interval has been calculated. The corresponding meson angles in the laboratory system are $95^{\circ}$ and $60^{\circ}$. The ratio of the cross sections at these two angles are

$$
\frac{\left(d \sigma / d \Omega_{\pi^{\circ}}\right)_{60^{\circ}}}{\left(d \sigma / d \Omega_{\pi^{\circ}}\right)_{95^{\circ}}}=1.45 \pm 0.25
$$

The corresponding ratio and angles in the center-of-mass system are

$$
\frac{\left(d \sigma / d \Omega_{\pi^{\circ}}\right)_{75^{\circ}}}{\left(d \sigma / d \Omega_{\pi^{\circ}}\right)_{110^{\circ}}}=1.0 \pm 0.2 .
$$

The results of the excitation function and the absolute cross sections are in reasonable agreement with the results of Panofsky, Steinberger, and Steller. ${ }^{2}$ They do not quote results on the angular distribution for a hydrogen target so that no comparison can be made.

The authors wish to express their gratitude to Professor J. W. DeWire for measuring the spectrum of the spread out beam, to Professor B. D. McDaniel for his assistance in measuring the efficiency of the $\gamma$-ray counter, and finally to Professor R. R. Wilson for his continued interest and guidance throughout the course of the experiment.

# Production of $\pi^{\circ}$ Mesons in Hydrogen and Deuterium by High Energy $\gamma$-Rays* 

G. Cocconi and A. Silverman<br>Cornell University, Ithaca, New York<br>(Received August 27, 1952)

The angular distribution of the $\pi^{\circ}$ s and the dependence of the cross sections on the energy of the $\gamma$-rays have been studied for the reactions

$$
\begin{aligned}
& \gamma+P \rightarrow \pi^{\circ}+P, \\
& \gamma+P \rightarrow \pi^{\circ}+D(P, N) .
\end{aligned}
$$

For the first reaction, the angular distribution in the center-of-mass system has the form $a+b \sin ^{2} \bar{\theta}$, with $a / b \simeq 1$, and the cross section is proportional to approximately the square of the excess energy of the $\gamma$-rays above the threshold; $\sigma_{\mathrm{D}} / \sigma_{\mathrm{H}} \simeq 2$ at all energies and angles.

## INTRODUCTION

PREVIOUS investigations of $\pi^{\circ}$ production by $\gamma$ rays have used detection techniques involving coincidence measurements between two of the products of the reaction, i.e., coincidences either between the two decay $\gamma$-rays ${ }^{1}$ or between one of the decay $\gamma$-rays and the recoiling nucleon. ${ }^{2}$ The difficulties involved in the latter technique for angular distribution measurements have been discussed in the preceding paper. The measurement of coincidences between the two decay $\gamma$-rays offers certain advantages for angular distribution measurements, but it has the rather serious disadvantage that the counting rates are quite small. As a consequence, the measurements on hydrogen and deuterium, particularly using the usual subtraction technique, become difficult to do with any reasonable statistical accuracy. We have, consequently, been led to investigate the production of $\pi^{\circ}$ mesons on hydrogen and deuterium by detecting only one of the decay $\gamma$-rays and thus increasing the counting rates by approximately a factor of 50 over the rates obtained by detecting both $\gamma$-rays in coincidence. The increased counting rate is purchased at the cost of some loss in angular definition,

[^0]since the decay $\gamma$-ray does not necessarily preserve the direction of the $\pi^{\circ}$. However, this disadvantage turns out to be not too serious because, as shown later, the $\gamma$-ray angular distribution reproduces quite faithfully the $\pi^{\circ}$ distribution for the energies with which we are concerned.

## APPARATUS

The experimental arrangement is shown in Fig. 1. The targets used were of cylindrical shape, 2 in . in diameter, 2 in . long, and made of $\mathrm{H}_{2} \mathrm{O}, \mathrm{D}_{2} \mathrm{O},\left(\mathrm{CH}_{2}\right)_{n}$, and C . The results for H and D were obtained with the subtraction method. The measurements consisted in recording the coincidences $(B+C-A)$ between crystals $B$ and $C$ with crystal $A$ in anticoincidence (hereafter called threefold coincidences) and coincidences ( $B+C$ $+D-A$ ) between crystals $B, C$, and $D$ again with crys$\operatorname{tal} A$ in anticoincidence (hereafter called fourfold coincidences) at various angles $\theta$ and for various maximum energies of the bremsstrahlung $\gamma$-ray beam of the Cornell sychrotron. An aluminum absorber $2 \frac{1}{4}$ in. thick was placed between the third and fourth crystal in order to make the fourfold coincidences insensitive to low energy $\gamma$-rays. The efficiencies of the threefold and fourfold coincidences as a function of $\gamma$-ray energy for a Pb converter $7 \mathrm{~g} / \mathrm{cm}^{2}$ thick are shown in Fig. 2. They were measured for $\gamma$-ray energies of 190,140 , and 100

Mev, using the monochromatic $\gamma$-ray beam available at this laboratory. ${ }^{3}$ The efficiency of the threefold coincidences was also calculated using the Monte Carlo method of Wilson. ${ }^{4}$ The experimental results are consistently lower by about 10 percent than the calculated ones. This is at least in part due to the fact that in the calculations the scattering was neglected. The efficiencies used in the analysis of the results are shown by the dotted curves in Fig. 2.

The evidence that the coincidences recorded were due to $\gamma$-rays arising from the decay of neutral mesons consists of the following observations:
(a) The dependence of the coincidence rates on the thickness and the material of the converters indicates that more than 90 percent of all counts were due to $\gamma$-rays.
(b) The counting rates decreased by more than a factor of 100 as the maximum beam energy was reduced from 310 Mev to 150 Mev .
(c) Assuming that all the counts arise from decay of $\pi^{\circ}$ mesons, one obtains a cross section for $\pi^{\circ}$ production on hydrogen of $1.5 \times 10^{-28} \mathrm{~cm}^{2}$, which is in good agreement with previous results. ${ }^{1,2}$
(d) The excitation function (see Fig. 4) measured on hydrogen is in reasonable agreement with previous results for $\pi^{\circ}$ production by $\gamma$-rays. ${ }^{2}$
(e) The $\gamma$-rays responsible for the fourfold coincidences have energies greater than about 80 Mev , the approximate threshold for this detector (see Fig. 2).

We believe that the above evidence is sufficient to establish that most of the coincidences recorded were due to decay $\gamma$-rays from neutral mesons. As far as we know, the only other process that could give rise to $\gamma$-rays scattered by H and D with properties similar to the ones listed before is Compton scattering. We have tried to measure the Compton scattering cross section of hydrogen at the $\pi^{\circ}$ threshold ( 150 Mev ) and at $90^{\circ}$ from the beam, and we found $d \sigma / d \Omega \leqslant 10^{-31} \mathrm{~cm}^{2}$ steradian ${ }^{-1}$. Thus, unless the cross section increases very rapidly with $\gamma$-ray energy so as to become several hundred times larger than the Thompson cross section, this process will contribute no appreciable background to our measurements. ${ }^{5}$

[^1]

## discussion of the results

## 1. Hydrogen

Before analyzing the experimental data, it is necessary to discuss the relation between the $\pi^{\circ}$ angular distribution and the angular distribution of their decay $\gamma$-rays. Let us suppose that, for a primary $\gamma$-ray of energy $\omega$, the cross section in the center-of-mass system for the reaction

$$
\gamma+P \rightarrow \pi^{\circ}+P,
$$

is given by the expression

$$
d \sigma_{\pi^{\circ}} / d \Omega=A+B \cos ^{2} \bar{\theta}
$$

where $\bar{\theta}$ represents the angle in the center-of-mass system at which the $\pi^{\circ}$ is emitted. Let $E$ and $\theta$ be the energy and the angle in the laboratory system of a decay $\gamma$-ray arising from the disintegration of the $\pi^{\circ}$. If $N(E, \theta) d E d \Omega$ is the intensity of the decay $\gamma$-rays with energy $E$ and angle $\theta$ in the intervals $d E$ and $d \Omega$, then


Fig. 2. Efficiency $\eta(E)$ of the threefold and fourfold coincidences as a function of the energy $E$ of the $\gamma$-rays. The solid line describes the results obtained with the Monte Carlo method. The circles represent the measured efficiencies. The dotted lines correspond to the expressions,

$$
\eta_{T}(E)=0.47\left[1-\exp \left(-\frac{E-25}{40}\right)\right]
$$

and

$$
\eta_{F}(E)=0.40\left[1-\exp \left(-\frac{E-81}{43}\right)\right],
$$

used in the calculation.


Fig. 3. Intensity $I(\theta)$ of threefold and fourfold coincidences as a function of the angle $\theta$ in the laboratory system. The curves show the expected $I(\theta)$ for various angular distributions of the $\pi^{\circ}$ in the center-of-mass system, for the reaction $\gamma+P \rightarrow \pi^{\circ}+P$. (Energy of the $\gamma$-rays $=280 \mathrm{Mev}$.) The curves have been normalized to unity at $0^{\circ}$. The ordinates of the experimental points have been multiplied by a constant in order to give the best fit to the $2+3 \sin ^{2} \bar{\theta}$ distribution.
one can show that $N(\theta, E)$ is given by ${ }^{6}$

$$
\begin{aligned}
& N(\theta, E)=\frac{1}{\bar{q} \gamma_{c}\left(1-\beta_{c} \cos \theta\right)}\left\{A+\frac{B \sin ^{2} \theta}{2 \gamma_{c}^{2}\left(1-\beta_{c} \cos \theta\right)^{2}}\right. \\
& \left.\quad+\frac{3}{2} B\left[\frac{1}{\bar{\beta}}-\frac{\mu^{2}}{2 q \gamma_{c}\left(1-\beta_{c} \cos \theta\right) E}\right]^{2}\left[\left(\frac{\cos \theta-\beta_{c}}{1-\beta_{c} \cos \theta}\right)^{2}-\frac{1}{3}\right]\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
M & =\text { mass of the nucleon, } \\
\mu & =\text { mass of the meson } \\
\bar{q} & =\left(\bar{\epsilon}^{2}-\mu^{2}\right)^{\frac{1}{2}} \text { with } \bar{\epsilon}=\frac{2 M \omega+\mu^{2}}{2\left(2 M \omega+M^{2}\right)^{\frac{1}{2}}} \\
\bar{\beta} & =\bar{q} / \bar{\epsilon} \\
\beta_{c} & =\omega /(\omega+M) \\
\gamma_{c} & =1 /\left(1-\beta_{c}^{2}\right)^{\frac{1}{2}}
\end{aligned}
$$

The intensity of the $\gamma$-rays observed at the angle $\theta$ is therefore given by

$$
I(\theta)=\int_{E_{\min }}^{E_{\max }} N(\theta, E) \eta(E) d E
$$

where

$$
E_{\min }=(1-\bar{\beta}) \frac{\bar{\epsilon}}{2 \gamma_{c}\left(1-\beta_{c} \cos \theta\right)}
$$

and

$$
E_{\max }=(1+\bar{\beta}) \frac{\bar{\epsilon}}{2 \gamma_{c}\left(1-\beta_{c} \cos \theta\right)}
$$

[^2]are, respectively, the minimum and the maximum energy of the photons emitted at $\theta$, and $\eta(E)$ represents the efficiency of the detector for recording $\gamma$-rays of energy $E$; the expressions used for $\eta(E)$ are those given in the caption of Fig. 2. The solid curves in Fig. 3 are the calculated $I(\theta)$ for both the threefold and fourfold coincidences for $\omega=280 \mathrm{Mev}$ and for the following $\pi^{\circ}$ angular distributions:
\[

$$
\begin{aligned}
d \sigma_{\pi^{\circ}} / d \Omega & =\text { constant } \\
& =\sin ^{2} \bar{\theta} \\
& =\cos ^{2} \bar{\theta} \\
& =2+3 \sin ^{2} \bar{\theta}
\end{aligned}
$$
\]

The value of 280 Mev chosen for the energy of the primary $\gamma$-ray represents the "average" $\gamma$-ray energy involved in this experiment. ${ }^{7}$ It is perhaps worth while observing that the results do not depend critically on the value of $\omega$, since a variation in $\omega$ affects them primarily through the value of the velocity of the $\pi^{\circ}$ in the center-of-mass system ( $\bar{\beta}$ ), and this velocity varies slowly with $\omega$.

The results for the threefold coincidences are essentially independent of the efficiency factor $\eta(E)$, since $\eta(E)$ is nearly constant over a large part of the relevant spectrum. One sees that, as stated in the introduction, the angular distribution of the decay $\gamma$-rays is quite similar to that of the $\pi^{\circ}$ for the case calculated, where $\bar{\beta}=0.807$. Actually the two distributions become identical for $\bar{\beta}=1$.

The experimental results for the angular distribution measurements are shown in Fig. 3. They are consistent with an angular distribution of the $\pi^{\circ}$ 's in the center-of-mass system of the following type:

$$
d \sigma_{\pi^{\circ}} / d \Omega=a+b \sin ^{2} \bar{\theta}, \quad \text { with } a / b \simeq 1
$$

In particular, the distribution $2+3 \sin ^{2} \bar{\theta}$ fits the data quite well. This result was predicted by Brueckner and Watson ${ }^{8}$ for a meson-nucleon state of angular momentum $J=\frac{3}{2}$ with isotopic spin $I=\frac{3}{2}$ or $\frac{1}{2}$. The same model gives quite good agreement with the experimental

TABLE I. Ratios $\sigma_{\mathrm{D}} / \sigma_{\mathrm{H}}$ and $\sigma_{\mathrm{C}} / \sigma_{\mathrm{H}}$ of the cross sections in deuterium carbon, and hydrogen, for production of $\pi^{\circ}$ s by $\gamma$-rays.

| Maximum <br> energ of <br> primary <br> $\gamma-$ rays <br> (Mev) | $\theta$ <br> (lab system) | $\sigma_{\mathrm{D}} / \sigma_{\mathrm{H}}$ | $\sigma_{\mathrm{C}} / \sigma_{\mathrm{H}}$ |
| :---: | :---: | :---: | ---: |
| 310 | $45^{\circ}$ | $1.76 \pm 0.20$ | $11.0 \pm 1.2$ |
| 310 | $70^{\circ}$ | $2.03 \pm 0.17$ | $10.0 \pm 0.7$ |
| 310 | $90^{\circ}$ | $1.91 \pm 0.09$ | $9.5 \pm 0.5$ |
| 310 | $120^{\circ}$ | $1.53 \pm 0.17$ | $9.2 \pm 1.0$ |
| 310 | $135^{\circ}$ | $9.0 \pm 1.6$ |  |
| 310 | $150^{\circ}$ | $2.11 \pm 0.20$ | $11.1 \pm 1.4$ |
| 273 | $90^{\circ}$ | $1.4 \pm \pm 1.3$ |  |
| 237 | $90^{\circ}$ | $1.88 \pm 0.37$ | $21.6 \pm 3.0$ |

[^3]excitation function and the ratio of the $\pi^{+}$to $\pi^{\circ}$ cross sections.
Kaplon ${ }^{9}$ and Francis and Marshak, ${ }^{10}$ using psuedoscalar theory in the weak coupling approximation including a Pauli term to describe the nucleon magnetic moment, obtained results for the $\pi^{+} / \pi^{\circ}$ cross section ratio and for the excitation function which are in fair agreement with the experiments. However, their calculated angular distribution seems to be in definite contradiction with the above results.
Figure 4 shows the results for the yield of $\pi^{\circ}$ 's as a function of the maximum energy of the bremsstrahlung beam. The solid curves are the expected yields under the assumption that $\sigma_{\pi^{0}}=K\left(\omega-\omega_{0}\right)^{n}$, where $\omega_{0}$ is the threshold energy. The different curves are calculated for $n=1,2$, and 3 .

One sees that, for hydrogen, $n$ is about 2.5 . This result is in fair agreement with the result of reference 2 in which $n=1.9 \pm 0.4$.

## 2. Deuterium

In column 3 of Table $I$ are given the ratios of the deuterium to hydrogen cross sections for various energies and angles. Within the accuracy of the measurements, $\sigma_{\mathrm{D}} \simeq 2 \sigma_{\mathrm{H}}$ independent of energy and angle. This result indicates that neutron and proton cross sections are quite similar and consequently that the strengths of the $\pi^{\circ}$ coupling to the proton and to the neutron are not very different. However, a quantitative measure of this coupling is not obtainable from the ratio $\sigma_{\mathrm{D}} / \sigma_{\mathrm{H}}$. The $\pi^{\circ}$ production cross section from deuterium cannot be considered as the simple sum of the cross sections from the proton and the neutron since interference effects can be present. On the other hand, the fact that the experimental ratio $\sigma_{\mathrm{D}} / \sigma_{\mathrm{H}}$ is found to be largely independent of energy and angle, leads one to infer that the interference does not contribute much at the energies where our measurements were performed.

## 3. Carbon

During the course of obtaining the results for hydrogen, the subtraction process provided some information on carbon. The yield curve for carbon is shown in Fig. 4. The production of $\pi^{\circ}$ 's by $\gamma$-rays on carbon appears to be a more slowly varying function of the $\gamma$-ray energy than for hydrogen. If the cross section versus $\gamma$-ray energy curve for hydrogen at energies above 300 Mev is less steep than for smaller energies, then the motion of the nucleons in carbon would tend to produce the

[^4]

Fig. 4. Excitation curves for H and C deduced from the coincicences observed at $90^{\circ}$ from the beam and assuming a distribution of the $\pi^{\circ}$ 's in the center-of-mass system proportional to $2+3$ $\sin ^{2} \bar{\theta}$. The calculated curves, referred to a constant energy content in the $\gamma$-ray beam, are obtained from the expression

$$
\text { yield }=\frac{K}{\omega_{\max }} \int_{\omega_{0}}^{\omega_{\max }}\left[\left(\omega-\omega_{0}\right)^{n} / \omega\right] d \omega
$$

The curves and the experimental points are normalized at 310 Mev.
observed effect. Another possible explanation involves the existence of charge exchange reactions in the carbon nucleus, according to the following schemes:

$$
\gamma+N \rightarrow \pi^{-}+P, \quad \pi^{-}+P \rightarrow \pi^{\circ}+N
$$

and

$$
\gamma+P \rightarrow \pi^{+}+N, \quad \pi^{+}+N \rightarrow \pi^{\circ}+P .
$$

However, since the energy dependence of the charge exchange reactions is not well known, and the effect of the motion of the nucleons is difficult to evaluate, a definite conclusion as to whether this explanation is adequate cannot be made.
In the last column of Table I are shown the ratios of the carbon to hydrogen cross sections at various energies and angles.

We are grateful to Professor A. Borsellino for the calculation of the angular distribution, to Professor B. D. McDaniel for his participation in making the efficiency measurements, and to Professor R. R. Wilson for making available to us some of the results of his Monte Carlo calculations.


[^0]:    * This work was supported in part by the ONR.
    ${ }^{1}$ Panofsky, Steinberger, and Steller, Phys. Rev. 86, 180 (1952).
    ${ }^{2}$ A. Silverman and M. Stearns, preceding paper [Phys. Rev. 88, 1225 (1952)].

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    ${ }^{5}$ It is, of course, possible that the Compton scattering has a resonance at some energy corresponding to a nucleon isobar level, and so its cross section is a very steep function of the energy. However, even in this case, the $\gamma$-ray emission must compete with meson emission and one would expect the meson widths to be larger than the $\gamma$-width approximately in the ratio of the mesic to the electric coupling constants.

[^2]:    ${ }^{6}$ We are indebted to Dr. A. Borsellino for the derivation of this formula.

[^3]:    ${ }^{7}$ By "average" $\gamma$-ray energy we mean an energy such that half of the coincidences are due to $\gamma$-rays with energies lower than it.
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[^4]:    ${ }^{9}$ M. F. Kaplon, Phys. Rev. 83, 712 (1951).
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