, Smith, Barkas, and Bishop, Phys. Rev. **77**, 462 (1950).

Figure 5 shows the 90° differential cross section $(d^2\sigma^-)/(d\omega dE)$ for π^- mesons. This spectrum consists of 29 observed meson-produced stars found in the identical areas scanned for positives.

The π^+ cross section, integrated over meson energy, is $(d\sigma^+)/(d\omega)=(5.3\pm1.1)\times10^{-28}$ cm² steradian⁻¹, and the corresponding negative cross section is $(d\sigma^-)/(d\omega)=(4.2\pm1.0)\times10^{-29}$ cm^2 $steradian^{-1}$.

The positive spectrum has a peak of 7×10^{-30} cm^2 $steradian^{-1}$ Mev^{-1} at about 70 Mev.

The negative cross section has a peak of about 10×10^{-31} cm^2 $steradian^{-1}$ Mev^{-1} in the neighborhood of 40 Mev. The statistics are not good enough to permit an accurate determination of shape, but the peak appears to be at a lower energy than the positive spectrum peak and the cutoff is also lower than that of the positive spectrum.

The energy resolution indicated is the energy spread due to the width of the regions scanned in the plate. The uncertainties indicated in the spectral data are statistical probable errors involved in the counting of mesons. The uncertainty in the differential cross sections $(d\sigma)/(d\omega)$ involves, in addition, the uncertainty in the absolute calibration, including the C^{11} monitoring and plate thickness measurements.

B. Hydrogen

The hydrogen cross section was obtained by a subtraction technique using a polyethylene target $(CH_2)_n$. Since Eq. (3) gives the cross section per (CH_2) group, the hydrogen meson production cross section is obtained by subtracting the carbon cross section, previously obtained, and halving the difference. Fortunately, the differential cross section for hydrogen is relatively large so that the subtraction errors are small.

A 0.23-g/cm² polyethylene target was exposed under the same conditions as the carbon target and in the same vertical geometry. The 90° meson energy spectrum from hydrogen, consisting of 250 π–μ decays, is shown in Fig. 6, where the statistical uncertainty indicated includes the errors incurred in subtraction.

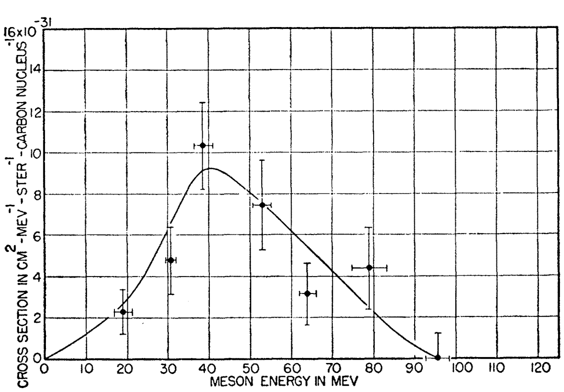

FIG. 5. Experimental results for the differential cross section for the production of π^- mesons in carbon at 90°±5° from a 381-Mev proton beam.

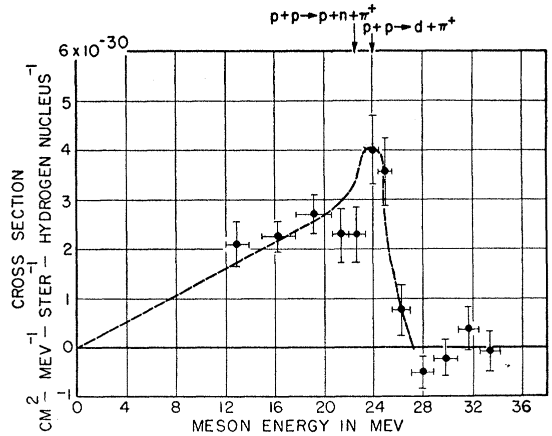

FIG. 6. Experimental results for the differential cross section for the production of π^+ mesons in hydrogen at 90°±5° from a 381-Mev proton beam.

The results for hydrogen show a pronounced maximum at 24 Mev and a sharp cutoff just above this energy.

The possible reactions leading to the production of π^+ mesons in hydrogen are $p+p\rightarrow\pi^++p+n$ or π^++d. In the latter case the mesons are produced in a line spectrum at the maximum energy available in the nucleon-nucleon collision. For the case of a 381-Mev proton striking a proton at rest and forming a π^+ meson and a deuteron, the maximum energy the meson would receive at 90° in the laboratory system would be 24 Mev. This curve is therefore consistent with the results at Berkeley,[6] which showed that the meson energy spectrum is mainly concentrated about the maximum energy constant with deuteron formation.

The curve cuts off sharply because the maximum energy the meson can receive is slightly above 24 Mev for the highest energy proton in the machine, and therefore, the poor resolution will not affect this cutoff. The curve falls off much less sharply on the low energy side because all the effects of the resolution, previously discussed, would tend to decrease the energy of the mesons.

Although the shape of the spectrum is distorted because of the poor resolution, the area under the curve should not be appreciably affected. Integration over the spectrum gives the cross section for production of mesons in hydrogen at 90°, $(d\sigma)/(d\omega)=(5.0\times1.2)\times10^{-29}$ cm^2 $steradian^{-1}$.

The results of angular distribution measurements at Berkeley[1] indicate that the mesons are emitted with a $\cos^2\theta$ distribution in the center-of-mass systems of the colliding nucleons. Using this angular distribution, the total cross section is found to be $\sigma=(7.3\pm2.3)\times10^{-28}$ cm² at 381 Mev.

C. Deuterium

A deuterated paraffin target, 60 mils thick, was prepared and mounted in the radial geometry. The composition of the paraffin was C_nD_{2n} (>96 percent) and C_nH_{2n} (<4 percent).[10] Consequently, for the process of meson production, the paraffin can be treated as pure C_nD_{2n}.

The deuterium contribution was obtained by subtracting the carbon differential cross section from the observed spectrum of C_nD_{2n}, and halving this result.

The positive spectrum $(d^2\sigma^+)/(d\omega dE)$, consisting of 174 π–μ decays, is shown in Fig. 7. It has a maximum in the region between 30 and 50 Mev of about 4.5×10^{-30} cm^2 Mev^{-1}. The integrated cross section $(d\sigma^+)/(d\omega) = (2.9\pm 1.2)\times 10^{-28}$ cm^2 $steradian^{-1}$.

In the same regions scanned for the positives, only $6\pi^-$ meson stars were observed. The statistical errors are too large to permit the drawing of a spectrum, but an estimate of the total 90° negative cross section $(d\sigma^-)/(d\omega)$ is about 1.1×10^{-29} cm^2 $steradian^{-1}$ D $nucleus^{-1}$.

D. Copper

A 1.4 g/cm^2 copper target was covered by a 13-mil polyethylene foil and exposed in the radial geometry.

The π^+ spectrum, of 157 π–μ decays, is shown in Fig. 8. The integrated cross section at 90°, $(d\sigma^+)/(d\omega) = (7.0\pm 1.7)\times 10^{-28}$ cm^2 $steradian^{-1}$. The total number of 11 π^- mesons observed was too small to permit the drawing of a spectrum, but an estimate of the integrated cross section at 90° is $d\sigma/d\omega = (1.6\pm 0.7)\times 10^{-28}$ cm^2 $steradian^{-1}$.

E. Lead

A 1.75 g/cm^2 lead target covered by a 13-mil polyethylene foil was exposed in the radial geometry for approximately 5 minutes in a beam current $\sim 10^{-10}$ amperes.

The π^+ spectrum, consisting of 173 π–μ decays, is shown in Fig. 9. The spectrum has a maximum at about

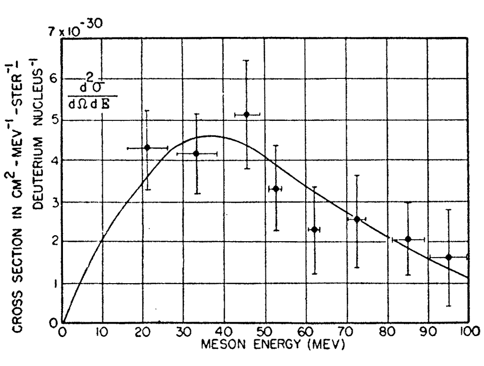

FIG. 7. Experimental results for the differential cross section for the production of π^+ mesons in deuterium at $90°\pm 5°$ from a 381-Mev proton beam.

[10] The deuterated paraffin was prepared by the Texas Oil Company, which gave a composition of 97.2 percent deuterium. A spectroscopic analysis of the particular batch used in the experiment was made by Professor T. I. Taylor of the Columbia University Chemistry Department, and he found a composition of 96 percent deuterium.

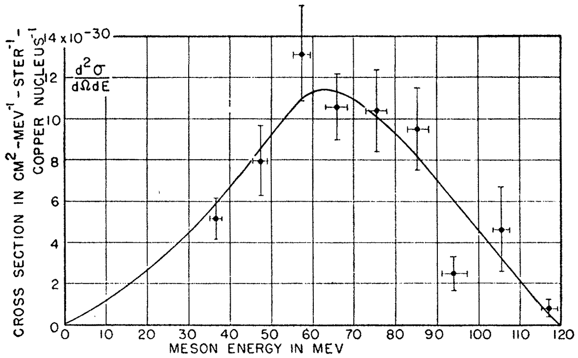

FIG. 8. Experimental results for the differential cross section for the production of π^+ mesons in copper at $90°\pm 5°$ from a 381-Mev proton beam.

70 Mev of 2.6×10^{-29} cm^2 $steradian^{-1}$ Mev^{-1}. The integrated cross section at 90° is $(d\sigma^+)/(d\omega) = (15\pm 3.7)\times 10^{-28}$ cm^2 $steradian^{-1}$.

In the identical region scanned for negatives, only 14 meson-produced stars were observed. The statistical errors are too large to give a reasonable spectral shape, so the results are not shown. An estimate of the integrated negative cross section at 90° is $(d\sigma^-)/(d\omega) = (3.5\pm 1.7)\times 10^{-28}$ cm^2 $steradian^{-1}$.

The experimental variation of π^+ production as a function of A is shown in Fig. 10(a). The positive meson cross section $(d\sigma)/(d\omega)$ at 90° increases much slower than the A variation that would be expected if each nucleon were to contribute equally or the $A^{\frac{2}{3}}$ variation if only nucleons in the surface were to contribute. The cross section for π^- protons *vs* A is shown in Fig. 10(b). In this case the cross section increases much more rapidly with A than for positive mesons.

VI. ANALYSIS OF MESON PRODUCTION

In order to calculate meson production cross sections in the heavy elements, the following basic assumptions are made:

(1) Meson production in a heavy nucleus occurs only through nucleon-nucleon collisions, i.e., the bombarding proton interacts with only one nucleon of the target nucleus. The only contribution of the remainder of the nucleons is to give the struck nucleon a momentum which is characteristic of the momentum distribution of the nucleus.

(2) In the meson production reaction there is a strong interaction between nucleons in the final state. This strong interaction requires that the meson receive the maximum available energy consistent with energy and momentum conservation. This is in accordance with the experimental results at Berkeley[1] which show that for a *p*-*p* collision meson production occurs principally via the reaction $p+p\rightarrow \pi^+ + d$. The assumption of strong interaction in the final state does not necessarily imply deuteron formation, since even an unbound final state in which the neutron and proton interact strongly will

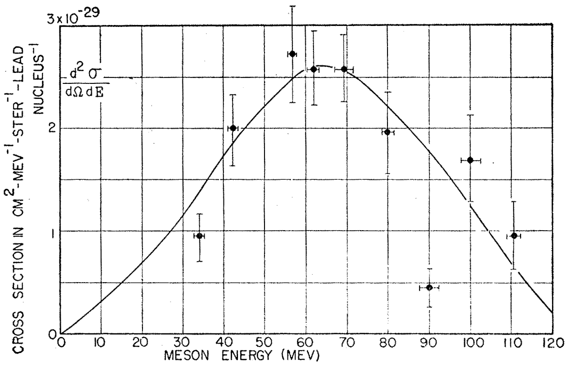

FIG. 9. Experimental results for the differential cross section for the production of π^+ mesons in lead at 90°±5° from a 381-Mev proton beam.

give the meson close to the maximum available energy and result in a high energy peak. This assumption considerably simplifies the analysis, since the kinematical reaction is now that of a two-body problem instead of that of the three-body problem. This assumption also automatically permits the Pauli principle to be satisfied for the case of meson production at 90°, as the final state nucleons will have a large energy in the laboratory system and will therefore be above any of filled momentum states of the heavy nucleus.

(3) There is no interaction between the escaping meson and the nucleus. The differential cross section for production of mesons in a nucleon-nucleon collision is given by

$$d\sigma/d\omega=(2\pi/\hbar)(|H_{AB}|^2/v_r)\rho, \qquad (4)$$

where H_{AB} is the matrix element that connects the initial state of two nucleons to the final state of two nucleons (or deuteron) plus a π-meson, ρ is the number of final states per unit energy interval, and v_r is the relative velocity of the two initial nucleons.

The matrix elements can be calculated by means of meson theory,[11] using the different interaction operators characteristics of the various types of meson fields. However, results based on meson theory are frequently in serious disagreement with experiment. For this reason, it seems safer, though less fundamental, to analyze the results phenomenologically, as did Watson and Brueckner.[12] Extending their treatment of meson production near threshold, a partial wave analysis is made which leads to six types of matrix elements that have the following energy and angular dependence in the c.m. system: (AI) constant; (AII) constant $\cos(\theta)$; (BI) $T_m^{\frac{1}{2}}$; (BII) $T_m^{\frac{1}{2}}\cos\theta$; (CI) T_m; and (CII) $T_m\cos\theta$, where T_m is the maximum available energy in the c.m. system consistent with deuteron formation and θ in the angle of emission of the meson in this system.

The energy dependence of the density of final states when combined with the relative velocity factor has been shown to be approximately equal to $\sqrt{T_m}$ [13] for the case of deuteron formation. If the energy and angle of emission of the meson in the c.m. system are known, then the energy of the colliding nucleons is completely specified. Thus, the differential production cross section for two colliding nucleons of energy E_g in the c.m. system can be calculated.

In order to determine the meson production cross section in a heavy nucleus the momentum distribution of the target nucleus must also be considered. The Fermi degenerate gas model was used in this calculation for this is the simplest model and is applicable to all nuclei of sufficiently high atomic weight.

The 90° laboratory differential cross section as a function of meson energy is calculated as follows. A meson energy at 90° in the laboratory system is selected. The corresponding meson energy T_m and angle θ in the c.m. system having energy E is calculated. The appropriate cross section is computed using T_m and θ and is multiplied by the probability of having a system with a c.m. energy E_g. The result is converted into a laboratory cross section and then integrated over all values of E_g consistent with kinematics. This results in a point by point numerical evaluation of the meson spectrum.

In view of the approximate nature of this calculation, in attempting to describe only the general characteristics of meson production in a heavy nucleus the momentum component of the target nucleon perpendicular to the direction of motion of the incident protons is neglected,

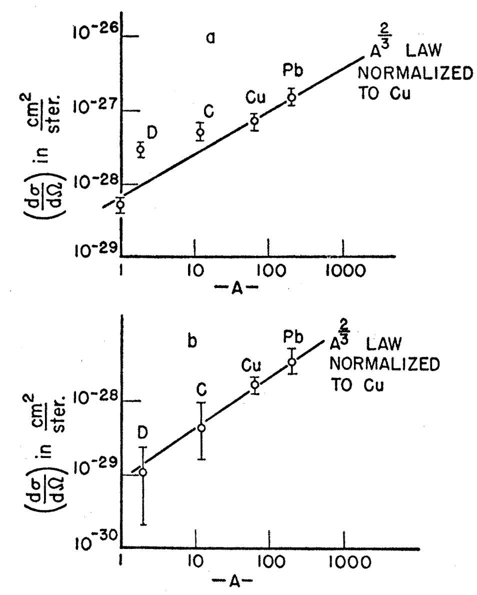

FIG. 10. (a) The differential π^+ production cross section at 90° to a 381-Mev proton beam is plotted *vs* the mass number A of the target. (b) The π^- production cross section at 90° to a 381-Mev proton beam is plotted *vs* the mass number A of the target.

[11] K. Brueckner, Phys. Rev. **82**, 598 (1951).
[12] K. Watson and K. Brueckner, Phys. Rev. **83**, 1 (1951).

[13] K. Brueckner, University of California Radiation Laboratory Report UCRL-630, 1950 (unpublished).

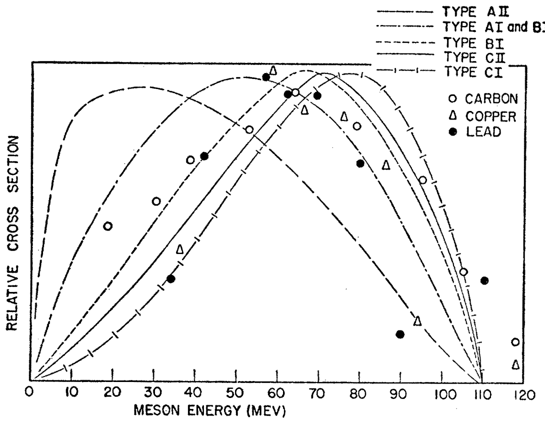

FIG. 11. The theoretical curves for the differential production cross section for the different matrix elements are compared to the experimental results for carbon, copper, and lead. The theoretical curves are arbitrarily normalized to the experimental curves to give the experimentally integrated cross section at 90°. The curves for the type AI and BII matrix elements are so similar that an average of both has been plotted.

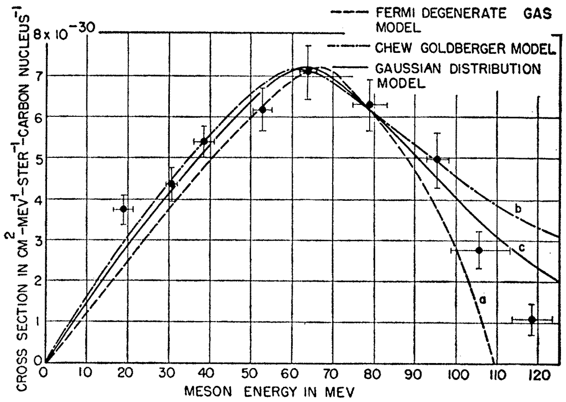

FIG. 12. The theoretical curves for the production of π^+ meson for three different nuclear models are compared with the experimental data for carbon; curve a is for a Fermi degenerate gas distribution with a 22-Mev cutoff, curve b is for the Chew-Goldberger model, curve c is for a Gaussian distribution with an average energy of 19.3 Mev.

since this has a relatively small influence on the calculated spectrum.

Utilizing matrix elements of types AI, AII, BI, BII, CI, CII combined with the energy dependence of the density of states and the probability factor for having a c.m. energy E_g, computations have been made for a proton bombarding energy of 385 Mev. The results of these calculations compared to the 90° experimental carbon, copper, and lead π^+ cross section data at 381 Mev are shown in Fig. 11.

The curves calculated from the matrix elements of type AI and BII were almost indistinguishable; therefore, only one curve was plotted. The curves were arbitrarily normalized to give the experimentally determined cross section at 90°. This arbitrary normalization obviates the need for correcting the production spectrum for meson reabsorption within the struck nucleus, providing this interaction is not strongly dependent on meson energy.

The calculations for the energy independent matrix element (AII) predict a maximum in the spectrum at too low a meson energy. The results with the AI and BII matrix elements are much too high at the low energy end of the meson spectrum, and the CI matrix elements are much too low at the low energy end of the meson spectrum. Matrix elements of type BI and CII give a better fit regarding the correct maximum of the spectrum and the low energy shape, but they do not explain the high energy tail. The results of the calculations have also been applied to deuterium and were reported previously.[14]

As a test of the sensitivity of these calculations to the assumption of a particular nuclear model, the calculations were repeated using the Chew-Goldberger[15] momentum distribution deduced from the pick-up effect and the Gaussian distribution having an average of 19.3 Mev, which also seems to fit the pick-up effect data.[16]

The results for the Fermi, Chew-Goldberger, and Gaussian distributions utilizing matrix element BI, arbitrarily normalized to fit the experimental data at 65 Mev, are shown in Fig. 12.

Recent evidence[15,16] indicates that the Chew-Goldberger distribution overemphasizes the high momentum tail and that the Gaussian distribution gives a better fit. This is confirmed in our calculations. The calculations fit the spectrum at the low energies and predict the correct maximum. They also give a high energy tail which is greater than the experimental results. This is to be expected since the matrix elements for all meson energies were assumed to vary as $T_m^{\frac{1}{2}}$. This relation holds only in the neighborhood of threshold, and the variation should be less rapid with increasing meson energy, an effect which would decrease the cross section for the production of very high energy mesons. Also, the recoil of the final nucleus has not been taken into account in the calculation. The neglect of this effect would raise the high energy end over its true value.

Since positive and negative mesons were treated symmetrically, the calculations should apply equally for negative mesons. However, the π^- spectra are *not* explained by the theory, as the maximum of the spectrum occurs at a lower meson energy than predicted. In addition, the absolute value of $(d\sigma^-)/(d\omega)$ is an order of magnitude below that of the corresponding π^+ cross section, whereas the assumption of equal n-p and p-p forces requires an *a priori* ratio $\pi^+/\pi^-=3$ for carbon. The large experimental π^+/π^- ratio ($\sim$25) in deuterium, which has been analyzed previously,[14] indicates that the main contribution to positive pions comes

[14] Passman, Block, and Havens, Phys. Rev. **85**, 370 (1952).
[15] G. Chew and M. Goldberger, Phys. Rev. **77**, 470 (1950).
[16] E. M. Henley and G. H. Huddlestone, Phys. Rev. **82**, 954 (1951).

from the reaction in deuterium: $p+p\rightarrow\pi^{+}+d$. However, the calculated ratios of the 90° π^{+} deuterium to the 90° π^{+} hydrogen cross sections are far too low (matrix elements CI, CII, BI, and BII predict a ratio of 2.2, 1.7, 1.6 and 1.2, respectively, whereas the experimental ratio is 5.6±2.3). This effect is difficult to understand, since it implies that a more energy-dependent excitation function than the $T_m^{\frac{1}{2}}$ or T_m matrix element dependence is required. This is not required, however, to explain the spectral shapes of the heavy elements. An alternative explanation, namely, that the two nucleons in deuterium do not act as independent particles in the process of meson formation, is also difficult to explain, since one might expect that, if the deuterium nucleus as a whole were to interact, a "tritium" peak (reaction $p+d\rightarrow t+\pi^{+}$) might be appreciable. Furthermore, the spectral shapes of the heavy elements were correctly predicted with the assumptions of independent nucleons. Certainly, in a deuterium nucleus where the binding is weak the assumption of independence should be better. Thus, one might suspect that negative mesons are produced by a different process from positive mesons.

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Excitation Function for Charged π-Meson Production in Hydrogen and Carbon by 345- to 380-Mev Protons*

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The 90° differential cross section for charged meson production in hydrogen and carbon is measured for incident proton energies 345 to 380 Mev. For hydrogen, the 90° π^{+} meson production energy spectra occur as peaks in the energy region predicted from the kinematics of the reaction $p+p\rightarrow\pi^{+}+d$. Ratios of these 90° π^{+} differential cross sections are $(d\sigma)/(d\omega)$ at 380 Mev: $(d\sigma)/(d\omega)$ at 365 Mev: $(d\sigma)/(d\omega)$ at 345 Mev = 2.6:1.6:1 with a statistical uncertainty of 15 percent. For carbon the π^{+} meson production spectra show an increase in the energy of the maximum of the broad distributions, with increasing incident proton energy. The $(\pi^{+})/(\pi^{-})$ ratio in carbon is found to be in the range (10±3):1 for all three proton energies studies. Qualitative agreement with these carbon π^{+} meson spectra is obtained from a phenomenological analysis based upon the assumptions of nucleon-nucleon collisions in which the target has a Gaussian momentum distribution.

INTRODUCTION

THE reaction $(p-p,\ \pi^{+})$ has been studied in great detail at Berkeley[1] with both liquid hydrogen targets and polyethylene-carbon subtraction techniques in conjunction with the external 340-Mev proton beam. The resulting π^{+} meson energy spectrum was observed as a clustering of the mesons about the maximum energy permitted by conservation laws, which indicates a strong nucleon-nucleon interaction in the final state of the reaction. Therefore, in the case of a triplet final spin state there is a strong possibility of deuteron formation. From the angular distribution of π^{+} mesons produced in $p-p$ collisions, Whitehead and Richman[2] conclude that the π^{+} production in the center-of-mass system has a $(0.07\pm0.07+\cos^{2}\theta)$ angular dependence.

Because of the difficulties associated with the quantum theoretical perturbation calculations of nucleon-nucleon meson production, one finds it more reliable if not as fundamental to apply the phenomenological theory of Watson and Brueckner.[3] In their method, the previously discussed qualitative features of the experimental results follow from the principles of conservation of parity and angular momentum together with an application of the exclusion principle, all contingent upon the pseudoscalar nature of the π^{+} meson. An important test for this theory is the dependence of the nucleon-nucleon meson production cross section upon the incident proton energy, since the energy dependence of the transition operator for this process is directly related to the type of meson-nucleon interaction assumed.

Experiments concerning the production spectra of charged mesons in carbon and the heavy elements[4]

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[2] Marian Whitehead and Chaim Richman, Phys. Rev. **83**, 97 (1951).
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[4] Richman, Weissbluth, and Wilcox, Phys. Rev. **85**, 161 (1952); Block, Passman, and Havens, Phys. Rev. **83**, 167 (1951).