The temperature dependence is

$$\chi_{\parallel} \propto T^{\frac{1}{2}} \exp[-S(24JK)^{\frac{1}{2}}/kT], \tag{24}$$

for $\zeta = S(24JK)^{\frac{1}{2}}/kT \gg 1$.

4. CONCLUSION

There is, as yet, very little information concerning the separate values of $\chi_{\parallel}$ and $\chi_{\perp}$ for an antiferromagnet. Griffel and Stout[11] have obtained values of $\chi_{\parallel}-\chi_{\perp}$ for MnF_2 at various temperatures, which, together with previous work by de Haas, Schultz, and Koolhaus[12] on the powder susceptibility of MnF_2, enable them to plot the values of $\chi_{\parallel}$ and $\chi_{\perp}$. It is quite clear from their work that $\chi_{\perp}$ is not constant below the Curie point but increases as T decreases toward zero. This is the type of behavior predicted by Hulthén, and differs from the prediction of the molecular field theory. $\chi_{\parallel}$ decreases as the temperature is lowered and seems to approach zero at $T=0$. The data are not sufficient to allow the temperature dependence of either $\chi_{\parallel}$ or $\chi_{\perp}$ to be determined very much more specifically.

It is a pleasure to thank Professor C. Kittel for the suggestion of the problem and for many discussions concerning it.

[11] M. Griffel and J. W. Stout, J. Chem. Phys. 18, 1455 (1950).
[12] de Haas, Schultz, and Koolhaus, Physica 7, 57 (1940).

PHYSICAL REVIEW VOLUME 88, NUMBER 5 DECEMBER 1, 1952

Ionization Loss and Straggling of Fast Electrons*

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(Received June 16, 1952)

The most probable energy loss of 9.6- and 15.7-Mev electrons in samples of about one gram per cm² of beryllium, polystyrene, aluminum, copper, and gold has been measured. The losses measured were of the order of one Mev, and the resolution of the apparatus made possible an accuracy of 20 kev. The observed distributions of energy losses are found to be in good agreement with the Landau straggling calculations for the light elements. For the heavier elements there is a spreading of the distribution introduced by radiation and K electron effects. Calculations made by Yang and Kennedy for gold, including these effects, check well with the experimental data.

Applying Fermi's correction for the polarization effect at extreme relativistic velocities to Landau's result for the most probable energy loss, one obtains for the predicted loss in Mev $\Delta_{pc}=0.1537D(\Sigma Z/\Sigma A)\times[19.43+\ln(D/\rho)]$, where D is the absorber thickness in g/cm² and ρ is the absorber density in g/cm³. Experimental results for the light elements are in excellent agreement with this theory. The heavier elements show losses somewhat smaller than those calculated.

INTRODUCTION

THERE have been a number of measurements of the ionization loss and energy straggling suffered by electrons in passing through matter, and the general processes appear to be well understood. A recent study of the energy loss distributions for electrons having initial energies up to 1 Mev has been reported by Chen and Warshaw.[1] They find that the observed most probable energy loss as well as the energy loss distributions are in good agreement with the calculation of Landau.[2] Further work with electrons having energies up to 5 Mev has been carried out by Paul and Reich[3] and is discussed by Schultz.[4] They find that the energy loss at these energies is less than that given by the theory and attribute the decrease to the effect of polarization in the material.

The present work represents an attempt to measure the energy losses and energy distributions at higher energies using highly monoenergetic electrons from the 22-Mev betatron.

EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The experimental arrangement for the removal of the electrons from the betatron and the focusing of the beam into the chamber has been described in previous work.[5,6] The modifications of the scattering chamber and the detector arrangements for the present experiments are shown in Fig. 1. The electron beam is brought to a focus upon the absorber samples, which were mounted on the remotely controlled sample holder at the center of the chamber. A horizontal slit one inch long and 0.020 inch high was mounted in the center of the chamber directly behind the absorbers. This slit was formed by two pieces of gold $\frac{1}{16}$ inch thick. The adjustable aperture and adjustable collimating slit in front of the ionization chamber were edged with pieces of gold $\frac{1}{8}$ inch thick, as described in the previous work.[6] Gold was chosen as the slit edging with the idea that electrons impinging on the slit edge would be scattered completely out of the beam or be so degraded in energy

* Work supported in part by the joint program of the ONR and AEC.
† AEC Predoctoral Fellow.
[1] J. J. L. Chen and S. D. Warshaw, Phys. Rev. 84, 355 (1951).
[2] L. Landau, J. Phys. (U.S.S.R.) 8, 201 (1944).
[3] W. Paul and H. Reich, Z. Physik 127, 429 (1950).
[4] Walter Schultz, Z. Physik 129, 530 (1951).
[5] Lyman, Hanson, and Scott, Phys. Rev. 84, 626 (1951).
[6] Scott, Hanson, and Lyman, Phys. Rev. 84, 638 (1951).

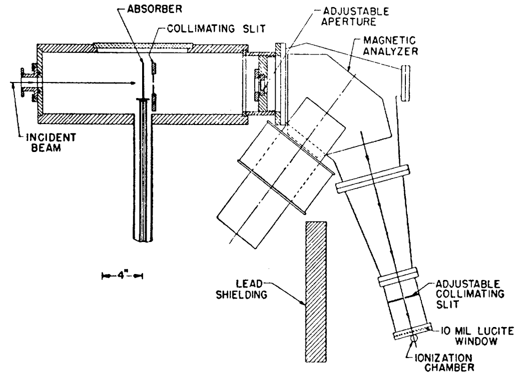

FIG. 1. Side view of experimental arrangement of absorber and analyzer.

that they would be removed from the energy range under consideration. The collimation defined by the aperture at the entrance to the analyzer was such as to limit the electrons entering the analyzer to those which were within two degrees of the direction of the incident beam.

The magnetic analyzer was the same as that used for the earlier experiments. The magnetic field in this instrument was measured and automatically regulated by comparing the emf generated in a rotating coil placed in the magnet gap with an accurately known fraction of the emf generated in a similar coil on the same shaft in the field of a permanent magnet. Special tests indicated that the regulator maintained the field at the position of the rotating coil constant to about 0.03 percent. Since the measuring coil could not be placed in the path traversed by the electrons passing through the analyzer, it was found that precautions had to be taken to insure that the magnet was on an established hysteresis cycle. This effect was not large, but if not kept in control it would easily introduce an error of about 0.2 percent.

The resolution of the analyzer as well as the absolute calibration of the instrument was checked by passing lithium ions of accurately known energies through the system. The lithium ions were evaporated from a platinum filament coated with a thin film of spodumene and accelerated through potentials of from 2 to 25 kilovolts. The momenta of these lithium ions lay in the same range as the momenta of the electrons used in this experiment and could be measured and controlled to about 0.01 percent.

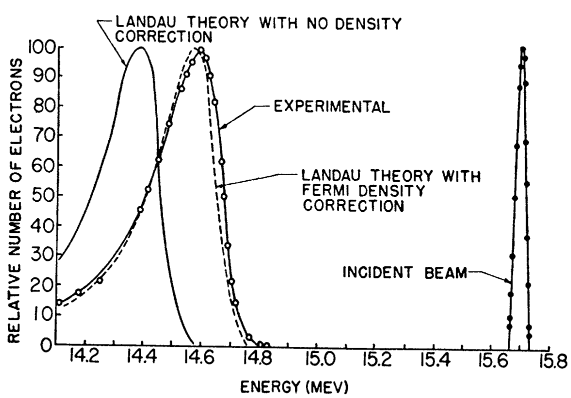

FIG. 2. Energy distribution of unobstructed electron beam and calculated and experimental distributions of electrons after passing through 0.86 g/cm² of aluminum.

The experimental procedure was simply to make alternate runs with and without the absorbers in the beam. In each run the analyzer field was varied through the range for which there was appreciable intensity detected by the ionization chamber, and the currents corresponding to the analyzer settings were recorded.

In the case of gold the intensity of the beam reaching the analyzer was greatly attenuated because of multiple scattering, and in this case a background of about 20 percent of the maximum intensity at the detector had to be subtracted. This background was apparently caused by x-rays produced in the vicinity of the absorber and entrance slit and was nearly independent of the analyzer setting.

Typical plots of the results are shown in Figs. 2 and 3, which show the energy loss spectrum of the electrons in passing through aluminum and gold.

It may be of interest to note that the observed width of the unobstructed beam could be accounted for by the widths of the entrance and exit slits. The energy of the electrons emerging from the betatron, therefore, must have an inherent energy spread that is less than the resolution of the equipment or about 0.1 percent. It was noticed, however, that the energy of the emerging electrons at a particular integrator setting varied by about 0.3 percent during a warming-up period of the order of an hour.

In order to be more certain of the observed energy losses a practice was made of checking the analyzer settings corresponding to the half-maximum intensity point on the steepest side of the observed straggling distribution and both the half-intensity points for the unobstructed beam. Since the half-intensity point of the straggling distribution can be located with greater precision than the maximum, it was often convenient to use this point in the calculation of the most probable energy loss. For light elements the relation between this point and the most probable energy loss is given accurately by the Landau theory, namely,

$$\Delta_p = \Delta_{\frac{1}{2}} + 1.58 S_0,$$

where S_0 has values as defined in the next section.

Since the energy band passed by the analyzer was less than the energy width of the steepest slope in the observed distributions, the distortion of the actual straggling distributions because of the resolution of the analyzer is very small and it is not necessary to make any correction to the observed widths at half height. There is, however, considerable error in the measurements of these widths since these are only about 0.25 Mev. An error of 0.1 percent (of the incident energy of

16 Mev) in the location of the two half-intensity points would therefore account for an uncertainty of about 20 kev or 8 percent of the width itself. The error in the most probable energy loss is considerably less than this, about 2 percent, since the observed losses are of the order of 1 Mev.

THEORY

The straggling in energy of electrons in passing through material has been calculated by Williams[7] and by Landau[2] on the assumption that the straggling is due only to collisions with essentially free electrons and that energy losses associated with the excitation of energy levels in the bound states are sufficiently small so as to effect only the total energy loss. Although the mathematical methods used are different, the resulting straggling distributions are almost identical. In treating the problem, Williams has pointed out that it is useful to classify the collisions suffered by the electrons into two groups. The first group includes all those collisions that occur with a frequency sufficiently great that the probability distribution of the losses due only to these collisions can be treated as a statistical problem of large numbers and therefore is a Gaussian distribution. The second group of collisions consists of the relatively large energy losses that are responsible for the long tail of the straggling distribution. The boundary between these two groups is chosen so that the integrated probability for an electron to make a collision in the second group is exactly unity. The value of this boundary energy S_0 is shown to be

$$S_0 = 2\pi e^4 nt/mv^2, \tag{1}$$

where e and m are the charge and rest mass of an electron, v is the velocity of the incident particle, n is the number of electrons per unit volume in the absorber, and t is the thickness of the absorber.

Williams finds a straggling distribution which has a full width at half-height $\Gamma = 3.86S_0$. The effect of the collisions in the second group is to displace the maximum of the Gaussian due to the energy losses less than S_0 by an amount $0.3S_0$. The most probable loss, which corresponds to the maximum of the straggling distribution, is given by the relation.

$$\Delta_p = \Delta_0 + 0.3S_0,$$

where Δ_0 represents the mean energy loss associated with individual energy losses of less than S_0.

Landau's calculations are based on the use of the transport equation and are carried out in a continuous manner rather than in Williams' stepwise fashion. The width of his distribution at half-height is $3.98S_0$, and the most probable energy loss is given as

$$\Delta_p = \Delta_0 + 0.37S_0. \tag{2}$$

The difference between the Williams and the Landau relations for the most probable energy loss amounts to

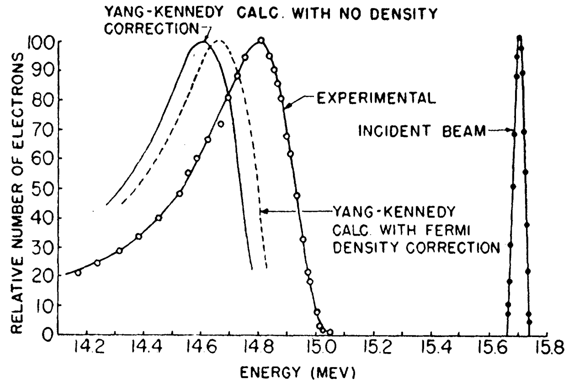

FIG. 3. Energy distribution of unobstructed electron beam and calculated and experimental distributions of electrons after passing through 0.97 g/cm² of gold.

only $0.07S_0$, which for our work (where S_0 is of the order of 60 kev) amounts to only 4 kev for losses of the order of 1 Mev.

The energy loss Δ_ξ, associated with individual losses less than ξ, is the same for all particles of unit charge moving with the velocity v and is given by the relation[8]

$$\Delta_\xi = S_0\left[\ln\frac{2mv^2\xi}{(1-\beta^2)I^2} - \beta^2\right]. \tag{3}$$

Thus, the loss due to interactions in which S_0 is the maximum energy transfer can be written

$$\Delta_0 = S_0\left[\ln\frac{2mv^2S_0}{(1-\beta^2)I^2} - \beta^2\right]. \tag{4}$$

TABLE I. Observed and calculated most probable energy losses. The values given for S_0, Δ, and Γ are all in Mev.

	Be	Poly	Al	Cu	Au
D (g/cm²)	0.7483	0.8090	0.8586	0.8400	0.9658
ρ (g/cm³)	1.839	1.055	2.694	8.808	19.51
			15.7-Mev data		
S_0	0.05094	0.06590	0.06353	0.05885	0.05945
Δ_p (Landau)	1.170	1.482	1.325	1.130	1.020
Δ_{pc} (Fermi)	0.950	1.270	1.135	1.005	0.965
Δ_p (exptl.)	0.945	1.235	1.110	0.980	0.902
Δ_{pc} (Yang-Kennedy)	—	—	—	—	1.045
Γ/S_0 (Landau)	3.98	3.98	3.98	3.98	3.98
Γ/S_0 (exptl.)	3.97	3.98	4.18	4.69	6.99
Γ/S_0 (Yang-Kennedy)	—	—	—	—	6.98
			9.6-Mev data		
S_0	0.05103	0.06602	0.06364	—	—
Δ_p (Landau)	1.125	1.422	1.264	—	—
Δ_{pc} (Fermi)	0.952	1.272	1.137	—	—
Δ_p (exptl.)	0.931	1.227	1.145		

[7] E. J. Williams, Proc. Roy. Soc. (London) **A125**, 445 (1929).

[8] A detailed discussion of the processes of ionization loss, Čerenkov radiation, and polarization effects has been given by M. Schönberg, Nuovo cimento **8**, 169 (1951); **9**, 372 (1952). Application of the theory to the problem at hand was made by P. Janssens and M. Huybrechts, Bull. Cent. Phys. Nucl. Brussels no. 29 (1951).

Since the electrons involved in the straggling distribution near the most probable energy loss are not those suffering violent collisions, the energy-loss distribution around the most probable energy loss is the same for heavy particles of unit charge moving with the same velocity. The only requirement is that the maximum energy which the incident particle can transfer to an electron $[W_m=2mv^2/(1-\beta^2)]$ is considerably larger than S_0.

The value of the average excitation energy I appearing in the equation is very nearly the same in this work as for the energy loss for fast protons, and one could use the experimental values of I determined by Mather and Segrè.[9] If, however, we consider the effect of polarization of the medium on the energy loss, we find by Fermi's treatment that the reduction in energy loss for particles having velocities less than $c/\sqrt{\epsilon}$ is $S_0 \ln\epsilon$, where ϵ is the effective dielectric constant.[10]

A formally more correct interpretation of the experimental values of I obtained from the proton range data is that these are equal to $\epsilon^{\frac{1}{2}}I'$, where I' is the value of the mean excitation potential without the polarization effect. The effective value of ϵ, however, is expected to be close to unity except for the lightest elements. Since we hope to investigate the polarization correction more specifically in future experiments, we have not examined the effect in detail. We have, in this work, used the simple expression $I=13.5Z$ in computing the energy losses from the Landau formula.

The most probable loss, corrected for the polarization effect for extreme relativistic electrons can be treated more simply, since it will not depend on the excitation energies of the material. This can be seen by considering the reduction in energy loss due to polarization, Δ_c, which is applicable to this case.[10,11]

$$\Delta_c=S_0\left[\ln\left(\frac{\epsilon-1}{1-\beta^2}\right)-\frac{1-\epsilon\beta^2}{1-\epsilon}\right] \text{ for } v>c/\sqrt{\epsilon}, \quad (5)$$

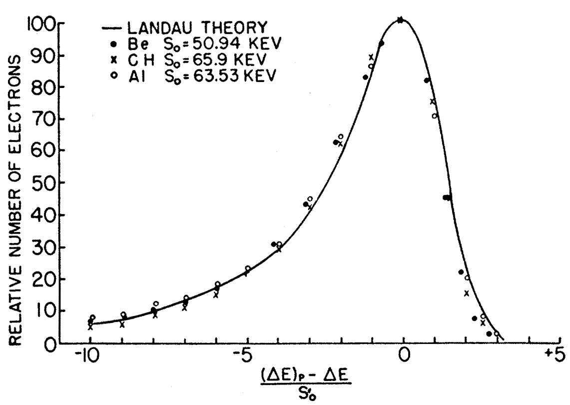

FIG. 4. Calculated and experimental straggling distributions of 15.7-Mev electrons after passing through absorbers of about 1 g/cm² of beryllium, polystyrene, and aluminum plotted *versus* energy in units of $S_0=2\pi e^4nt/mv^2$.

[9] R. Mather and E. Segrè, Phys. Rev. **84**, 191 (1951).
[10] E. Fermi, Phys. Rev. **57**, 459 (1940).
[11] O. Halpern and H. Hall, Phys. Rev. **73**, 477 (1948).

where $\epsilon=1+(4\pi ne^2\hbar^2/mI^2)$;

$$\Delta_c\simeq S_0\left[\ln\left(\frac{4\pi e^2n\hbar^2}{(1-\beta^2)mI^2}\right)-1\right] \text{ for } v\rightarrow c. \quad (6)$$

This reduction in energy loss involves only remote collisions which give rise to small energy transfers of the order of the excitation potentials. These collisions are therefore not expected to change the observed straggling distributions in the cases where the simple straggling theory is applicable. Substituting Eq. (4) in Eq. (2) and subtracting Eq. (6), we find that the corrected most probable energy loss in condensed materials for extreme relativistic particles is given by

$$\begin{aligned}\Delta_{pc}&=\Delta_p-\Delta_c=S_0[\ln(e^2mt/\hbar^2)+0.37],\\ &=S_0[\ln(t/a_0)+0.37],\end{aligned} \quad (7)$$

where a_0 is the Bohr radius of the hydrogen atom. This formula may be written in another way, which states explicitly the dependence on the volume density ρ and the surface density D of the sample. Putting in the constants with ρ and D in units of grams per cm³ and grams per cm², respectively, one obtains

$$\Delta_{pc}=0.1537(\Sigma Z/\Sigma A)D[19.43+\ln(D/\rho)] \text{ Mev}. \quad (8)$$

The simple formulas and straggling distributions do not hold very accurately for elements of higher atomic number for several reasons. One important complication is the fact that when the K or L excitation energies are of the same order as S_0 these energy losses must be treated separately in evaluating the straggling. Such a treatment of the straggling for lower energy electrons has been made by Blunck and Leisegang,[12] who found that they could account for the now classic straggling distributions of White and Millington.

The effect of bremsstrahlung on the most probable energy loss is very small, even in the case of gold where the mean radiation loss is about twice the mean ionization loss. The reason for this is that the radiation loss represents the effect of a small number of large energy transfers. An order of magnitude estimate of the fraction of the radiative losses which may contribute to the shift and broadening of the distribution curve can be made by considering the total radiation loss associated with energy losses less than $2S_0$. This fraction of the radiative loss is about 30 kev for the present gold sample for which the mean loss is about 2 Mev and only about 5 kev for the aluminum sample. From such an estimate it is seen that the radiation losses must be included in the case of gold, if one is to obtain results having a precision equal to that of the experimental measurements, but that such considerations are unnecessary for aluminum and elements of lower Z.

Multiple scattering can have only a small effect in most of the samples used in this work. Yang has shown that, in the case of electrons which enter and leave the

[12] O. Blunck and S. Leisegang, Z. Physik **128**, 500 (1950).

absorber normal to the surface, the effective thickness of the sample is increased by the factor $(1+t_r/6.9)$, where the thickness t_r is in radiation lengths.[13] This results in less than a 3 percent increase in the path length in the gold absorber and about $\frac{1}{2}$ percent in the aluminum absorber. The effect of multiple scattering is therefore completely accounted for by simply increasing the thickness of the absorber by the above factor.

The three above complications have been taken into account for our particular gold sample by Yang and Kennedy who have made numerical calculations for the expected straggling distributions suffered by 15.7-Mev electrons neglecting the effect of polarization.[14] This curve is shown with the experimental data for gold in Fig. 3.

DISCUSSION OF RESULTS

The results for the observed most probable energy losses in the various samples are given in Table I. The samples of lower atomic number were used with both 9.6- and 15.7-Mev electrons, while the gold and copper samples were used only at 15.7 Mev. The losses and the widths of the straggling distributions as calculated by the Landau equation are included in each case. For gold we have also included the values calculated by Yang and Kennedy.

The nature of the agreement between experiment and the calculations for the light elements is apparent in Figs. 2 and 4. The observed most probable energy loss is about 20 percent less than that calculated by the Landau equation without the density correction, but it is in good agreement with the simple asymptotic equation which includes the effect of polarization. The agreement between the observed and calculated distributions shown in Fig. 4 is well within the probable error of the experiments.

The corresponding results for gold are shown in Figs. 3 and 5. It can be seen that the shape of the straggling distribution is in very good agreement with the Yang-Kennedy calculation. The observed energy loss is less than that calculated without the polarization effect by about 0.20 Mev. The expected reduction due to the polarization effect as calculated by the asymptotic equation is 0.06 Mev. The reduction (as obtained from the more detailed calculations of Halpern and Hall for lead), increased by the density term

$$\{S_0 \ln[(\rho/ZA)_{\mathrm{Au}}(AZ/\rho)_{\mathrm{Pb}}]\},$$

is 0.10 Mev.

The distribution for copper is also given in Fig. 5,

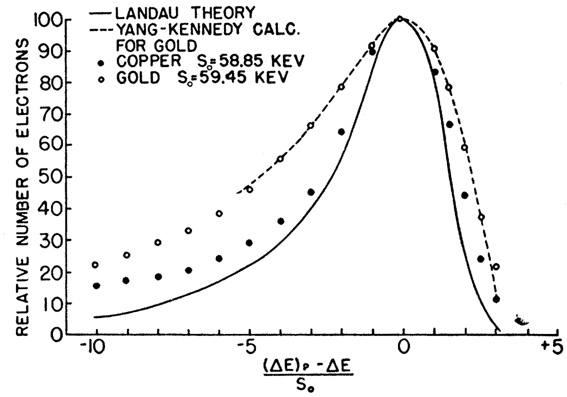

FIG. 5. Calculated and experimental straggling distributions of 15.7-Mev electrons after passing through absorbers of about 1 g/cm² of copper and gold plotted *versus* energy in units of $S_0=2\pi e^4 nt/mv^2$.

and it is apparent that the straggling distribution is somewhat wider than the Landau curve. The observed energy loss is somewhat less than that given by the asymptotic formula. No more exact calculations are available for copper.

The most probable energy loss has been measured for cosmic-ray mesons by Bowen and Roser.[15] They find that the most probable energy loss of fast mesons in a 3-cm crystal of anthracene ($C_{14}H_{10}$, density = 1.25) is 6.15 Mev. This is in good agreement with the value of 6.25 Mev calculated by the asymptotic formula [Eq. (8)].

SUMMARY

The agreement between experiment and theory is very good for the light elements both with respect to the magnitude of the energy loss and to the energy distribution.

The broadening of the energy-straggling distribution in gold is completely accounted for in a numerical calculation which includes the effect of the high K and L excitation energies and the additional straggling introduced by the radiation losses.

The only difficulty is the failure to account for an apparent polarization effect in gold, which is somewhat larger than the theoretical estimates of this effect.

It is a pleasure to express our thanks to T. J. Keegan for his cooperation in operating and maintaining the betatron and to J. C. Buchta, J. H. Malmberg, and G. E. Modesitt for their contributions in the development of the equipment and for their assistance in obtaining the data and organizing the results. We are very much indebted to Dr. C. N. Yang for valuable discussions and his stimulating interest in this work.

[13] C. N. Yang, Phys. Rev. **84**, 599 (1951).

[14] C. N. Yang and J. M. Kennedy, private communication. (Schultz in reference 4 carried out similar calculations for the data of Paul and Reich.)

[15] T. Bowen and F. X. Roser, Phys. Rev. **85**, 992 (1952).