The Effect of Charge Symmetry on Nuclear Reactions*

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For a charge symmetric nuclear Hamiltonian, the operator which changes neutrons into protons and protons into neutrons (charge parity operator) commutes with the Hamiltonian and is therefore a constant of the motion. Since the charge parity operator anticommutes with the "3" component of the total isotopic spin, for nuclei with $T_3=0$ (self-conjugate nuclei) the charge parity is a good quantum number and in the absence of degeneracy the eigenstates of such nuclei have either odd or even charge parity. This leads to strong selection rules in nuclear reactions involving self-conjugate nuclei in the initial and final states which may reasonably be invoked to explain recent experimental results on such reactions. Since states of even total isotopic spin have even charge parity and states of odd total isotopic spin have odd parity, the selection rules arising from charge symmetry often coincide with those of charge independence and in such cases a definitive test of the charge independence hypothesis by the use of these selection rules is impeded. Some other applications of the charge symmetry principle are discussed.

THAT the neutron-neutron and proton-proton forces are equivalent apart from electromagnetic interactions is strongly indicated by the energy differences in the ground states and the general similarity of energy levels in the various mirror nuclei. It is the purpose of this note to point out that the assumption of such a charge symmetry of nuclear forces implies certain strong selection rules[1] which apply in a small but experimentally interesting group of nuclear reactions.

These selection rules arise in self-conjugate systems, that is, systems having equal numbers of neutrons and protons. For such systems under the assumption of charge symmetry the Hamiltonian is invariant under interchange of the neutron and proton space and spin coordinates. It is therefore possible to define an operator P which performs this interchange and can appropriately be called the charge parity operator.[2] It is clear that P is a constant of the motion with eigenvalues 1 and -1. The wave functions of the system can always be selected so as to be simultaneous eigenfunctions of the Hamiltonian and charge parity and therefore may be characterized as charge even or charge odd. Of course in the degenerate case where two states of opposite charge parity have the same energy a state of the system of this energy may be a superposition of charge even and charge odd states.

A formal exposition of the above described concept can be conveniently obtained in terms of the isotopic spin formalism. The operator P can be represented as a rotation in isotopic spin space of 180° about the 1 axis. That is, for a system of A particles the total isotopic spin $\mathbf{T}$ is given by

$$\mathbf{T}=\sum_{j=1}^{A}\boldsymbol{\tau}^{(j)}.$$

A rotation of 180° about the 1 axis is then given by

$$P=\exp\{i\pi T_1/2\}=\prod_{j=1}^{A}\exp\{i\pi\tau_1^{(j)}/2\}=i^A\prod_{j=1}^{A}\tau_1^{(j)},$$

since $\exp\{i\pi\tau_1^{(j)}/2\}=i\pi\tau_1^{(j)}$.

The charge symmetry of the Hamiltonian implies that it is a symmetric function of the A particles whose dependence upon the isotopic spin[3] can be expressed in terms of $\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}$ and $\tau_3^{(i)}\tau_3^{(j)}$. Since $[\boldsymbol{\tau}^{(i)}\cdot\boldsymbol{\tau}^{(j)}, P]=[\tau_3^{(i)}\tau_3^{(j)}, P]=0$, it is clear that P commutes with the Hamiltonian and is a constant of the motion. This property is not, in fact, restricted to self-conjugate systems. On the other hand, nontrivial applications of the concept are in fact so restricted. This arises from the fact that $[P, T_3]=2PT_3$ that is, P and T_3 anticommute. Thus a system can be in a simultaneous eigenstate of P and T_3 only for states of $T_3=0$, which corresponds to the case of equal numbers of neutrons and protons. The physical meaning of the above is quite

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[1] The selection rules referred to would be rigorous were it not for the neutron-proton mass difference and the fact that the electromagnetic interactions between nucleons are not charge symmetric. This limitation on the universality of the charge symmetry hypothesis sets practical limits on all derived consequences of the hypothesis. The fact that these nonsymmetric interactions are generally weak compared to specifically nuclear interactions for the cases of interest implies, however, that the selection rules obtained have considerable potency.

[2] The earliest reference to the existence of a good quantum number for self-conjugate nuclei associated with the charge parity operator appears to be in a paper of E. Feenberg and E. P. Wigner, Phys. Rev. **51**, 95 (1937). Recently the concept of charge parity was discussed by L. Trainor, Phys. Rev. **85**, 962 (1952), in connection with its application to the problem of electric dipole radiation from self-conjugate nuclei. It has the consequence here of forbidding electric dipole radiation in a transition between two states of the same charge parity.

[3] This form of the Hamiltonian includes (in addition to the equality of *nn* and *pp* forces) the assertion that the interaction energy between neutrons and protons involves the neutron and proton space and spin coordinates symmetrically. It is clear that usually asserted consequences of charge symmetry with respect to the mirror nuclei are valid only when this additional assumption is included.

obvious. P simply transforms protons into neutrons and neutrons into protons. Since T_3 is simply $\frac{1}{2}(N-Z)$, the application of P to an eigenstate of T_3 with eigenvalue t_3 simply changes it to an eigenstate of T_3 with an eigenvalue $-t_3$.

We note that $P^2=(-1)^A$ so that for even A, P has eigenvalues $+1$ and -1. A further property of P which is of interest for the discussion to follow is the fact that $[T^2, P]=0$, so that T^2 and P have simultaneous eigenstates. Furthermore, a $T_3=0$ eigenstate of T^2 with eigenvalue $t(t+1)$ must, in fact, be an eigenstate of charge parity and odd or even as t is odd or even. This latter result follows from the fact that under a rotation in isotopic spin space, the eigenfunctions of T^2 corresponding to a given t must transform like the spherical harmonic of order t under the homologous rotation in coordinate space.

The principal application of the concept of charge parity arises in the case of reactions between nuclei for which the incident and product nuclei are individually self-conjugate. In just these cases, the initial and final states of the system will be eigenstates of charge parity. From the fact that P is a constant of the motion, the initial and final states must both be charge even or charge odd. As a specific example one might consider the $O^{16}(d,\alpha)N^{14}$ reactions recently investigated by various research groups.[4] In this case one finds prominent α-particle groups corresponding to the ground state and various excited states of N^{14} but none corresponding to the 2.3-Mev excited state. A reasonable interpretation of this result is simply that this particular state has charge parity opposite to that of the states observed.† It is almost certain that the ground state of N^{14} is charge even, while if the nuclear forces are only approximately charge independent one would expect a low-lying charge odd state.

In order to make clear the relevancy of having both the incident and product nuclei in charge conjugate states one might note that a reaction like $O^{16}(d,p)O^{17*}$ will never be forbidden by charge parity conservation in spite of the fact that the initial state is charge even. The requirement that the final state be charge even as well simply implies that the reactions $O^{16}(d,p)O^{17*}$ and $O^{16}(d,n)F^{17*}$ occur with equal probability, where the final states of O^{17*} and F^{17*} are mirror states.

On the other hand, the observation of nuclear resonances associated with compound nucleus formation can be affected by charge parity considerations when only the product nuclei or only the incident nuclei are self conjugate. For example, the observation of a resonance in the reaction $C^{12}(d,p)C^{13}$ due to the formation of an excited state of N^{14} implies that the state of N^{14} is charge even, although in this case the product nuclei are not self conjugate.

The existence of charge parity has certain interesting connections with the problem of charge independence of nuclear forces. As has been pointed out by Adair,[5] the assumption of charge independence also implies selection rules in nuclear reactions, which, while including those implied by charge symmetry, are considerably more far reaching. This is a consequence of the fact that total isotopic spin as well as charge parity must be conserved. It is unfortunate, however, that the role of isotopic spin is obscured in many reactions by the coincidence of its predictions with those of charge parity. It has, for example, been proposed that the 2.3-Mev N^{14} state is a member of an isotopic spin multiplet with $T=1$, while the ground state is $T=0$. It is important to realize that it is then not possible for these states to have the same charge parity. It follows that it is difficult to find suitable reactions involving these states ($T=0$ and $T=1$) which are allowed by charge parity conservation and prohibited by isotopic spin conservation. Thus the observed prohibition of the previously mentioned reaction, $O^{16}(d,\alpha)N^{14*}$, yields no direct information on the strength of np forces as compared with nn and pp forces.[6]

Another application of the charge parity operator is in determining the terms which may be admixed in a particular state of a nucleus. Thus consider the case of a neutron and a proton both in the same p orbital in a nucleus such as in the case of Li^6. The terms which may be constructed from this configuration with total angular momentum unity are ${}^3S_1$, ${}^3D_1$, ${}^1P_1$, and ${}^3P_1$. These terms will not all be admixed (except again in the case of a degeneracy), since the first three of these have even charge parity and the last has odd charge parity.

Our final application of charge parity will be to the problem of the β-decay[7] of O^{14} to the 2.3-Mev excited state of N^{14}. Definite identification of the excited state as belonging to the same isotopic spin triplet to which the ground state of O^{14} belongs (under the assumption of charge independence) would confirm the angular momentum assignment of this state as $I=0$ and thus allow an estimation of the β-decay coupling constant associated with Fermi selection rules. Blatt[8] has recently pointed out that Adair's interpretation of the aforementioned $O^{16}(d,\alpha)N^{14*}$ result would indeed confirm

[4] Ashmore and Raffle, Proc. Phys. Soc. (London) **A64**, 754 (1950); Burrows, Powell, and Rotblat, Proc. Roy. Soc. (London) **A209**, 478 (1951); Van de Graaff, Sperderto, Beuchner, and Enge, Phys. Rev. **86**, 966 (1952).

† *Note added in proof:* E. Feenberg (private communication) has pointed out that the negative results obtained by the Van de Graaff, etc. group using 2.1-Mev deuterons might possibly be attributed entirely to the effect of the Coulomb barrier on the outgoing α-particles combined with the effects of angular momentum and space parity conservation associated with the probable assignment of spin zero and positive space parity to the N^{14} state. These effects appear, however, to be unimportant in the other experiments, which made use of higher energy deuterons.

[5] R. K. Adair, Phys. Rev. **87**, 1044 (1952). The authors wish to acknowledge the fact that their thoughts on the substance of this article were initiated by Adair's work, and to thank V. F. Weisskopf for calling it to their attention.

[6] On the other hand, if one assumes the nuclear forces to be approximately charge independent then this result does contribute to the identification of the symmetry character of the state. The energy of the state is then entirely consistent with charge independence.

[7] Sherr, Muether, and White, Phys. Rev. **75**, 282 (1949).

[8] J. Blatt (to be published).

this identification. Furthermore, the fact that the transition is super-allowed implies that the matrix element has its maximum value (two).[9] The relevance of charge parity arises from the fact that it weakens the dependence of these conclusions on the assumption of charge independence and, therefore, strengthens the conclusions drawn. Thus if one considers the deviation from charge independence as a perturbation, one notes that there are no nearby states of N^{14} with which the $T=1$ state can mix. To elaborate, the energy level diagram of O^{14} indicates a separation of about 6 Mev between the ground state and first excited state. The several states in the vicinity of the $T=1$ state of N^{14} must, therefore, all be $T=0$ states. If one now considers the effect of a deviation from charge independence on the $T=1$ state of N^{14} it is clear that the charge even character of $T=0$ states prohibits the mixing (which might otherwise be large) of these states with the $T=1$ state. Thus the expectation that the matrix element of the $O^{14}\rightarrow N^{14*}$ transition is two is only weakly affected.

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[9] G. L. Trigg, Phys. Rev. **86**, 506 (1952).

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Elementary Particles at Rest

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The object of this paper is to present a theory of the structure of elementary particles at rest. Some applications are included; in particular a solution to an unsolved problem in astronomy is presented.

I. THE UNCHARGED PARTICLE

In order to study this particle, let us imagine something with spherical symmetry, associated with which there is a gravitational field **F**. About this field we make some assumptions. In the first place, we assume that at each point there is a density of energy given by the scalar product $\mathbf{F}\cdot\mathbf{F}/8\pi\gamma$, where γ is the gravitational constant.[1] Next we assume, in accord with the principle of the inertia of energy,[2] that this density of energy is equivalent to a density of mass ρ given by the divergence $-\nabla\cdot\mathbf{F}/4\pi\gamma$.

We therefore have the following equation, where c is the speed of light in free space:

$$4\pi\gamma\rho=-\nabla\cdot\mathbf{F}=\mathbf{F}\cdot\mathbf{F}/2c^2. \tag{1}$$

If we imagine the particle at the origin of a system of spherical coordinates (r, ϑ, φ) then, on account of the spherical symmetry assumed, we can take the components of **F** in this system to be $(-F, 0, 0)$. Equation (1) then becomes one in F with the solution

$$F=r^{-2}\bigg/\left(A+\frac{1}{2c^2r}\right), \tag{2}$$

where A is an integration constant. This is a rather decent function. It has a pole of order one at the origin (instead of a pole of order two, as in Newton's law) and is integrable square over all space. At the origin $F\sim 2c^2/r$, independent of the mass of the particle.

In general, if **F** is assumed to be the gradient of a scalar function ϕ, Eq. (1) will become one in ϕ with solution $2c^2$ times the natural logarithm of a harmonic function.[3]

In order to find the value of A in Eq. (2), we compute ρ by Eq. (1), integrate over all space, and equate the result to the mass m of the particle. In this way we find $A=1/\gamma m$, and Eq. (2) becomes

$$F=\frac{\gamma m}{r^2}\bigg/\left(1+\frac{\gamma m}{2c^2r}\right)\sim\frac{\gamma m}{r^2}\quad\text{(for large } r). \tag{3}$$

The density of mass results:

$$\rho=\frac{\gamma m}{8\pi c^2r^4}\bigg/\left(1+\frac{\gamma m}{2c^2r}\right)^2. \tag{4}$$

The particle is thus seen to invade all space.[4] Therefore, interaction between two particles can be conceived as taking place by "intimate" contact, since each must be immersed in the other.

A potential ϕ defined by $\mathbf{F}=\nabla\phi$ is found to be $2c^2\ln(1+\gamma m/2c^2r)$. The potential energy of a "concentrated" mass m' in such a potential field would clearly not be symmetric in the masses m and m'. What then of the principle of equality of action and reaction? Of course what happens is that in this theory there are no such concentrated masses, and therefore this con-

[1] W. D. MacMillan, *Theory of the Potential* (McGraw-Hill Book Company, Inc., New York, 1930).

[2] G. Joos, *Theoretical Physics* (Blackie and Son, London, 1951), second edition.

[3] This observation is due to Professor John A. Wheeler, for the resultant field in the case of many particles.

[4] Things like this already appear in literature. See, e.g., the section on H. Weyl's Action Principle in *Mathematical Theory of Relativity*, A. S. Eddington (Cambridge University Press, Cambridge, 1930), second edition.