to the decay energy is only *ca* 1.5 kev (20.5−19.0), but the spectrum shape and *ft* value are appropriate to a first forbidden, unfavored transition with $E_0=20.5$ kev. It is interesting to note that the experimental errors and the theoretical uncertainty in the value of the total orbital electronic binding energy may be such as to allow for a zero or even a negative value for the specifically nuclear contribution to the decay energy. Thus a stripped Pu^{241} nucleus is probably only slightly (1.5 kev) unstable with respect to beta-minus-decay, or a stripped Am^{241} nucleus may even be energetically unstable with respect to a stripped Pu^{241} nucleus, although neither positron emission nor electron capture processes could occur.

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The Nuclear Moments of Ta^{181}

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An analysis of the hfs pattern of several Ta II lines has made it possible to determine the separations of the three levels of the hfs multiplet associated with the $5d^36s\ {}^5F_1$ state of Ta II. For this multiplet, the constants A and B appearing in the energy expression $W=W_J+\frac{1}{2}AK+BK(K+1)$ have the numerical values $A=(-0.079\pm0.001)$ cm^{-1} and $B=(-0.77\pm0.4)\times10^{-3}$ cm^{-1}. Calculations are carried out in order to evaluate the magnetic and quadrupole moments of the Ta^{181} nucleus. On the basis of the above measurements the magnetic moment as calculated by the Goudsmit-Fermi-Segrè formula has the value of 1.9 nuclear magnetons. Taking into account effects due to the spatial extension of the nucleus, this result is raised to 2.1 nuclear magnetons when the correction factor of 12 percent as listed by Klinkenberg is applied. The result for the quadrupole moment as calculated by the Casimir formula is $+5.9\times10^{-24}$ cm^2. According to Sternhermer this moment should be increased by a factor of 10 percent in order to include the effect of an induced quadrupole moment in the closed shell electrons. With this correction the quadrupole moment has the value of $+6.5\times10^{-24}$ cm^2.

INTRODUCTION

IT was deemed desirable to undertake an investigation of the hyperfine structure in the spectrum of Ta II taking advantage of recent identification of levels[1] arising from the $5d^36s$ configuration. These levels should exhibit a large hyperfine splitting due to the presence of a single s electron. For a term with a small J-value optimum conditions are provided for the measurement of deviations due to the presence of an electric nuclear quadrupole moment both from the point of view of pattern size and the number of hfs components. These considerations led to the study of the structure of the ${}^5F_1$ level—the lowest state arising from the $5d^36s$ configuration. Previous researches[2–4] indicate that the Ta nucleus has a spin of 7/2 and a magnetic moment of 2.1 nuclear magnetons. In addition, Schmidt[5] has assigned a value of $\sim6\times10^{-24}$ cm^2 to the nuclear quadrupole moment on the basis of hfs measurements in connection with the lowest states (${}^4F_{3/2}$ and ${}^4F_{5/2}$) of the Ta I spectrum. Values of the quadrupole moment obtained by him from the study of two other levels were found to disagree with the above value. The purpose of the present measurements was to attempt an independent determination of the nuclear magnetic and quadrupole moments.

TABLE I. Wavelengths and transitions of Ta II lines whose hyperfine patterns were investigated. In each case the final state arises from the $5d^36s$ configuration. The term values of the ${}^3P_0$ and ${}^5F_1$ states are equal to 4124.77 and 0.00, respectively.

Wavelength in air in angstroms	Transition
3379.49	$(33706.50)_1\rightarrow{}^3P_0$
3042.06	$(36987.73)_1\rightarrow{}^3P_0$
2965.92	$(33706.50)_1\rightarrow{}^5F_1$
2965.13	$(33715.15)_2\rightarrow{}^5F_1$
2763.37	$(36177.12)_2\rightarrow{}^5F_1$
2702.80	$(36987.73)_1\rightarrow{}^5F_1$
2595.59	$(38515.55)_2\rightarrow{}^5F_1$

EXPERIMENTAL

The Ta II spectrum was excited in a hollow cathode discharge tube containing argon. The hfs patterns were obtained by the use of a Fabry-Perot interferometer crossed with a 21-ft concave grating in a modified Wadsworth mounting. The interferometer plates were coated with aluminum and three separators having thicknesses of 3, 3.7, and 4 mm were utilized. Measurements were carried out on the seven lines whose wave-

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[1] Kiess, Harrison, and Hitchcock, J. Research Natl. Bur. Standards **44**, 245 (1950).

[2] J. H. Gisolf and P. O. Zeeman, Nature **132**, 566 (1933).

[3] N. S. Grace and E. McMillan, Phys. Rev. **44**, 949 (1933).

[4] J. H. Gisolf, dissertation, Amsterdam (1935) (unpublished).

[5] T. Schmidt, Z. Physik **121**, 63 (1943).

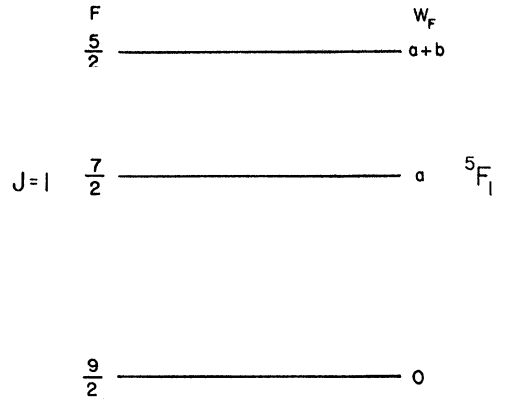

FIG. 1. Hyperfine multiplet belonging to the $5d^36s\ ^5F_1$ ground state of Ta II.

lengths in air and atomic transitions are listed in Table I. An additional line at 2249.8A was also investigated but the fringes were not sufficiently well resolved. It is fortunate that Ta has but one isotope; otherwise, the study of patterns in a spectrum which is already so rich would have been prohibitively difficult.

LINE STRUCTURES AND THE DETERMINATION OF THE CONSTANTS *A* AND *B*

In a hfs multiplet, the energy of a level with total quantum number F is given by[6]

$$W_F=W_J+\tfrac{1}{2}AK+BK(K+1), \quad (1)$$

where W_J is the center of gravity of the multiplet, A is the splitting factor due to the interaction of the nuclear magnetic moment with atomic electrons, and B is a factor determined by the interaction of the nuclear quadrupole moment and the electrons. The quantity K is defined by $K=F(F+1)-I(I+1)-J(J+1)$, where J and F are the electronic and atomic quantum numbers and I is the nuclear spin.

The study of the various line structures was directed towards the determination of the hfs separations of the $^5F_1$ state. The intervals of interest are designated by a and b on the level diagram shown in Fig. 1. The constants A and B were evaluated from the relations:

$$A=-(70a+54b)/504; \quad B=-a/144+b/112, \quad (2)$$

by substituting in Eq. (1) the best experimental values of the intervals a and b. The nuclear moments were then calculated by the use of appropriate theoretical expressions involving A and B.

We now turn to a discussion of the procedure followed in obtaining the numerical values of the intervals a and b from observations of hfs patterns. As indicated in the level diagram shown in Fig. 2, the lines 2702A and 3042A are due to transitions which have the same upper level with $J=1$. The hfs intervals of this common upper level were measured directly from the triplet

[6] H. Kopfermann, *Kernmomente* (Edwards Brothers, Ann Arbor, 1945).

TABLE II. The hfs intervals of the $^5F_1$ state and the associated splitting factors derived from the structures of lines listed in the first column. All quantities are expressed in cm^{-1}.

(*A*)	*a*	*b*	(*a*+*b*/2)	*A*	*B*
2702 3042	0.395	0.220	0.505	−0.078	-0.77×10^{-3}
2966 3379	...	...	0.500	...	...

structure associated with the pattern of line 3042A. The magnitudes of these intervals as designated in Fig. 2 were found to be $a_u=0.327$ cm^{-1} and $b_u=0.262$ cm^{-1}. In the case of line 2702A, the component pairs labeled as 2, 3 and 5, 6 were not resolved. These components have equal intensities and the center of gravity of each pair is located midway between the two components. Component 7 was so weak that its position could not be determined accurately. Hence, from the line 2702A, it was possible to measure only the two intervals of magnitude equal to 0.361 cm^{-1} and 0.602 cm^{-1} as designated in Fig. 2. From these considerations, the desired intervals of the $^5F_1$ are found to be: $a=0.395$ cm^{-1} and $b=0.220$ cm^{-1}.

A partial confirmation of the above values for a and b may be obtained by investigating the patterns of two other lines, namely, $\lambda=2965.9$A ($J=1\rightarrow J=0$) and $\lambda=3379$A ($J=1\rightarrow J=1$). Here, the situation is similar

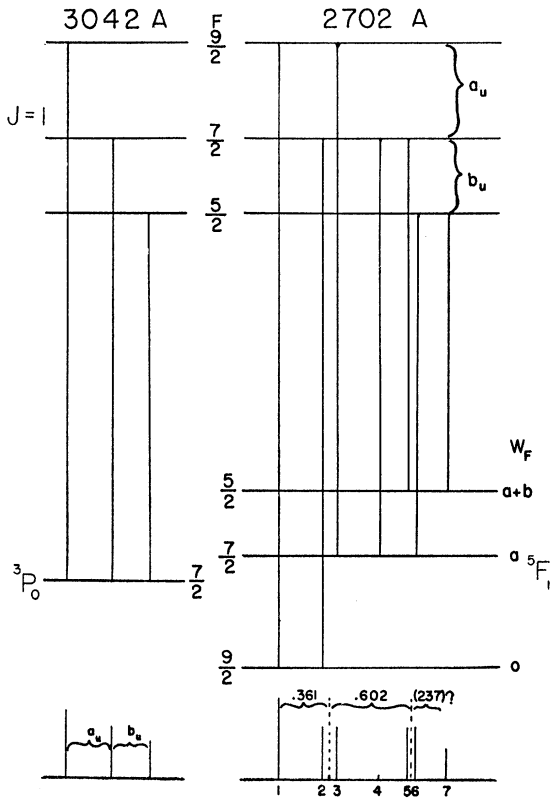

FIG. 2. Level diagram and line structure for λ2702 and λ3042. Intervals are expressed in cm^{-1}. The spin of Ta^{181} is 7/2.

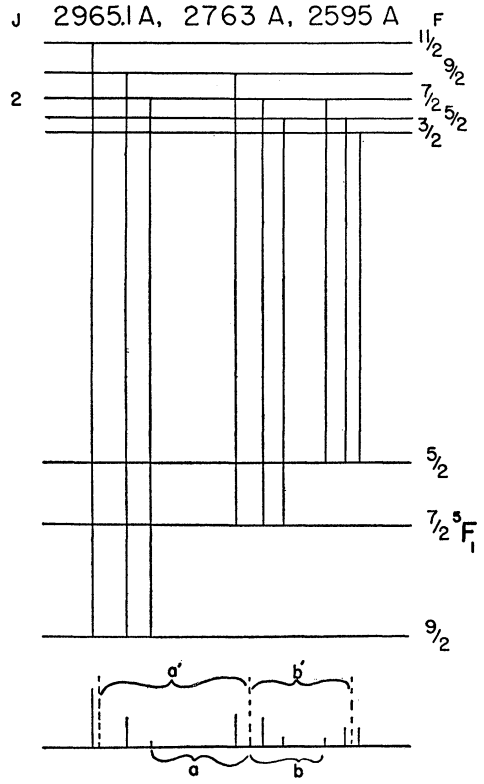

FIG. 3. Line structure arising from transitions into the lowest $^5F_1$ level from a higher level with $J=1$.

to the case of the preceding pair of lines, except that the common upper level with $J=1$ is inverted. From a study of the expected pattern it is observed that $a+b/2=0.500$ cm^{-1}. This result is within 1 percent of the corresponding value for $(a+b/2)$ obtained from the previous pair of lines. A summary of these measurements along with the values of A and B as computed from Eq. (2) is presented in Table II.

The structures of other lines which combine with the ground level are in general too complex to yield reliable measurements. However three other lines (λ2595, 2763, 2965) were examined to see if additional information might be obtained regarding the values of A and B. The above-mentioned lines represent transitions into the $^5F_1$ level from upper states whose J value is 2. The level diagram and the hfs pattern shown in Fig. 3 is typical of this group of lines. Due to the small splitting of the upper levels, the individual components of the multiplet could not be resolved and the intensity distribution of the observed pattern resembled that shown in the lower portion of Fig. 3. If the intervals between the centers of gravity of the observed groupings are designated as a' and b', then, the desired intervals of the $^5F_1$ level are related by

$$a=a'-6.75A_u-114B_u;\quad b=b'-5.25A_u+148B_u,$$

where A_u and B_u are the constants which determine the splitting of the upper level involved. The expressions above are arrived at by applying Eq. (1) to the hfs levels of the upper state and by taking into account the relative intensities of components as given by tables.[7] Upon substitution in Eq. (2) of the values of a and b specified by the above relations one obtains,

$$\begin{aligned} A&=-\frac{70a'+54b'}{504}+1.50A_u,\\ B&=-\frac{a'}{144}+\frac{b'}{112}+2.11B_u. \end{aligned} \tag{3}$$

For each line of this group a' and b' were measured from the patterns and the value of A_u was estimated from the width of the unresolved components. This method of arriving at the value of A_u is sufficiently accurate since a 10 percent error in the measurement of the width introduces only a 2 percent error in the calculated value of A. Corresponding estimates of the value of B_u cannot be obtained. For a level with $J=2$, the constant B is expected to be only $\frac{1}{6}$ as large as the one deduced from a level with $J=1$. The information gleaned from a study of these transitions $(J=2\rightarrow J=1)$ is presented in Table III. Since the magnitude of the correction B_u is not known this quantity is included in the results for B.

The agreement between the values of A appearing in Table II and Table III is satisfactory. In obtaining the average value of A, the mean of the three values shown in Table III was weighted equally with the determination listed in Table II. Two of the values listed for B in Table III are in fair agreement with the result given in Table II. The reason for the discrepancy in the case of λ2965 is not known. Kiess[8] has indicated that this wavelength may satisfy two atomic transitions. The discrepancy may also be ascribed to the presence of a large quadrupole effect associated with the structure of the upper level involved in this particular transition. The individual values of B listed in Table III cannot be regarded as reliable as the value of B obtained from measurements on the 2702A line. Consequently, these determinations were disregarded in the calculation of the quadrupole moment. The various considerations presented in the preceding discussion lead to the following values of the constants:

$$\begin{aligned} A&=(-0.079\pm0.001)\text{ cm}^{-1};\\ B&=(-0.77\pm0.04)\times10^{-3}\text{ cm}^{-1}. \end{aligned} \tag{4}$$

TABLE III. Values of the splitting factors deduced from the partially resolved structures of the lines listed in the first column. These represent transitions from upper levels with $J=2$ into the $^5F_1$ state. All quantities are expressed in cm^{-1}.

λ(A)	a'	b'	A_u	A	B
2595	0.521	0.347	0.018	−0.083	$(-0.52+2.1B_{u1})\times10^{-3}$
2763	0.510	0.306	0.017	−0.079	$(-0.81+2.1B_{u2})\times10^{-3}$
2965	0.500	0.426	0.024	−0.079	$(+0.33+2.1B_{u3})\times10^{-3}$

[7] H. E. White and A. Y. Eliason, Phys. Rev. 44, 753 (1933).
[8] C. C. Kiess (private communication).

THE CALCULATION OF THE MAGNETIC MOMENT

According to the calculations of Breit and Wills,[9] the magnetic splitting factor A of the $^5F_1$ state may be written as

$$A=(5/4)a_F-\tfrac{1}{4}a_s,$$

where a_s and a_F are the coupling coefficients for the s electron and the d electron complex, respectively. It is necessary to evaluate a_s for the calculation of the magnetic moment. The value of A is already available, but a_F must still be determined. The expressions for the magnetic coupling coefficient for the d electron complex as given by Fermi and Segrè,[10] yield the ratio

$$\frac{a_F}{a'_F}=\frac{\langle 1/r^3\rangle_{\text{Av Ta II}}}{\langle 1/r^3\rangle_{\text{Av Ta I}}}.$$

In the above ratio a'_F is the magnetic coupling coefficient of the d complex in Ta I and is known to have the value of 0.0167 cm^{-1} according to measurements carried out by Schmidt[5] on the structure of the $^4F_{3/2}$ state of Ta I. The averages indicated in the ratio on the right-hand side are for a d electron. The numerical value of the ratio may be obtained from a study of a series of spectra where a similar situation exists; that is, where the ionization involves the removal of one of a pair of s electrons and where the remainder of the configuration has the form d^ns. Such a procedure sets the average value of the ratio at 1.05. It should be mentioned that the ratio has the value of 1.00 when calculated from the fine structure intervals of the spectra of Ta I and Ta II. However, the low-lying ^4F multiplet in Ta I shows a large deviation from the interval rule, so that the value of 1.05 as deduced from the spectra of neighboring elements was considered as more reliable. On the basis of this information, the factor a_F comes out to be 0.0175 cm^{-1}, and consequently $a_s=0.405\pm0.005$ cm^{-1}.

The connection between the magnetic moment μ (in nuclear magnetons) and the coupling factor a_s (in cm^{-1}) is given by [10,11]

$$a_s=\frac{8R\alpha^2}{3I}\frac{Z(1+z)^2}{n^{*3}}\left(1-\frac{d\sigma}{dn}\right)1836\mathfrak{C}(Z,j)\mu. \qquad (5)$$

The various quantities in Eq. (5) have the meaning and numerical values listed below:

R=Rydberg constant, $\alpha=1/137$; Z=the interior charge number here equal to 73 for an s electron; z=the stage of ionization, hence for Ta II, $(1+z)^2=4$; $I=7/2$, n^*=the effective quantum number$=(n-\sigma)$ where σ is the quantum defect; $\mathfrak{C}$=relativity correction factor. The value of n^{*3} was determined by plotting n^{*3} against Z for a large number of singly ionized atoms throughout the periodic table. The effective quantum number n^* is the one associated with the lowest term in a series originating from the excitation of an s which is shielded by p, d, and f electrons. For Ta II, the value of n^{*3} as read from the resulting smooth curves is 6.6. The quantity $[1-(d\sigma/dn)]$ was found to have the average value of 1.05 as determined from a study of similar spectra. Here $d\sigma/dn$ represents the change in the quantum defect observed in the lowest pair of s series terms in the fine structure levels. As tabulated by Kopfermann,[6] the relativity correction factor $\mathfrak{C}$ is 1.92. Upon substitution in Eq. (5) the nuclear moment of Ta^{181} is found to be

$$\boxed{\mu_{181}=4.6a_s=1.9 \text{ nuclear magnetons.}}$$

CALCULATION OF THE QUADRUPOLE MOMENT

The evaluation of the quadrupole moment was carried out by the use of the Casimir[12] formula:

$$q=-\frac{8}{3}\frac{I(2I-1)J(2J-1)}{e^2\left[\sum_i\left\langle\frac{3\cos^2\theta_i-1}{r_i^3}\right\rangle_{\text{Av}}\right]_{M_J=J}}B. \qquad (6)$$

The indicated summation is carried out over the electron coordinates i for those electrons which contribute to the quadrupole interaction with the nucleus for the state $M_J=J$. In order to include the effects of relativity corrections dependent on the j-values of individual electrons, the wave function of the $^5F_1$ state was written as the zero-order Russell-Saunders sum of products of one electron functions in the $m_l{}^{m_s}$ representation. Following the scheme used by Schmidt,[5] the $m_l{}^{m_s}$ matrix elements were then expressed in terms of the $d_j{}^{mj}$ elements.[13] The result obtained for the quantity in brackets found in Eq. (6) is

$$\left[\sum_i\left\langle\frac{3\cos^2\theta_i-1}{r_i^3}\right\rangle_{\text{Av}}\right]=\frac{8}{875}(13R_2'+14R_2''-12S_2)\langle 1/r_d^3\rangle_{\text{Av}},$$

where R_2', R_2'', and S_2 are the relativity corrections for d electrons. From the fine-structure splitting of the two lowest ^5F intervals of Ta II, the average inverse cube of the radius of $5d$ electron in Ta II was estimated to be 0.37×10^{26} cm^{-3} by the method of Fermi and Segrè.[10] (See also Condon and Shortley.[14]) From tables prepared

[9] G. Breit and L. A. Wills, Phys. Rev. 44, 475 (1933).
[10] E. Fermi and E. Segrè, Z. Physik 82, 729 (1933).
[11] S. A. Goudsmit, Phys. Rev. 43, 636 (1933).

[12] H. Casimir, *On the Interaction between Atomic Nuclei and Electrons* (Teylers Tweede Genootschap, XI, Haarlem, 1936).

[13] Two typographical errors exist in the 5th and 7th elements as listed by Schmidt. These should read:

$$(0^\pm|3\cos^2\theta-1|0^\pm)=\frac{4}{175}(12R_2'+7R_2''+6S_2);$$

$$(0^\pm|3\cos^2\theta-1|\pm1^\mp)=\frac{2\sqrt{6}}{175}(8R_2'-7R_2''-S_2).$$

[14] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1951), p. 196.

by Kopfermann,[6] one finds $R_2'=S_2=1.07$ and $R_2''=1.24$. Hence for the bracketted expression above one obtains the numerical value of 6.3×10^{24} cm^{-3}. Combining this factor with the experimental value for B already determined, the nuclear quadrupole moment of Ta^{181} is evaluated with the aid of Eq. (6). The result is

$$\boxed{q=+5.9\times10^{-24}\text{ cm}^2.}$$

DISCUSSION OF RESULTS

A source of error in the determination of the magnetic moment may lie in the procedure followed in obtaining the coupling coefficient of the s-electron. This step involves the removal of the contribution of the d-electrons to the coupling coefficient of the $^5F_1$ state. Since the term due to the d-electrons represents 20 percent of the $^5F_1$ coupling coefficient, the error introduced in a_s cannot be in excess of this amount and probably is not as high. It is believed that this is the best approach to the evaluation of the magnetic moment, since, results derived from s-electron coupling coefficients are in general expected to be most reliable in spectroscopic determinations.

A second error may arise from the method adopted in the calculation of a_F, the coupling coefficient associated with the d-electron complex. This involves the transfer of measurements of the hfs of the $^4F_{3/2}$ state of Ta I to the Ta II structure by means of the ratio of $\langle r_d^{-3}\rangle_{Av}$ terms. The error here cannot be greater than that involved in the evaluation of $\langle r_d^{-3}\rangle_{Av}$ by the use of fine structure splitting factors and is estimated to be of the order of a few percent. It is gratifying to note that magnetic moment of 1.9 nuclear magnetons as obtained from the present measurements is in agreement with the value of 2.1 nuclear magnetons arrived at by Gisolf.[4]

In a review article[15] Klinkenberg points out that the value of μ derived from optical hfs on the basis of the original Goudsmit-Fermi-Segrè formula should be increased by 12 percent in the case of Ta. The correction is introduced to take into account two effects due to the spatial extension of the nucleus. The greater of the two is the influence upon the electronic wave functions exerted by the uniform electric charge distribution. The second effect is the result of the presumed uniform distribution of the magnetic dipole moment over the nuclear volume. With this correction, the value of the magnetic moment is increased to 2.1 nuclear magnetons.

On the basis of observations carried out on the $^5F_1$ level of Ta II, the present investigation yields a value of $+5.9\times10^{-24}$ cm^2 for the quadrupole moment of Ta^{181}. Previous measurements on the $^4F_{3/2}$ level of Ta I reported by Schmidt lead to the result of $+6\times10^{-24}$ cm^{-2} for the same quantity. It must be borne in mind that the determination of the quadrupole moment does depend on the detailed form of the coupling between the three d-electrons. Although the Landé interval rule is obeyed more closely by the fine structure levels of Ta II than those of Ta I, significant deviations from the rule still exist in the spectrum of the ion. Hence, the present calculation of q based on R-S coupling cannot be justified completely. The agreement which appears in the values of q as obtained from two different stages of excitation is not necessarily a reliable index as to the correctness of the type of coupling assumed. In the electron configurations $5d^36s^2$ and $5d^36s$ which give rise to the $^4F_{3/2}$ and $^5F_1$ levels of Ta I and Ta II, respectively, the s electron gives no quadrupole interaction with the nucleus. Hence, the errors due to a departure from R-S coupling in the case of Ta I may be similar to those encountered in the present determination dealing with Ta II. It is difficult to assign values to the expected errors, since the main source lies in the assumptions which must be made in order to approximate the electronic averages.

Recent calculations by Sternheimer[16] indicate that the value of the nuclear quadrupole moment as deduced above should be increased by 10 percent due to an induced quadrupole moment in the electrons belonging to closed shells. The value of the quadrupole moment then becomes

$$+6.5\times10^{-24}\text{ cm}^2.$$

The authors gratefully acknowledge the generous assistance of Mr. C. C. Kiess of the National Bureau of Standards in matters relating to the spectrum of Ta II. They also wish to thank Dr. N. Schwartz and Mr. G. Sprague for aid rendered in the course of these calculations.

[15] P. F. A. Klinkenberg, Revs. Modern Phys. 24, 63 (1952).

[16] R. Sternheimer, Phys. Rev. 86, 316 (1952).