have been proposed to describe the p-p interaction, has been to modify strongly the previous predictions for those potentials which are singular. Indeed, the use of Born approximation trial functions in the variational procedure is, in itself, inadequate for these cases.

The singular tensor-force model of Christian and Noyes, which was treated exactly, now presents only moderate agreement with experiment for the choice of sign $V_t = +18$ Mev, but only for angles greater than 45°. The predominance of singlet scattering at lower angles introduces too much anisotropy, so that deviations from experiment become considerable. The large corrections, introduced by the variational treatment of the **L·S** force model of Case and Pais, lead to an even less encouraging picture. While it is conceivable that more exact calculations would remove the large anisotropies, there is no *a priori* reason to expect this. On the other hand, the predictions of Jastrow's hard-core model are essentially unchanged on performing a variational calculation for the triplet scattering (which is reliable for this case). The cross section is reasonably flat, although its magnitude is somewhat low.

It must be emphasized that the preceding remarks in regard to the singular potentials are based on calculations performed with a zero cutoff at distances of the order of the nucleon Compton wavelength. It is always possible that a better fit with experiment might be obtained with a different choice of cutoff. In any case, the exact nature of the cutoff must be taken seriously.

The asymmetry in a double p-p scattering experiment was calculated in order to provide an additional means for distinguishing among the potential models. On this basis, the hard-core model which is found to yield small asymmetries is quite different from the singular tensor-force potential which predicts large asymmetries. The **L·S** potential also predicts large asymmetries, but it must be emphasized that this model, particularly, gives a poor fit with experiment. Finally, we should like to reiterate that the foregoing analysis is completely non-relativistic, and it is quite possible that polarization effects, in particular, would be modified in a more consistent relativistic theory.

We should like to thank Professor R. E. Marshak for useful discussions.

PHYSICAL REVIEW VOLUME 88, NUMBER 5 DECEMBER 1, 1952

An Analysis of the Energy Levels of the Mirror Nuclei, C^{13} and N^{13}

R. G. THOMAS*
Kellogg Radiation Laboratory, California Institue of Technology, Pasadena, California
(Received February 19, 1951)
(Revised manuscript received August 15, 1952)

An analysis employing the recent nuclear reaction theories of Wigner and others is given of the experimental data on the low energy interactions of s, p, d orbital neutrons and protons with C^{12} and s neutrons and protons with O^{16}. Assuming the equality of nn and pp nuclear interactions, it is possible to account for the data on the s interactions if the level spacing is considered in addition to the customary two resonance parameters: reduced width and level position; in particular, the displacement of conjugate levels can be attributed to the difference of the external wave functions for the odd particle, although with an uncertainty of about 25 percent which is due primarily to the lack of precise knowledge of the internal Coulomb energy of the excited states. The large magnitudes of the reduced width and level spacing indicate that two-body potential interactions exist between the odd particle and the C^{12} and O^{16} cores, and the values of the respective logarithmic derivatives indicate that these interactions are of about equal strengths. The energy dependence of the radiative capture cross section of s neutrons and protons with C^{12} can be understood if an additional quantity, the final-state reduced width, is included in the theory to take into account the energy-dependent external contribution to the transition moment. The experimental data are only sufficient to treat the p and d interactions in the one-level approximation; a reasonable explanation can be given of the observed displacements of conjugate levels in terms of the differences of the electromagnetic properties of the odd particle such as: external wave functions, spin-orbit interactions, and variations of the internal Coulomb energy. There is some indication from the data on radiative transitions that the independent-particle model also prevails in the p states; on the other hand, the small reduced widths of these states suggest a many-body description. Derivations based on the recent theories are given of the one-channel formulas and of the general one-level formulas which include the negative-energy alternatives. The radial dependences of the resonance parameters are discussed.

I. INTRODUCTION

SINCE there is considerable experimental material on the low levels of the mirror nuclei, N^{13} and C^{13}, it seems worth while to attempt a detailed investigation of such matters as the extent of the validity of the independent-particle model, the assumption of equality of nn and pp nuclear forces, and the applicability of the recent theories of nuclear reactions. The analysis is carried out by means of the theories due to Wigner and others,[1–9] and we are therefore concerned with the de-

* Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.

[1] E. P. Wigner, Phys. Rev. **70**, 15 (1946).
[2] E. P. Wigner, Proc. Am. Phil. Soc. **90**, 27 (1946).
[3] E. P. Wigner, Phys. Rev. **70**, 606 (1946).
[4] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947), referred to as FPW.
[5] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947), referred to as W-E.
[6] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).

termination from the data of the characteristic quantities: partial reduced level widths, unshifted level positions, displacements of corresponding levels, and level spacings; we are also concerned with radiative transition moments. The displacement of corresponding s states[10] has already been investigated by Ehrman;[11] the present treatment offers several new aspects to this problem. Section II presents the terminology and derivations of the formulas used for the interpretations of the data given in Secs. III and IV. The derivations include the negative-energy alternatives which enter in the consideration of level displacements. A formula is given for the electric dipole radiative capture which includes an energy-dependent contribution from the external moment in addition to the resonance contribution from the internal moment, the former being important for the treatment of transitions between levels with large reduced widths. Some relations between the quantities of the nuclear reaction theory in the one-channel approximation and the effective range theory are discussed as the latter theory is appropriate for the analysis of the scattering of low energy neutrons by C^{12}.

The available experimental data are surveyed in the introduction to Sec. III, and the subsequent analysis is given in three parts covering the interaction of s, p, d neutrons and protons with C^{12}. The s interaction data are rather extensive and permit some consideration of the level spacing in addition to the two resonance parameters (of the one-level approximation): reduced width and level position; the treatment of the p and d interactions, however, is limited to the one-level approximation. A comparison of the low energy interactions of s neutrons and protons with C^{12} and O^{16} is made in Sec. IV.

II. THEORY

A. Terminology

The terminology of Wigner and Eisenbud[5,6] is used to describe the nuclear configuration space. The external region of this space is subdivided into channels which comprise alternative pairs c of two bodies with an energy of relative motion ϵ_c, which may be positive or negative, relative angular momentum $l\hbar$, and reduced mass M_c; the members of the pair are separated by distances r_c which are greater than or equal to the channel radii a_c. We attempt to choose the a_c greater than the range of nuclear forces between the bodies in order that the external wave functions reflect the solutions of the wave equation containing only the Coulomb interaction. It is desirable, on the other hand, to choose the a_c as small as possible, consistent with the above requirement, so that the characteristic quantities of the resonance theory which are determined from the data will contain primarily information concerning the nuclear interactions. A common prescription for the channel radii (which we shall follow) is $a_c=(e^2/2mc^2)\times(A_{1c}^{\frac{1}{3}}+A_{2c}^{\frac{1}{3}})$, A_{1c} and A_{2c} being the mass numbers of the bodies of the pair. The letter c is used to describe all of the features of the channel, unless it is necessary to distinguish the positive-energy ($\epsilon_{c+}>0$) from the negative-energy ($\epsilon_{c-}<0$) channels in which case the symbols $c+$ and $c-$ will be used, respectively.

The rest of the nuclear configuration space is the internal region τ which includes not only that part wherein all particles are close together but also that part representing pairs c with separation $r_c<a_c$ and the total subspace with three or more bodies in relative motion. The hypersurface which separates the internal from the external region is denoted by S. In order to keep the number of characteristic quantities of the theory to a minimum, one can usually ignore the $c-$ channels by allowing their radii to be infinite in which case they become a part of the internal region. However, in the treatment of level displacements and, as emphasized by Wigner,[6] in the consideration of the behavior of cross sections near threshold, it is necessary to include the near-threshold channels in the external region.

If the channel radii are large enough, the external wave function can be factored as

$$P_l(\Omega_c)\psi_c\mathfrak{F}_{cl}(r_c)r_c^{-1}, \tag{1}$$

where $\mathfrak{F}_{cl}(r_c)r_c^{-1}$ and $P_l(\Omega_c)$ describe the relative radial and angular distributions, and ψ_c is the product of the wave functions for the internal distributions of the pair. We will simplify the equations by absorbing the factors $P_l(\Omega_c)$ and r_c^{-1} into ψ_c; the subscript c will then also be used to characterize the l and will be omitted from all but one of the symbols in an equation, provided there is no ambiguity.[12] It is assumed that on S

$$\int_S \psi_c\psi_{c'}dS=\delta_{cc'}. \tag{2}$$

B. External Wave Functions

The radial factor in (1) satisfies the wave equation

$$\mathfrak{F}_c''+(2M_c/\hbar^2)(\epsilon_c-\mathcal{V}_c)\mathfrak{F}_c=0, \tag{3a}$$

(where a prime signifies differentiation with respect to r) with the interaction

$$\mathcal{V}_c=Z_{1c}Z_{2c}e^2r_c^{-1}+(\hbar^2/2M_c)l(l+1)r_c^{-2}. \tag{3b}$$

In the notation of Yost, Wheeler, and Breit,[13] the posi-

[7] T. Teichmann, dissertation, Princeton University (1949), (unpublished).
[8] T. Teichmann, Phys. Rev. **77**, 506 (1950).
[9] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).
[10] The letters s, p, d are used throughout to denote the relative orbital angular momentum of the channel (C^{12} or O^{16}+proton or neutron) which is a good quantum number since C^{12} has zero spin; the $\frac{1}{2}$-integer subscript denotes the total nuclear spin. It is not intended that this notation imply that configuration is a good quantum number in the internal region.
[11] J. B. Ehrman, Phys. Rev. **81**, 412 (1951).
[12] The letter s (c in this paper) is used by W-E to characterize the channel spin. For the present problem this is not necessary since the channel spin is always $\frac{1}{2}$.
[13] Yost, Wheeler, and Breit, Phys. Rev. **49**, 174 (1936).

tive-energy solution which is regular at the origin is designated by $F(kr)$ and has the asymptotic form for large r,

$$F_{c+}\sim\sin(x-\tfrac{1}{2}l\pi-\eta\log 2x+\sigma), \tag{4a}$$

and a solution which is linearly independent of F and irregular at the origin is conveniently taken with the asymptotic form for large r,

$$G_{c+}\sim\cos(x-\tfrac{1}{2}l\pi-\eta\log 2x+\sigma); \tag{4b}$$

the quantities entering (4) are

$$x_{c\pm}=kr;\quad k_{c\pm}=(2M|\epsilon|/\hbar^2)^{\frac{1}{2}};$$
$$\eta_{c\pm}=MZ_1Z_2e^2/\hbar^2k;\quad \sigma_{c+}=\arg\Gamma(1+l+i\eta).$$

The general solution $\mathfrak{F}(r)$ will be a linear combination of F and G. The Wronskian relation for these two particular solutions, which directly follows from (3) and (4), is

$$F'G-G'F=k_{c+}. \tag{5}$$

Extensive tables have been prepared by Breit and his collaborators[14] for obtaining F and G and their derivatives when $\eta>0$; they are used here. If there is no Coulomb interaction, F and G are related to the Bessel functions

$$F_{c+}=(\pi x/2)^{\frac{1}{2}}J_{l+\frac{1}{2}}(x),$$
$$G_{c+}=(-1)^l(\pi x/2)^{\frac{1}{2}}J_{-l-\frac{1}{2}}(x). \tag{6}$$

For the $c-$ channels only the solution to (3) vanishing at large distances from the origin can occur; it is the Whittaker function[15]

$$W_{-\eta,\,l+\frac{1}{2}}(2x_{c-})=\frac{\exp(-x-\eta\log 2x)}{\Gamma(1+l+\eta)}\int_0^\infty t^{l+\eta}e^{-t}\left(1+\frac{t}{2x}\right)^{l-\eta}dt. \tag{7}$$

When $\eta>0$, this form is convenient for obtaining the radial dependence of W; for obtaining its logarithmic derivative to the required accuracy, it was expedient to use the power series expansion,[15] which is too lengthy to be given here. If there is no Coulomb interaction in a $c-$ channel, we have from (7) for s, p, d orbitals

$$W_{0,\frac{1}{2}}(2x)=\exp(-x), \tag{8s}$$

$$W_{0,\frac{3}{2}}(2x)=(1+x^{-1})\exp(-x), \tag{8p}$$

$$W_{0,\frac{5}{2}}(2x)=(1+3x^{-1}+3x^{-2})\exp(-x). \tag{8d}$$

Another convenient set of linearly independent solutions to (3) consists of a unit-flux emerging wave E_{c+} and a unit-flux incident wave E_{c+}^*; in terms of F and G, they are

$$E_{c+}=(M/k\hbar)^{\frac{1}{2}}(G+iF)\exp i(\tfrac{1}{2}l\pi-\sigma), \tag{9a}$$

and

$$E_{c+}^*=\text{complex conjugate } E_{c+}.$$

We also put

$$E_{c-}=(M/k\hbar)^{\frac{1}{2}}W_{-\eta,\,l+\frac{1}{2}}(2x). \tag{9b}$$

According to (5), (9) the flux equation is

$$(\hbar/2iM_{c+})(E^*E'-EE'^*)=1, \tag{10}$$

and it is, of course, zero for the $c-$ channels.

In discussing conditions at the nuclear surface, we shall need to evaluate the real and imaginary parts of the logarithmic derivatives, $g_c=E'/E$. From (5) and (9), these are

$$g_{c+}^{\mathrm{Re}}=(FF'+GG')(F^2+G^2)^{-1},$$
$$g_{c+}^{\mathrm{Im}}=k(F^2+G^2)^{-1}=M/\hbar|E|^2, \tag{11}$$
$$g_{c-}^{\mathrm{Re}}=W'W^{-1},\quad g_{c-}^{\mathrm{Im}}=0,$$

where

$$g_c=g^{\mathrm{Re}}+ig^{\mathrm{Im}},\quad r_c=a_c.$$

As there are no tables of $W_{-\eta,\,l+\frac{1}{2}}$ and as it is difficult to evaluate g_{c-}^{Re} from the power series expansion for W, the WKB approximation was investigated, *viz.*,

$$ag_{c-}=-\xi+\tfrac{1}{2}[\eta x+l(l+1)]\xi^{-2}, \tag{12}$$

where

$$\xi=[l(l+1)+2\eta x+x^2]^{\frac{1}{2}}.$$

In three different cases (12) was compared with exact evaluations and found to be quite satisfactory if $(l+\frac{1}{2})^2$ is substituted for $l(l+1)$ as observed by Yost, Wheeler, and Breit for the positive-energy solutions.[13,16] Expression (12) with this substitution is used hereafter for the evaluation of g_{c-} when $\eta\neq 0$.

The quantity $dg_{c+}^{\mathrm{Re}}/d\epsilon$ enters in the determination of the reduced width from the experimental width. In the barrier region where $G\gg F$, $G'\gg F'$, it can be obtained by approximating $g_{c+}^{\mathrm{Re}}=G'/G$ and taking the slope of a plot of G'/G as a function of ϵ. However, in the case of the s proton interaction with C^{12}, this slope could not be obtained with sufficient accuracy so that we used the expression[17]

$$d(aG'/G)/d\log\epsilon=x_c\Theta_0^{-2}(x_c)\left[H(\eta)-\int_0^{x_c}\Theta_0^2(x)dx\right], \tag{13}$$

[14] Bloch, Hull, Broyles, Bouricius, and Breit, Phys. Rev. **80**, 553 (1950), and Revs. Modern Phys. **23**, 147 (1951).

[15] W. Magnus and F. Oberhettinger, *Formeln und Sätze für die Speziellen Funktionen der Mathematische Physik* (J. Springer, Berlin, 1948), second edition, Chap. VI, Sec. 2.

[16] As an illustration of the validity of this approximation, the table below lists the various evaluations for the first excited s states of O^{17} and F^{17}, which are treated in IV. The difference of the logarithmic derivatives of the mirror levels enters in the determination of the level displacement, and it is apparent from this example that it can be obtained accurately from the WKB—$(l+\frac{1}{2})^2$ approximation for F^{17} and the simpler, exact expression (8s) for O^{17}.

Values of $-ag_{c-}$; $a=4.60\times10^{-13}$ cm.

Channel	Exact	WKB—$(l+\frac{1}{2})^2$	WKB—$l(l+1)$
$O^{16}+n$, $x=1.770$, $\eta=0$	1.770	1.802	1.770
$O^{16}+p$, $x=0.275$, $\eta=4.36$	1.387	1.384	1.330

[17] This expression can be obtained from Eqs. (49) and (50) of reference 24 by setting $\delta_1=\delta_2=\frac{1}{2}\pi$ and then taking the limit as $E_2\rightarrow E_1$.

where

$$\Theta_0(x)=C_0G(x),\quad x_c=ka,$$

$$C_0=\left(\frac{2\pi\eta}{e^{2\pi\eta}-1}\right)^{\frac{1}{2}},\quad H(\eta)=\eta-2\eta^3\sum_{\nu=1}^{\infty}\frac{\nu}{(\nu^2+\eta^2)^2},$$

the quantity $\Theta_0(x)$ being tabulated as an auxiliary function for G.[14]

C. One-Level, Many-Channel Formulas

The essential elements of the one-level nuclear reaction formula, which will be needed in the later discussion, can be obtained quite simply from a consideration of the boundary conditions which must be satisfied by the external wave functions at the surface S and by an application of Green's theorem. The present steady-state derivation draws mainly from the first of Wigner's recent papers[1] but includes the negative-energy alternatives as indicated in later papers;[6, 9] as we require less general results than those obtained by him, a somewhat simpler treatment will suffice. A more general derivation employing the R matrix formalism of Wigner and Eisenbud[5] leads to essentially the same results. Some aspects of the present treatment were suggested by the work of Feshbach, Peaslee, and Weisskopf.[4]

On the surface S the value and normal gradient of any general solution to $H\Psi=E\Psi$ are represented as

$$\Psi=\textstyle\sum_c\psi_cV_c, \tag{14a}$$

$$\text{grad}_n\Psi=\textstyle\sum_c\psi_c(D_c-a_c^{-1}V_c), \tag{14b}$$

and the logarithmic derivatives as

$$f_c=D_c/V_c; \tag{14c}$$

the quantities V_c and D_c denote the value and derivative of $\mathfrak{F}_c$ at $r_c=a_c$ (note that ψ_c contains the factor r_c^{-1}). We consider now the particular solution Ψ_e which has an incident wave, normalized to unit flux, only in the e (entrance) channel, and emerging waves in the positive-energy r (reaction) channels, and exponentially-decaying wave functions in the $c-$ channels ($c+$ will denote any or all positive-energy channels if it is unnecessary to distinguish the e from the r channels). In the external region this particular solution can be represented as

$$\Psi_e=\psi_e(E_e^*-U_{ee}E_e)-\textstyle\sum_{c\neq e}\psi_cU_{ec}E_c. \tag{15}$$

The submatrix $U_{c+,c+}$ is the unitary, symmetric collision matrix giving the coefficients of the unit-flux emerging waves in terms of the coefficients of the unit-flux incident waves; the submatrix $U_{e,c-}$ gives the coefficients of the E_{c-}. The important quantities are the logarithmic derivatives on S which, for the r and $c-$ channels, are the g_c of (11); the e channel, however, contains a scattered wave in addition to the incident wave so that

$$f_e=f_e^{\text{Re}}+if_e^{\text{Im}}=(E_e'^*-U_{ee}E_e')(E_e^*-U_{ee}E_e)^{-1}. \tag{16a}$$

Expressing U_{ee} in terms of f_e, we have

$$U_{ee}=(E_e^*/E_e)(f_e-g_e^*)(f_e-g_e)^{-1}, \tag{16b}$$

all external wave functions being evaluated at $r_c=a_c$. Comparing (14) with (15) and using (11) and (16b), we find

$$|V_e|^2=(4M_e/\hbar)g_e^{\text{Im}}/|f_e-g_e|^2, \tag{17a}$$

$$|V_r|^2=(M_r/\hbar)|U_{er}|^2/g_r^{\text{Im}}, \tag{17b}$$

$$|V_{c-}|^2=|U_{e,c-}|^2\cdot E_{c-}^2. \tag{17c}$$

The reduced widths are defined here by the relations

$$\gamma_c^2\equiv(\hbar^2/2M_c)|V_c|^2/\int_\tau|\Psi_e|^2d\tau \tag{18}$$

each of which is a measure of the probability that the pair of particles c appears at the nuclear surface; the reduced widths and the logarithmic derivatives contain all the information needed concerning conditions within the internal region. Introducing (18) into (17b), the reaction cross sections and the coefficients of the exponentially-decaying wave functions are expressed in terms of the reduced widths:

$$\begin{aligned}\sigma_{er}&=\pi k_e^{-2}(2l_e+1)|U_{er}|^2,\\ |U_{er}|^2&=4\gamma_r^2g_r^{\text{Im}}g_e^{\text{Im}}/\gamma_e^2|f_e-g_e|^2,\\ |U_{e,c-}|^2&=4\gamma_{c-}^2k_{c-}W_{c-}^{-2}g_e^{\text{Im}}/\gamma_e^2|f_e-g_e|^2.\end{aligned} \tag{19}$$

By applying Green's theorem it may be shown that the width term in the denominator of the resonance formula arises from the imaginary part of (f_e-g_e) and that the energy term and the level shift arise from the real part.

For two general solutions $\Psi_1(E_1)$ and $\Psi_2(E_2)$ of the equation $H\Psi=E\Psi$ we have, using Green's theorem, (14), and (2):

$$\begin{aligned}(E_2-E_1)\int_\tau\Psi_1\Psi_2^*d\tau&=\int_\tau[\Psi_1(H\Psi_2)^*-\Psi_2^*H\Psi_1]d\tau\\ &=\int(\hbar^2/2M_c)(\Psi_2^*\,\text{grad}_n\Psi_1-\Psi_1\,\text{grad}_n\Psi_2^*)dS\\ &=\textstyle\sum_c(\hbar^2/2M_c)V_{1c}V_{2c}^*(f_{1c}-f_{2c}^*).\end{aligned} \tag{20}$$

By defining quantities analogous to (18),

$$\gamma_{12,c}^2\equiv(\hbar^2/2M_c)V_{1c}V_{2c}^*\Big/\int_\tau\Psi_1\Psi_2^*d\tau \tag{21}$$

(20) may be simplified to

$$E_2-E_1=\textstyle\sum_c\gamma_{12,c}^2(f_{1c}-f_{2c}^*). \tag{22}$$

If we set $E_1=E_2$, the imaginary part of (22) becomes

$$\gamma_e^2f_e^{\text{Im}}+\textstyle\sum_r\gamma_r^2g_r^{\text{Im}}=0, \tag{23a}$$

which is equivalent to the expression for conservation of flux,

$$|U_{ee}|^2+\sum_r|U_{re}|^2=1. \tag{23b}$$

The $c-$ channels do not appear in (23) since $g_{c-}^{\mathrm{Im}}=0$. Because $g_{c+}^{\mathrm{Im}}\geqslant 0$, (23a) requires that $f_e^{\mathrm{Im}}\leqslant 0$. Adding and subtracting a term $\gamma_e^2 g_e^{\mathrm{Im}}$ in (23a), we can write

$$g_e^{\mathrm{Im}}-f_e^{\mathrm{Im}}=\Gamma/2\gamma_e^2 \tag{23c}$$

where

$$\Gamma=\sum_{c+}\Gamma_c, \quad \Gamma_{c+}=2\gamma^2 g^{\mathrm{Im}};$$

Γ is the total width and the Γ_{c+} are the partial widths.

In order to treat the real part of (22), we introduce the one-level approximation which is to consider that $\gamma_{12,c}^2=\gamma_c^2=$constant for a finite range of energies and of boundary conditions on S. This amounts to the assumption that over the resonance region, which must be narrow compared with the level spacing, the shape of the internal wave function does not change in first approximation [according to (20) its logarithmic derivatives on S will change in second approximation—see reference (1)], its amplitude, however, depends critically on the external conditions. Variations of the reduced widths are attributed to other levels and expected to be of order $(E-E_{\mathrm{res}})/D$, where D is the level spacing. We let $E_1=E_{\mathrm{resonance}}$ be the energy at which $f_e^{\mathrm{Re}}=g_e^{\mathrm{Re}}$ and let $E_2=E$ be the variable energy, then the real part of (22) becomes

$$(f_e^{\mathrm{Re}}-g_e^{\mathrm{Re}})=(E_{\mathrm{res}}+\Delta-E)/\gamma_e^2 \tag{24a}$$

where

$$\Delta=\sum_{c\pm}\Delta_c=-\sum_{c\pm}\gamma_c^2[g_c^{\mathrm{Re}}(E)-g_c^{\mathrm{Re}}(E_{\mathrm{res}})]. \tag{24b}$$

Inserting (24a) and (23c) into (19) we obtain the absolute square of the components of the collision matrix:

$$|U_{e,c\neq e}|^2=\Gamma_e\Gamma_{c\neq e}[(E_{\mathrm{res}}+\Delta-E)^2+\tfrac{1}{4}\Gamma^2]^{-1}, \tag{25}$$

this expression also being applicable to the $c-$ channels with $\Gamma_{c-}=2\gamma^2 kW^{-2}$. (The Γ_{c-} are not, however, real widths since they do not contribute to the total width Γ.) A relation typifying the resonance theory, which follows from (18) and (17a), is

$$\int_\tau|\Psi_e|^2 d\tau=\hbar\Gamma_e[(E_{\mathrm{res}}+\Delta-E)^2+\tfrac{1}{4}\Gamma^2]^{-1}. \tag{26}$$

The quantity Δ, which as defined above is zero at $E=E_{\mathrm{res}}$, is generally not constant, even in the one-level approximation, but a function of energy through the external wave function factors. In many cases, a linear expansion of Δ with respect to E about the resonance is adequate. With such an expansion the cross section given by (19) and (25) becomes[18]

$$\sigma_{er}=\pi k_e^{-2}(2l_e+1)\Gamma_e{}^\dagger\Gamma_r{}^\dagger[(E-E_{\mathrm{res}})^2+\tfrac{1}{4}\Gamma^{\dagger 2}]^{-1} \tag{27a}$$

with

$$\begin{aligned}\Gamma_{c+}{}^\dagger&=\Gamma_{c+}[1+\Sigma_{c\pm}\gamma_c^2(dg_c^{\mathrm{Re}}/dE)]^{-1}{}_{E=E_{\mathrm{res}}},\\ \Gamma^\dagger&=\Sigma_{c+}\Gamma_{c+}{}^\dagger.\end{aligned} \tag{27b}$$

[18] R. G. Thomas, Phys. Rev. **81**, 148 (1951).

Equation (27a) is the type of expression with which experimental resonance data are usually compared so that the $\Gamma_{c+}{}^\dagger$ may be regarded as the observed widths. As a consequence of the distinction between Γ_{c+} and $\Gamma_{c+}{}^\dagger$, it is not strictly correct to obtain reduced widths by merely dividing the observed width by the barrier factor, $2g_c^{\mathrm{Im}}$, more detailed considerations being required. Generally $dg^{\mathrm{Re}}/dE\lesssim Ma\hbar^{-2}$ (it is, however, zero for s neutron positive-energy channels) so that those terms in the sum of (27b) should be considered for which $\gamma_c^2\gtrsim\hbar^2/10Ma$. It is apparent from (20) that the inclusion of the $c-$ channels in the sum in (27b) is simply a consequence of not including them in the volume integral τ appearing in (20) and (21). Provided E_{res} is not near a threshold for a $c-$ channel, these channels may be ignored by a redefinition of τ and then the only concern is the $c+$.

The above expression for Δ differs from that given by W-E,[19] but this distinction is purely formal, lying entirely in the choice of the reference energy. We can obtain their expression for Δ_λ by assuming the existence of a solution $X_\lambda(E_\lambda)$ satisfying the equation $HX_\lambda=E_\lambda X_\lambda$ and the boundary condition[9]

$$f_{\lambda c}=-\bar{b}_c \tag{28}$$

for all c, the $\bar{b}_c$ being an arbitrary set of finite real numbers. Substituting $X_\lambda(E_\lambda)$ for $\Psi_2(E_2)$ and $\Psi(E)$ for $\Psi_1(E_1)$, and assuming that $\gamma_{1\lambda,c}^2=\gamma_c^2=$constant, the real part of (22) reduces again to (24a) but with

$$\Delta_\lambda=\sum_{c\pm}\Delta_{\lambda c}=-\sum\gamma_c^2(g_c^{\mathrm{Re}}(E)+\bar{b}_c) \tag{24c}$$

and with E_{res} replaced by E_λ, the reference energy; Δ_λ is interpreted as the total level shift and the $\Delta_{\lambda c}$ as the partial level shifts, referred to the boundary condition (28).[19]

If a level is bound, all channels are then $c-$ and the cross section formula is of no significance; nevertheless, Δ_λ signifies the shift of the level from its position with the reference boundary condition given by (28).

The same type of argument as that which led to Eqs. (24) can be used to compare corresponding levels of mirror nuclei. In Eq. (20) we replace $\Psi_1(E_1)$ by $\Psi_p(E_p)$, a solution to $H_p\Psi_p=E_p\Psi_p$, and $\Psi_2(E_2)$ by $\Psi_n(E_n)$, a solution to $H_n\Psi_n=E_n\Psi_n$, where the subscripts n and p denote the nucleus with the odd neutron or proton, respectively. If we assume equality of nn and pp nuclear interactions, then $H_p-H_n\equiv V$ is the difference of the Coulomb, electromagnetic spin-orbit, and mass energies. Furthermore, in the one-level approximation we assume that, irrespective of the boundary conditions, in the internal region Ψ_p and Ψ_n are the same to within a multiplicative constant. Then the real part of (22) reduces to

$$E_n-E_p=-\langle V\rangle_\tau+\Delta_{\lambda n}-\Delta_{\lambda p}, \tag{30a}$$

[19] In W-E the boundary condition on the X_λ is $\bar{b}_c=l_c/a_c$, where l_c is the channel orbital angular momentum; in reference 6 the condition $\bar{b}_c=-1/a_c$ is used. A detailed discussion on the choice of $\bar{b}_c$ is given in reference 9.

if we take $\bar{b}_{nc}=\bar{b}_{pc}$. The difference,

$$\Delta_{\lambda n}-\Delta_{\lambda p}=\sum_{c\pm}\gamma_c^2(g_{pc}^{\mathrm{Re}}-g_{nc}^{\mathrm{Re}}), \qquad (30b)$$

is referred to as the boundary-condition level displacement[20] and $\langle V\rangle_\tau$ is the mean value of V in the internal region.

D. One-Channel Approximation

In the one-channel approximation, we disregard the terms in (22) with $c\neq e$ as regards their effect on f_e but otherwise consider the reactions if there are r channels. That is, the total level shift and width are attributed entirely to the e channel. This approximation is suited to the treatment of the levels of C^{13} and N^{13} below 6 Mev since the channel for emission of the odd neutron or proton can be taken as the e channel and, aside from radiation, all others are $c-$ channels of high binding energy; these $c-$ channels may be eliminated from (22) by allowing their radii to be infinite in which case the γ_{c-}^2 vanish. Expression (30b) for the displacement of corresponding levels will depend only on the boundary conditions on the e channels; (in III, however, we consider the possible effect of the $c-$ on this displacement). Although the formulas of the one-channel case may be well known from the treatments of the two-body interactions, they are repeated here in order to indicate the notation, only those formulas being given which are needed in the later discussion. Here we are not restricted to the one-level approximation.

In the one-channel case the imaginary part of (22) is assumed to be zero so that $f_e^{\mathrm{Im}}=0$, and in the limit as $E_2\rightarrow E_1$ the real part of (22) is

$$-\gamma_e^{-2}=df/dE, \qquad (31a)$$

and (27b), which relates the reduced widths to the observed widths in the one-level approximation, becomes

$$\gamma_e^{-2}=(2g^{\mathrm{Im}}/\Gamma^\dagger)-(dg^{\mathrm{Re}}/dE),\quad E=E_{\mathrm{res}}. \qquad (31b)$$

In connection with (24a), the resonance energy was defined as the energy at which

$$f=g_e^{\mathrm{Re}}; \qquad (31c)$$

the maxima of the reaction cross sections may, however, be shifted slightly from the resonance energy on account of the energy dependences of the factors g_e^{Im}, g_r^{Im}, k_e^{-2} in (19). If the e channel has negative energy, the bound levels will appear where (31c) is satisfied.

According to (16b) the ee component of the collision matrix for positive energy e channels has modulus one and can be written

$$U_{ee}=(-1)^{l_e}\exp 2i(\sigma_e+\delta_e), \qquad (32a)$$

the nuclear phase shift being

$$\delta_e=\tan^{-1}[g^{\mathrm{Im}}/(f-g^{\mathrm{Re}})]-\tan^{-1}(F/G). \qquad (32b)$$

If the incident part of the wave function for the e channel arises from an incident plane wave beam of unit flux, the particular solution (15) is to be multiplied by $(-1)^{l_e}\pi^{\frac{1}{2}}k^{-1}(2l_e+1)^{\frac{1}{2}}$; in the external region, r times the radial part of its wave function is then given by

$$-i^{l_e+1}(4\pi M/\hbar k^3)^{\frac{1}{2}}(2l_e+1)^{\frac{1}{2}}\exp[i(\sigma_e+\delta_e)] \times(F\cos\delta+G\sin\delta). \qquad (32c)$$

In the interaction of s nucleons with C^{12}, the one-level approximation is shown to be inadequate by the observed variation of γ_e^2 by about a factor of three in the energy range investigated; according to (31a) this variation will also be indicated by a deviation of the energy dependence of f_e from linearity. The effects of other levels can be included by means of the R-matrix formalism; in the one-channel case the R-matrix is simply the R function and equal to the reciprocal of f if we set $\bar{b}_e=0$:[9]

$$f^{-1}=R=\sum_\lambda\gamma_\lambda^2(E_\lambda-E)^{-1}. \qquad (33)$$

The γ_λ^2 are the energy-independent reduced level widths. They are to be distinguished from the energy-dependent quantity $\gamma^2(E)$ which we have designated as the reduced width; according to (33) and (31a)

$$\gamma^2(E)=R^2/\sum_\lambda\gamma_\lambda^2(E_\lambda-E)^{-2}, \qquad (34)$$

and

$$\gamma^2(E=E_\lambda)=\gamma_\lambda^2.$$

It is γ^2 which can be determined from the width of a resonance level rather than γ_λ^2 unless E_{res} should happen to coincide with E_λ; thus, we are primarily concerned with the former quantity. However, in the one-level approximation γ^2 is considered to be constant, only one term in the R function being considered, and there is then no distinction. Expressions (33) and (34) involve too many parameters for applications. A satisfactory way of treating deviations from the one-level approximation with but one additional parameter is to approximate

$$R=R_\infty+\gamma_\lambda^2(E_\lambda-E)^{-1}, \qquad (35)$$

R_∞ being a constant which is the order of magnitude of the average reduced width divided by the level spacing. Wigner[1] has shown that this form will give accurately the cross section for scattering over a wide range of energies provided there are no intervening resonances. Another way of introducing the effects of the other levels is, as suggested by FPW, to represent the energy dependence of f by

$$f(E)=-K\tan Z(E), \qquad (36)$$

where K is a wave number for the incident particle

[20] We distinguish here between "shift" and "displacement." The former is used to denote the shift of a level for any particular reason and the latter denotes the difference of the corresponding shifts of conjugate levels. For example, the Δ_λ are the level shifts which contribute an amount $\Delta_{\lambda n}-\Delta_{\lambda p}$ to the displacement of conjugate levels. The net displacement is the difference of the displacements of a conjugate pair of excited states from that of the ground states; it is the displacement which is apparent when the energy level diagrams of the mirror nuclei are placed side by side with their ground states at the same level, as in Fig. 1.

within the nucleus, $\sim 1\times 10^{13}$ cm^{-1}, and $Z(E)$ is a monotonic increasing function which is anticipated to have a fairly smooth energy dependence. This representation is particularly convenient and will be used in III and IV. Wigner[21] has shown that every R function can be put in this form with a convergent series representation for $Z(E)$.

In comparing mirror interactions we assume that V is sufficiently uniform in the internal region so that

$$f_n(E_n)=f_p(E_p-\langle V\rangle_\tau). \qquad (37)$$

The net displacements of mirror levels are obtained by matching the ground states, as in Fig. 1, thus correcting for the energy difference given by (30a) for the ground state. However, (37) and (30a) indicate that in order to have a common scale for the nuclear excitation energy of the excited states, the ground state of the odd-neutron nucleus should be displaced by an amount equal to the ground-state boundary-condition displacement, $\Delta_{\lambda n}-\Delta_{\lambda p}$, above the ground state of the odd-proton nucleus, assuming that the quantity $\langle V\rangle_\tau$ is the same in the excited state as in the ground state.

It has been shown by Teichmann[22] that nuclear resonance levels and their widths may be interpreted in terms of the quantities of the effective range theory, such as scattering length and effective range. This theory has been applied with considerable success to the analysis of the s np and pp interactions;[23,24] the s $(C^{12}+n)$ interaction will be shown to be similar in some respects to the $^3S_1$ np interaction so that it is appropriate to use the effective range theory in the former case in order to bring out the similarity. Moreover, we will be able to use the graphs given by Blatt and Jackson[24] for interpreting the interaction parameters in terms of the strengths and intrinsic ranges of various potentials. In dealing with broad levels, the nuclear resonance theory is considered objectionable on account of the sensitivity of its characteristic parameters to the channel radius; the effective range theory, on the other hand, does not contain a channel radius.[25]

If there is no Coulomb or centrifugal interaction, the comparison-function potential is conveniently taken to be zero everywhere, and the expansion of the effective range theory is

$$k\cot\delta_0=-\alpha+\tfrac{1}{2}r_0k^2-Pr_0{}^3k^4+\cdots \qquad (38)$$

where r_0 is the effective range, α^{-1} the scattering length, and P the shape factor. Expression (38) holds for a bound state with the substitutions: $\cot\delta=-i$, $k=-i(2MB/\hbar^2)^{\frac{1}{2}}$; B is its binding energy. According to (32c) and (38) the relation between the logarithmic derivative and the scattering length at a nuclear energy E_0 corresponding to the neutron threshold is given by

$$f^{-1}(E_0)=a(1-\beta^{-1}),\quad \beta=\alpha a, \qquad (39a)$$

where a is the channel radius, which is not to be mistaken here for the scattering length. The reduced width at E_0 is given by

$$\gamma^2(E_0)=(\hbar^2/2Ma)(1-\beta)^2(1-\beta+\tfrac{1}{3}\beta^2-\tfrac{1}{2}r_0a^{-1})^{-1}, \qquad (39b)$$

which follows from (31a), (32c), and (38).

E. Radiative Capture

Channels for radiation were not included in the considerations of II–C. It is evident that Eq. (23a), expressing conservation of flux, must include a term in $f_e{}^{\mathrm{Im}}$ to take account of the radiation and there will thus be terms in the total width for partial radiative widths, as already shown by FPW. No investigations have been reported concerning the effect of radiation on the real part of f_e, leading to terms in the total level shift from the partial radiative shifts; such terms are expected to be negligible owing to the smallness of radiative widths.

An expression is given here for the electric dipole s-wave capture cross section, obtained from the usual formula for the transition rate between states, which separates the contributions to the transition moment from the external and internal regions. The reduced width of the final state, to which the external contribution is proportional, enters as a parameter additional to those contained in the familiar resonance capture formulas. The contribution to the radiation from the internal region is assumed to be proportional to $\int_\tau|\Psi|^2d\tau$ as given by (26) in the one-level approximation; the contribution from the external region is calculable in terms of the known wave functions. It is known that the contribution to the dipole transition moment in the photodisintegration of the deuteron arises predominantly from beyond the range of nuclear forces so that it should not be surprising that in the treatment of radiative capture in light nuclei, some of which have rather large reduced widths, it is also necessary to include the external contribution.

Only the one-channel case is considered and in order to simplify the notation a two-body type of wave function is used to describe the internal as well as external regions. Neglecting exchange effects, the cross section for electric dipole emission is given by[26]

$$\sigma_\gamma(e\rightarrow f)=64\pi^4(h\nu)^3e^2A^2q_eq'\left|\int_0^\infty ru_eu_fdr\right|^2/3h^4c^3, \qquad (40a)$$

[21] E. P. Wigner, Ann. Math. 53, 36 (1951).

[22] T. Teichmann, Phys. Rev. 83, 141 (1951); Th. Sexl, Naturwiss. 19, 454 (1951).

[23] J. M. Blatt and J. D. Jackson, Phys. Rev. 76, 118 (1949).

[24] H. A. Bethe, Phys. Rev. 76, 38 (1950).

[25] However, an attempt to adapt this theory to the $(C^{12}+p)$ s interaction encounters the difficulty, due to the strong Coulomb barrier, that the comparison function, usually designated ψ, is in the internal region much larger than the actual wave function, and as a result the effective range has little significance and the expansion which is analogous to (38) has poor convergence. One may overcome this difficulty by suitably modifying the comparison-function potential, but then the computation becomes unnecessarily difficult and available graphs cannot be used for interpretation.

[26] G. Breit and D. M. Yost, Phys. Rev. 48, 203 (1935).

where $h\nu$ is the transition energy, q_e is the statistical factor for the formation of the initial state $((l_e+\frac{1}{2}\pm\frac{1}{2})/(2l_e+1)$ for the state with spin $J=l\pm\frac{1}{2})$, $A=(Z_1A_2-A_1Z_2)/(A_1+A_2)$ is the reduced charge number, q' is the angle-spin integration factor ($s_{\frac{1}{2}}\rightarrow p_{\frac{3}{2}}$, $q'=\frac{2}{3}$; $s_{\frac{1}{2}}\rightarrow p_{\frac{1}{2}}$, $q'=\frac{1}{3}$) and u_e and u_f ($\int_0^\infty u_f^2dr=1$) are r times the radial parts (the angle-spin part being normalized to unity) of the initial- and final-state wave functions. As we are not concerned here with interference from the various partial waves, u_e can be taken real and is given by the modulus of (32c). Equation (40a) can be put in a convenient form by separating the radial integration into internal and external parts and by introducing the quantities

$$\theta_e^2=au_e^2(a)\Big/\int_0^a u_e^2dr,\quad \theta_f^2=au_f^2(a)\Big/\int_0^a u_f^2dr,$$

$$\mathfrak{M}=a^{-1}\int_0^a ru_eu_fdr\Big/\left[\int_0^a u_e^2dr\cdot\int_0^a u_f^2dr\right]^{\frac{1}{2}};\quad (40b)$$

$\mathfrak{M}$ is a dimensionless measure of the transition moment from the internal region (if the radiation is due to the nucleon from the entrance channel, $\mathfrak{M}\leq 1$); θ_e^2 and θ_f^2 are dimensionless measures of the reduced widths, $\gamma^2=(\hbar^2/2Ma)\theta^2$, for the two states. With the quantities (40b) and (17a) for evaluating $u_e^2(a)=\pi k_e^{-2}(2l_e+1)\times|V_e|^2$, we find

$$\left|\int_0^\infty ru_eu_fdr\right|^2=4\pi Mk_e^{-2}\hbar^{-1}(2l_e+1)a^3g_e^{\mathrm{Im}}$$
$$\times(\mathfrak{M}+\theta_e\theta_fJ)^2|f_e-g_e^{\mathrm{Re}}|^{-2}N^{-1}\theta_e^{-2}\quad (40c)$$

where

$$J=a^{-2}\int_a^\infty rw_ew_fdr,\quad w(r)=u(r)/u(a),$$

$$N=1+\theta_f^2a^{-1}\int_a^\infty w_f^2dr.$$

In the case of proton capture, N and the energy-dependent quantity J are found by numerical integration, the quantity w_f being obtained from (7). In the radiative capture of thermal neutrons to a final p state with binding energy B, they are

$$N=1+\theta_f^2(x_f+2)/2(x_f+1)^2,\quad J=\beta Q(\beta-1)^{-1},$$
$$Q=[(x_f^2+3x_f+3)-x_f(x_f+2)\beta^{-1}]/x_f^2(1+x_f),\quad (40d)$$
$$x_f=(2MBa^2/\hbar^2)^{\frac{1}{2}}.$$

The dimensionless quantity J is a function of the energy both because w_e, which is a multiple of (32c), contains the energy-dependent phase shift δ_e and because the external wave functions F and G are functions of the energy. By means of (32b) the energy dependence contained in δ can be factored out in the form

$$J=k_e^{-1}(f-g^{\mathrm{Re}})FGJ'+J''$$

where

$$J'=a^{-2}\int_a^\infty[F(r)F(a)^{-1}-G(r)G(a)^{-1}]rw_fdr,$$

$$J''=a^{-2}(F^2+G^2)^{-1}{}_{r=a}\int_a^\infty[G(r)G(a)+F(r)F(a)]rw_fdr.\quad (40e)$$

Although the critical energy dependence of F and G in the barrier region, which is attributed to the factor C_0 of (13), does not appear in J', J'', and (FG), the latter quantities are nevertheless expected to be mildly energy dependent.

If the radiative capture formula is written in the usual one-level form

$$\sigma_{e\gamma}=\pi k_e^{-2}\omega\Gamma_e\Gamma_\gamma[(E_{\mathrm{res}}+\Delta-E)^2+\tfrac{1}{4}\Gamma^2]^{-1},\quad (40f)$$

$\omega=(2l_e+1)q_e$ being the statistical factor, then by comparing (40a, c, e) with (40f) and introducing the one-level formula, Eq. (24a), we find that

$$\Gamma_\gamma=\frac{4}{3}\left(\alpha\frac{h\nu}{m_0c^2}\right)^3A^2q'\left(\frac{a}{r_0}\right)^2N^{-1}$$
$$\times\left[\mathfrak{M}'+2\theta_e\theta_f\left(\frac{E_{\mathrm{res}}+\Delta-E}{\Gamma_e^*}\right)J'\right]^2m_0c^2,$$

where

$$\mathfrak{M}'=\mathfrak{M}+\theta_e\theta_fJ'',\qquad \alpha=e^2/\hbar c,$$
$$\Gamma_e^*=2\gamma_e^2(FG)^{-1}{}_{r=a},\quad r_0=e^2/m_0c^2.$$

The energy dependence of the interference between the internal and external contributions is thus clearly evident. This dependence is also expected to occur with radiations of other multipolarities.

III. THE ENERGY LEVELS OF C^{13} AND N^{13}

Experimental Data

The experimental data on the energy levels of C^{13} and N^{13} published prior to July, 1950 have been surveyed in "Energy Levels III."[27] Considerable additional information has subsequently been gathered, so that it is necessary to give a brief summary.

Levels in N^{13} shown in Fig. 1 have been observed at 2.37 and 3.52 Mev from the $C^{12}(p,\gamma)N^{13}$ resonances. By reason of the large width-without-barrier of the 2.37 resonance, it is attributed to s capture.[28] The resonance at 3.52 Mev was first observed by Van Patter and its energy dependence found to be adequately described by a one-level Breit-Wigner formula.[29] These two levels were also observed by Grosskreutz[30] in the

[27] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).
[28] Fowler, Lauritsen, and Lauritsen, Revs. Modern Phys. **20**, 236 (1948).
[29] D. M. Van Patter, Phys. Rev. **76**, 1264 (1949).
[30] J. C. Grosskreutz, Phys. Rev. **76**, 482 (1949).

energy spectrum of neutrons from $C^{12}+d$ and by Goldhaber and Williamson[31] in the elastic scattering of protons by C^{12}. A partial wave analysis[32] of the elastic scattering at the 2.37 resonance shows it to be due to s-waves; however, the anomalous energy-dependence of the cross section in the vicinity of 3.5 Mev is interpreted as due to a doublet, rather than a single level, with one component at 3.50 which is $p_{\frac{3}{2}}$ and the other at 3.55 which is $d_{\frac{5}{2}}$. No evidence for any other levels below 6-Mev excitation was found in the elastic scattering experiment. The fact that only a single level was observed at 3.52 from $C^{12}(p, \gamma)$ presents no difficulty because the radiation from a $d_{\frac{5}{2}}$ level to the $p_{\frac{1}{2}}$ ground state would be magnetic quadrupole which is expected to be considerably weaker than magnetic dipole radiation from the $p_{\frac{3}{2}}$ component. The angular distribution of the radiation from the 2.37 level was found to be isotropic by Devons and Hine in agreement with an $s_{\frac{1}{2}}$ assignment.[33] The angular distribution of the radiation from the 3.52 level, as measured by Day and Perry, was found to be anisotropic with the coefficient of $\cos^2\theta$ at resonance agreeing with the calculated value for a level spin $\frac{3}{2}$; however, the energy dependence of this coefficient indicates that the level may be formed by d-waves with an s-wave background.[34] The angular distribution of neutrons from $C^{12}+d$ also indicates that the 2.37 level is formed by the capture of an s neutron in the stripping process;[35] the level at 3.5 Mev, though not resolved, indicates d, rather than p capture or a mixture, which may be due to the fact that the reduced width, which enters as a yield factor in this process, is 5-times greater for the $d_{\frac{5}{2}}$ component than it is for the $p_{\frac{3}{2}}$ component (see below).

The level of C^{13} at 3.10 Mev is well established from measurements of proton groups and gamma-radiation from $C^{12}+d$.[27] The measurement of Rotblat[36] and the analysis, by means of the stripping theory due to Butler,[37] of the angular distribution of protons from this reaction, associated with this level and the ground state, indicate that they are $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$, respectively, and the fact that the resulting radiation is electric dipole, as determined from the measurement of the internal pair formation coefficient for the transition,[38] is consistent with these assignments. Rotblat[39] has also observed in photographic emulsions proton groups associated with levels at 3.7 and 3.9 Mev, and from their angular distribution he determined that the levels are formed by the capture of p and d neutrons, respectively. This finding is in accord with the mirror nuclei hypothesis and the observed doublet in N^{13}, comprising $p_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ components, provided that the additional splitting in C^{13} can be accounted for as a result of the differences of the electromagnetic properties of the odd nucleons. The 3.68-Mev level is most clearly evident in the $N^{15}(d, \alpha)C^{13}$ reaction which was investigated with magnetic analysis of the reaction products by Malm and Buechner[40] who, however, failed to observe the 3.9 level but noted that a weak group associated with such a level might be obscured by the presence of a broad group associated with the 9.6 level in C^{12} formed from the competing $N^{14}(d, \alpha)C^{12}$ reaction.

TABLE I. Q values of the $B(\alpha, p)$ reactions. From the compilation of reaction energies, (see reference 45) the ground-state Q-values are: $B^{10}(\alpha, p)C^{13}$, $Q=4.071\pm0.010$ Mev; $B^{11}(\alpha, p)C^{14}$, $Q=0.778\pm0.007$ Mev.

Reference (41)	(42)	(43)	(44)	Remarks
3.8	4.07	4.08±0.12		$B^{10}(\alpha, p)C^{13}$-(ground state)
		3.35±0.25		Weak
0.75±0.01	0.85	0.65±0.15	0.63	$B^{11}(\alpha, p)C^{14}$-(ground state)
0.24±0.02	0.31			(2nd state in C^{13})
		0.15±0.15	0.00	
−0.22	0.07			(3rd state in C^{13})
	−0.31	−0.57±0.15	−0.55	$B^{10}(\alpha, d)C^{12}$?
	−1.57		−1.75	Weak

Additional results concerning the C^{13} levels have been obtained from the $B^{10}(\alpha, p)C^{13}$ reaction. Because they are not easily interpreted, we list the more recent Q values[41–45] in Table I. Reference 41 used magnetic analysis of the particle groups, the others range measurements. References 43 and 44 apparently do not resolve the second and third excited states. The first excited state of C^{13} is evidently not formed in this reaction. The group with $Q=3.35$ found by reference 43 may come from another reaction as Blundell and Rotblat[46] report that there is no level in C^{13} in the vicinity of 1 Mev. We assume that the weak groups with Q values of −1.57 and −1.75 Mev belong to another reaction since no levels appear in C^{13} from the scattering by C^{12} of neutrons with an energy less than 2 Mev,[47] except for the broad background which is attributed to the s interaction associated with the 3.10 bound level.[18] A complete survey of these and earlier $B(\alpha, p)$ investigations has been given by Slätis, Hjalmar, and Carlsson.[48] A 3.8-Mev gamma-ray has been observed with 1.4-Mev

[31] G. Goldhaber and R. M. Williamson, Phys. Rev. **82**, 495 (1951).
[32] H. L. Jackson and A. I. Galonsky, Phys. Rev. **84**, 401 (1951).
[33] S. Devons and M. G. N. Hine, Proc. Roy. Soc. (London) **A199**, 56 (1949).
[34] R. B. Day and J. E. Perry, Jr., Phys. Rev. **81**, 662 (1951) and R. B. Day, Ph.D. thesis, California Institute of Technology (1951), (unpublished).
[35] El-Bedewi, Middleton, and Tai, Proc. Phys. Soc. (London) **A64**, 1055 (1951).
[36] J. Rotblat, Nature **167**, 1027 (1951).
[37] S. T. Butler, Phys. Rev. **80**, 1095 (1950) and Proc. Roy. Soc. (London) **A208**, 559 (1951).
[38] R. G. Thomas, Phys. Rev. **80**, 138 (1950).
[39] J. Rotblat, Phys. Rev. **83**, 1271 (1951).
[40] R. Malm and W. W. Buechner, Phys. Rev. **81**, 519 (1951).
[41] G. M. Frye and M. L. Wiedenbeck, Phys. Rev. **82**, 960 (1951).
[42] R. J. Creagan, Phys. Rev. **76**, 1769 (1949).
[43] J. L. Perkin, Phys. Rev. **79**, 175 (1950).
[44] Slätis, Hjalmar, and Carlsson, Phys. Rev. **81**, 641 (1951).
[45] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).
[46] M. Blundell and J. Rotblat, Phys. Rev. **81**, 144 (1951).
[47] D. W. Miller, Phys. Rev. **78**, 806 (1950).
[48] Slätis, Hjalmar, and Carlsson, Arkiv. Fys. **17**, 315 (1951).

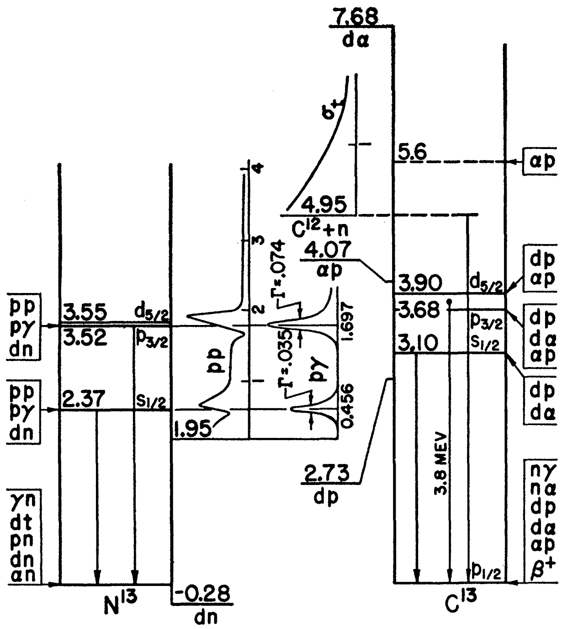

FIG. 1. The energy levels of C^{13} and N^{13} below 6 Mev.

alpha-particles;[49] this gamma-ray may be emitted from either or both the 3.7- and 3.9-Mev levels. In Fig. 1, which shows the levels of C^{13} and N^{13} below 6 Mev, we have adopted Rotblat's value of 3.9 for the position of the second excited level of C^{13}; the $B^{10}(\alpha, p)$ data, however, favor a somewhat higher energy.

Thus, there appear at low excitation energies levels in C^{13} and N^{13} which may be characterized as s, p, d neutrons and protons interacting with a C^{12} core. We turn to the study of the widths, displacements, spacings, and radiative properties of these levels.

s Interaction

According to (31c), the known positions of the $C^{12}+p$ resonance at 2.37 and the C^{13} bound level at 3.10 determine $f(E)$ and therefore $Z(E)$ from (36) at the respective excitation energies; these data are plotted in Fig. 2(a) for $Ka=2.5$ and 3.5 and with a channel radius $a=4.9$.[50] Tentatively we assume that the nuclear energy of N^{13} is the same as that of C^{13} when both energies are referred to their ground states. A third value of Z at $E=4.95$ is obtained from the epithermal neutron scattering length for carbon using (39a). This scattering length, as measured by Havens and Rainwater,[51] is $\alpha^{-1}=(\sigma/4\pi)^{\frac{1}{2}}=6.11\times10^{-13}$ cm; this datum is in agreement with that of Jones.[52] Since there is no measurable spin or isotopic effect in the coherent scattering of thermal neutrons,[53] the presence of a small amount of C^{13} can be disregarded. We obtain dZ/dE at 2.37 from the width, $\Gamma^{\dagger}=35$ kev, of the $C^{12}+p$ resonance using (31a, b). This slope appears as a solid line in Fig. 2.

The sensitivity of the computation from (31b) of γ^2, and likewise of dZ/dE, to the choice of a for the resonance at 2.37 is illustrated in Fig. 3 by plotting $\theta^{-2}=\hbar^2/2Ma\gamma^2$ as a function of a, using (13) for obtaining dg^{Re}/dE. The radial dependence is due to the positive quantities g^{Im} and dg^{Re}/dE; g^{Im} decreases with decreasing a more than dg^{Re}/dE and to such an extent that for $a<2.7$ the positive-definite θ^2 becomes negative. This difficulty arises from the fact that we assume that only the Coulomb interaction exists in the external region ($r>a$) when, evidently, nuclear interactions are also present. The net effect of ignoring these (attractive) nuclear interactions is that a value of g^{Im} is used which is too small and a value of dg^{Re}/dE which is too large; consequently the value obtained for θ^{-2} is low and, for sufficiently small a, can even have the wrong sign. From the plot of Fig. 3 and the sum rule for reduced widths,[54] one can infer that the range of the nuclear interaction is $\gtrsim 3.6$. The determination of γ^2 at this resonance is also especially sensitive to the experimental value of $\Gamma^{\dagger}$; this sensitivity is an indication that the strong Coulomb

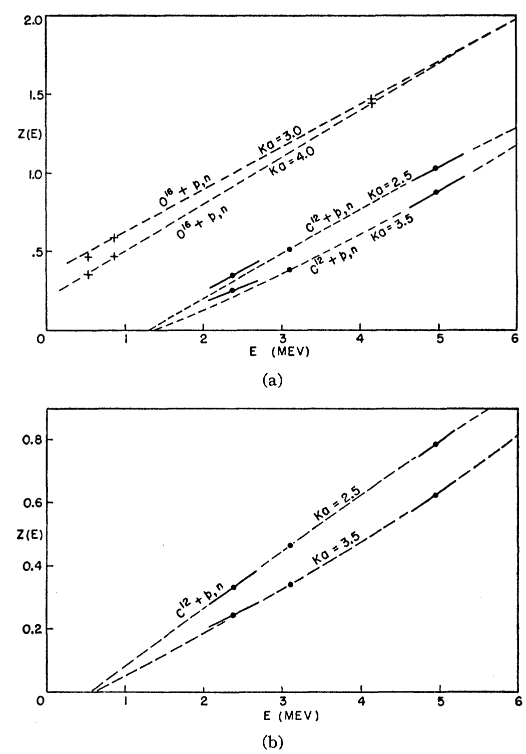

FIG. 2. A plot of $Z=\tan^{-1}(-f/K)$ as a function of E for the s interactions: (a) $a=4.9$ for $C^{12}+p, n$; $a=5.27$ for $O^{16}+p, n$; (b) $a=4.4$ for $C^{12}+p, n$.

[49] Bennett, Roys, and Toppel, Phys. Rev. **82**, 22 (1951).
[50] The energy of relative motion, ϵ, will be given in laboratory units unless stated otherwise. The nuclear excitation energy E is referred to the ground state and given in Mev in the center-of-mass system. Nuclear radii are given in units of 10^{-13} cm.
[51] W. W. Havens, Jr., and L. J. Rainwater, Phys. Rev. **75**, 1296 (1949).
[52] W. B. Jones, Phys. Rev. **74**, 364 (1948).
[53] E. O. Wollan and C. G. Shull, Phys. Rev. **73**, 830 (1948); Burgy, Ringo, and Hughes, Phys. Rev. **84**, 1164 (1951); W. C. Koehler and E. O. Wollan, Phys. **85**, 491 (1952).
[54] $\sum_{c\pm}\gamma_{\lambda c}^2\approx 3\hbar^2/2Ma$; E. P. Wigner, Am. J. Phys. **17**, 99 (1949).

barrier for the incident proton makes it difficult to obtain accurate information concerning the nuclear interaction. Published values of nuclear radii of carbon, which have been obtained in various ways, are indicated at the top of the graph.

The evaluation of reduced widths from (31b) is also particularly sensitive to the value of l which is assumed for the incident particle, and this fact can be used advantageously to set an upper limit on l. As shown by Ehrman,[11] assignments of $l>0$ to the 2.37 resonance level in N^{13} are excluded because γ^2 would be negative for any reasonable choice for a, thus conclusively establishing that this resonance is due to s protons.

A value for dZ/dE at 4.95 Mev can be obtained from the plot of $k\cot\delta_0$ for the low energy scattering of neutrons by carbon shown in Fig. 4. The data obtained by Miller, although of high relative accuracy (the $k\cot\delta_0$ plot considerably exaggerates the scatter of the points), is not claimed to be precise in absolute magnitude;[55] since it is in excellent agreement with the absolute measurements of Williams,[56] it is assumed to be absolute throughout.

In order to compute $k\cot\delta_0$, it is necessary to subtract the scattering contribution from the higher angular momenta,

$$4\pi k^{-2}\sum_{l\neq 0}(2l+1)\sin^2\delta_l, \quad (41a)$$

δ_l being given in terms of f and the external functions by (32b). The subtraction was actually carried out for scattering from a hard sphere, $f_{l\neq 0}=\infty$, of radius 3.9. Subsequently, however, Wigner[57] has shown that as a consequence of the finite contribution to the R function

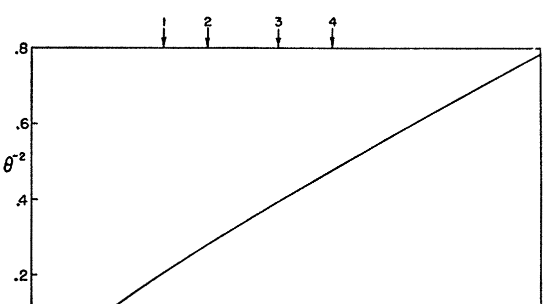

FIG. 3. Reciprocal of the reduced width, in dimensionless units, plotted as a function of the channel radius for the $C^{12}+p$ resonance at $\epsilon_p=0.456$ kev. Nuclear radii obtained in various ways are indicated at the top of the graph: (1) Neutron scattering at 95 Mev (theory: *transparent nucleus*)—J. De Jurenand N. Knable, Phys. Rev. **77**, 606 (1950); (2) From the ground-state Coulomb energy difference, $R_c=1.46A^{\frac{1}{3}}$; (3) Neutron scattering at 25 Mev (theory: *schematic*)—H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949); (4) Total neutron cross section at 42 Mev (theory: *opaque nucleus*)—P. H. Hildebrand and C. E. Leith, Phys. Rev. **80**, 842 (1950).

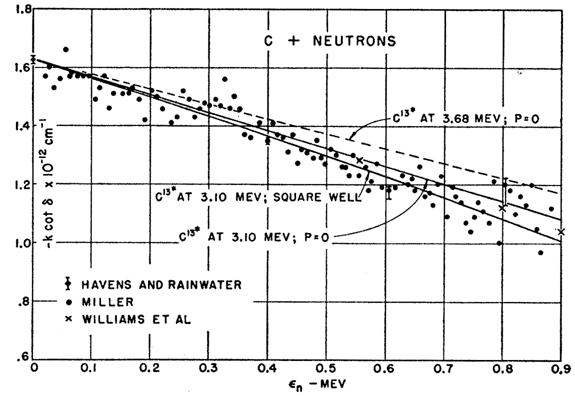

FIG. 4. Plot of $k\cot\delta_0$ for the scattering of low energy neutrons by carbon. The points at $\epsilon_n=0.4$, 0.6, 0.8 Mev have vertical lines through them to indicate the possible uncertainty of the correction for the scattering of the higher partial waves.

from distant high energy levels, the value

$$f^{-1}{}_{l\neq 0}=S_R(E)\log(X^{\frac{1}{2}}+T^{\frac{1}{2}})/(X^{\frac{1}{2}}-T^{\frac{1}{2}}) \quad (41b)$$

is to be preferred. Here $E=\epsilon+B$, $X=E+T$, B being the binding energy of the incident neutron (or proton), T its kinetic energy in the nucleus, and $S_R(E)=\langle\gamma_\lambda^2\rangle_{Av}D^{-1}$ is the average reduced level width divided by the level spacing, evaluated at an excitation energy E. With this expression for f_l, (41a) yields a negligibly small contribution, but since (41b) is only an estimate, we have indicated the possible uncertainty of the $k\cot\delta$ plot by drawing vertical lines through the points at 0.4, 0.6, 0.8 Mev; the tops of these lines correspond to scattering from a hard sphere of radius 4.1 and the bottoms to no correction, as indicated by (41b). In any case, the correction is almost negligible below 0.8 Mev and resonances in the scattering do not occur below $\epsilon_n=2.1$ Mev[58] so that the plot should be of some significance.

From the plot, together with the datum from the bound s state at 3.10, we find that the interaction parameters lie between: $[r_0=2.9\times 10^{-13}$ cm, $P=0.02]$ and $[r_0=3.6\times 10^{-13}$ cm, $P=-0.08]$. From (39b) we then find $\theta^2(E=4.95)=0.55\pm 0.25$ with $a=4.9$, and the value of $dZ/dE=2MaK/\hbar^2\theta^2(f^2+K^2)$ corresponding to it is represented on Fig. 2(a). This value of θ^2 is clearly smaller than the value $\theta^2(E=2.37)=1.4$ obtained from Fig. 3 for the same a. This variation of θ^2 indicates that the one-level approximation is inadequate for treating the s interaction over a $2\frac{1}{2}$-Mev range.

The curves given by Blatt and Jackson enable one to say something about the nature of the neutron interaction; according to Figs. 6 and 10 of their paper, the long-tailed interactions are excluded and a square well would have a radius of about 4.7 and a depth of about 11 Mev. In connection with the use of these curves, we note that according to almost any model, a node is

[55] D. W. Miller (private communication).
[56] Lampi, Freier, and Williams, Phys. Rev. **80**, 853 (1950); Freier, Fulk, and Williams, Phys. Rev. **78**, 508 (1950).
[57] E. P. Wigner, Proc. Cambridge Phil. Soc. **47**, 790 (1951).
[58] Bockelman, Miller, Adair, and Barschall, Phys. Rev. **84**, 69 (1951).

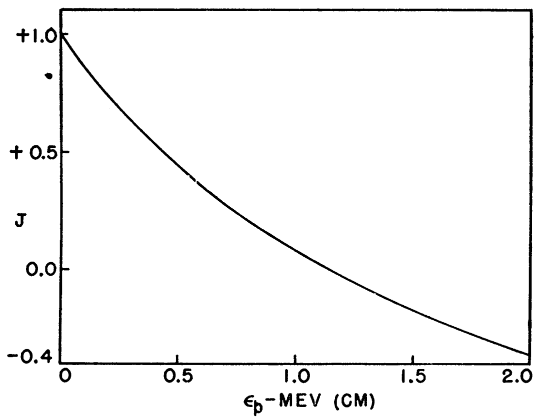

FIG. 5. The quantity J of Eq. (40c) as a function of the proton energy for an $s\rightarrow p$ transition in the capture of protons by C^{12} using a channel radius of 4.9×10^{-13} cm.

predicted for the internal wave function. Consequently, these may not apply exactly to our case although we would not expect very much change, other than in the determination of well depths, if the node were included. We have verified that a square well wave function with a node gives a $k\cot\delta$ plot that is nearly identical with that shown in Fig. 4 for the case without the node. However, the radius of the well is 4.0 and the depth is 35 Mev.

Also shown in Fig. 4 are the straight lines corresponding to $P=0$ with either the 3.68 or 3.10 levels as the bound s state. If the 3.68 level is taken as the bound s state, we find $P=-0.14$ with r_0 about 3.0; judging from Fig. 10 of reference 23, these values are unreasonable.

Returning to Fig. 2(a), it appears that in order to have the points and slopes determine a smooth curve, it is necessary to increase E for N^{13} with respect to C^{13} by about 150 kev. In the ensuing analysis of the p-interaction, it is shown that due to the level shift terms Δ_λ and the differences of the electromagnetic spin-orbit interactions of the odd particle, the ground-state excitation energy of N^{13} is shifted downwards with respect to C^{13} by about (39−27)-kev. Consequently, either the quantity $\langle V\rangle_\tau$ is (150+12)-kev less in the s state than in the ground state or else the $C^{12}+n$ and the $C^{12}+p$ nuclear interactions differ by this much. This diminution is only 5 percent of the total Coulomb energy due to the odd proton, and it is reasonable to expect this amount for a state having a large reduced width compared to that of the ground state (which will be shown to be about 15 times smaller). From a compilation of various theoretical calculations of Coulomb energies,[59] it appears that ±200 kev may be a reasonable estimate of possible variations of the internal Coulomb energy in different states of N^{13}.

The energy at which $Z(E)=0$ is the unshifted level position in the theory of W-E, corresponding to the boundary condition $\bar{b}=0$ in (28); thus, $E_\lambda=1.3$ for C^{13} and 1.1 Mev for N^{13}. The reduced width at E_λ is $\theta_\lambda^2=1.5$. The energy required to increase Z by π is the level spacing, which is about 12 Mev. This large reduced width (the upper estimate given by the sum rule[54] is 3.0) and level spacing demonstrate that the interaction is essentially a two-body type with no amalgamation of the odd particle within the C^{12} core; (the number of nuclear traversals of the incident particle is θ^{-2} in the classical sense).

There is an additional datum on the s-wave interaction from the angular distribution of the scattering of 4.2-Mev protons by C^{12} ($E=5.9$ Mev.[60] From a phase shift analysis it was concluded that s-waves alone could account for this distribution with $\delta_0=127°$.[60] By means of (32b) we determine that $f=9.6$ and thus $Z=1.9$ for $Ka=3.0$, a value which is considerably displaced from the curves of Fig. 2a. This discrepancy may be due to the omission of the higher partial waves in the phase shift analysis or to the presence of narrow s resonances at higher energies causing Z to behave erratically (see IV on the $O^{16}+p, n$ s interactions where such erratic behavior is observed).

By introducing the neutron scattering data into the analysis and the use of (36), which effectively amounts to including the level spacing as a third parameter in the analysis, it has been possible partially to overcome the critical dependence on the channel radius in the level displacement computation, which was observed by Ehrman.[11] Figure 2(b) gives a plot of Z using $a=4.4$ wherein the N^{13} datum at 2.37 is displaced by less than 50 kev from the smooth curve determined by the $C^{12}+n$ data. Although a 150-kev discrepancy was observed with $a=4.9$, we would in fact expect $\langle V\rangle_\tau$ to decrease with increasing a. The resonance parameters as obtained from Fig. 2(b) are: $\theta_\lambda^2=2.0$, $E_\lambda=0.6$, $D=19$ Mev; these differ from those found with $a=4.9$. The quantities θ_λ^2 and E_λ are, however, by definition functions of a. This dependence has been investigated by Teichmann who shows that[7]

$$dE_\lambda/da=(\mathcal{V}-\epsilon_\lambda)\theta_\lambda^2/a \tag{42a}$$

and

$$d\gamma_\lambda^2/da=[2(\epsilon_\lambda-\mathcal{V})R_\infty-\gamma_\lambda^2]\theta_\lambda^2/a, \tag{42b}$$

where

$$R_\infty=\textstyle\sum_{\mu\neq\lambda}\gamma_\mu^2/(E_\mu-E_\lambda),\quad \theta_\lambda^2=2Ma\gamma_\lambda^2/\hbar^2;$$

$\mathcal{V}$ and ϵ are the quantities entering (3), and $R_\infty\approx\langle\gamma_\lambda^2\rangle_{\rm Av}/D$. These two relations are a consequence of (3) and the definitions of E_λ, γ_λ^2. The observed radial dependences of θ_λ^2 and E_λ are in qualitative agreement with these derivatives. However, the variation of D with a is more than expected, which is probably, to a certain extent at least, because we are trying to obtain more information from the data than entitled to.

Having determined the s interaction parameters, we should be able to account for the energy dependence of the electric dipole radiative capture cross section for transitions to the ground state. The reduced charge number entering in (40a) is 6/13 for both neutron and proton capture so that, aside from a common E de-

[59] W. E. Stephens, Phys. Rev. **57**, 938 (1940).

[60] Heitler, May, and Powell, Proc. Roy. Soc. (London) **190**, 180 (1947).

pendence, the differences of the cross sections should be due to differences of the external wave functions. The $C^{12}(p, \gamma)$ resonance cross section, as measured by Fowler, Lauritsen, and Lauritsen[28] and also by Seagrave,[61] is $(1.25\pm0.15)\times10^{-4}$ barn. Hall and Fowler[62] as well as Bailey and Stratton[63] measured this cross section below resonance and found deviations from the usual Breit-Wigner formula, which increased with decreasing proton energy. In particular, the cross section at $\epsilon_p=120$ kev is $(6.1\pm0.9)\times10^{-10}$ barn, which is about two-times greater than the value predicted by the extension of the Breit-Wigner curve fitted to the resonance data (and taking into account the $(h\nu)^3$ dependence of the radiative width).

The thermal neutron capture cross section of graphite is listed as 4.5 mb and that of C^{13} as about 100 mb, so that the C^{12} isotopic cross section is about 3.5 mb[64] and perhaps less considering the possibility of contributions from impurities. This cross section is about five times smaller than that calculated by extending the $C^{12}(p, \gamma)$ resonance data, taking account of the differences of the external wave functions and the $(h\nu)^3$ factor but neglecting the external contribution to the transition moment.

Because of the large reduced width and level spacing of the initial s-state, these deviations are attributed to the omission in the usual resonance formula of the external contribution to the transition moment, rather than to large variations of the internal transition moment $\mathfrak{M}$ of (40b). Contributions to the capture from higher partial waves are estimated to be negligible.

When the values of f_e and θ_e obtained from Fig. 2(a) are substituted into (40c), the possible values of $\mathfrak{M}$ and θ_f which will satisfy the $C^{12}(p, \gamma)$ measurements at $\epsilon_p=456$ and 120 kev can be determined. Considering the experimental uncertainties, these values lie in the region of Fig. 6 common to the two sets of parallel lines for each measurement. The values of J required in (40c) are given in Fig. 5. The possible range of values for $\mathfrak{M}$ and θ_f can be considerably reduced by including the thermal neutron capture cross section to the ground state of C^{13} (for which $J=-0.70$ according to (40d)) which we assume to be between 1.8 and 3.5 mb considering the possibility that a part of the cross section may be due to transitions to excited states, as indicated below; (there are two sets of parallel lines because of the ambiguity in sign of $\mathfrak{M}+\theta_e\theta_f J$). With the assumption that $\mathfrak{M}$ can be treated as constant over a 2.9-Mev range of nuclear excitation energy (about one-fifth of the level spacing), the possible range of values is narrowed to the regions in Fig. 6 labeled I and II. Further comments on Fig. 6 follow the discussion of the p interaction.

Qualitatively, the anomalous behavior of the $C^{12}(p, \gamma)$ cross section is due to the fact that the external contribution to the moment is of the same sign as the internal contribution and increases with decreasing proton energy. Since the neutron scattering length of C^{12} is positive and only slightly larger than the channel radius, the external contribution to the $C^{12}(n, \gamma)$ process is opposite in sign to the internal contribution thus yielding a small cross section. In contrast, in the capture of thermal neutrons by Li^7 the negative scattering length provides a large external electric dipole transition moment, which is qualitatively capable of explaining the anomalously large cross section.[65]

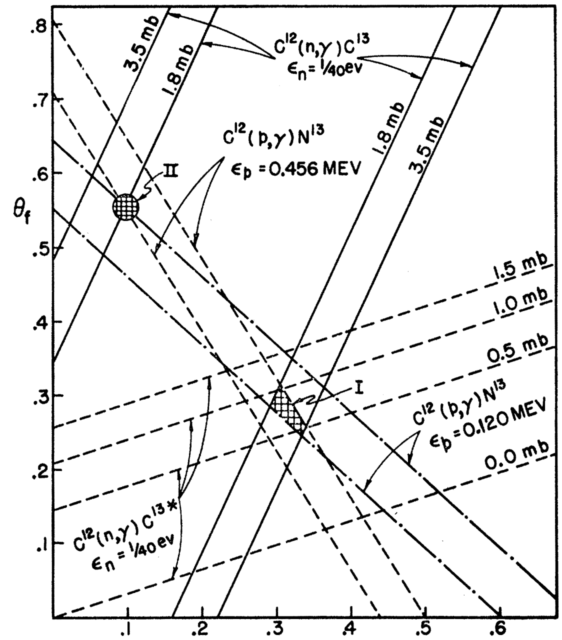

FIG. 6. Square root of the final-state reduced width plotted as a function of the internal transition moment $\mathfrak{M}$ for various transitions in the capture of s protons and neutrons by C^{12}.

In connection with the energy production in stars by means of the carbon cycle, it is required to know the C^{12} proton capture cross section at stellar temperatures. This cross section can be obtained with sufficient accuracy by writing (40f) as

$$\sigma_{e\gamma}=\pi k_e^{-2}\omega(h\nu/h\nu_{\text{res}})^3\Gamma_{\gamma-\text{res}}^\dagger\Gamma_e^\dagger \times[1+C(E_{\text{res}}-E)]^2/[(E_{\text{res}}-E)^2+\tfrac{1}{4}\Gamma_e^{\dagger 2}], \quad (43)$$

the parameters $\Gamma_{\gamma-\text{res}}^\dagger$, $\Gamma_e^\dagger$ being found from the behavior of the cross section in the vicinity of the resonance and the constant C from the cross section at $\epsilon_e=120$ kev. At proton energies which are sufficiently below the resonance so that the width term in the denominator of (43) can be ignored, the expression obtained by this procedure is

$$\sigma_{e\gamma}=6.8\times10^{-6}(h\nu)^3\epsilon_e^{-1}[(0.42-\epsilon_e)^{-1}+2.7]^2\exp(-5.74\epsilon_e^{-\frac{1}{2}})$$

with $h\nu$ and ϵ_e in units of Mev, measured in the c.m. system. For example, at $\epsilon_e=28$ kev, $\sigma_{e\gamma}=6.1\times10^{-17}$ barn.

p Interaction

The magnetic moment of C^{13} falls very close to the Schmidt line for $J=l-\frac{1}{2}=\frac{1}{2}$ suggesting that an inde-

[61] J. Seagrave, Phys. Rev. **84**, 1219 (1951).
[62] R. N. Hall and W. A. Fowler, Phys. Rev. **77**, 197 (1950).
[63] C. L. Bailey and W. R. Stratton, Phys. Rev. **77**, 194 (1950).
[64] M. Ross and J. S. Story, Repts. Progr. Phys. **12**, 291 (1948).
[65] R. G. Thomas, Phys. Rev. **84**, 1061 (1951).

TABLE II. Level shifts in kev due to the electromagnetic spin-orbit interaction. $R_c=3.5\times10^{-13}$ cm.

Level	N^{13}	C^{13}
$d_{5/2}$	−21	18
$d_{3/2}$	32	−27
$p_{3/2}$	−11	9
$p_{1/2}$	21	−18

pendent-particle type of interaction prevails between the odd neutron and the C^{12} core. Moreover, the radiative width for a magnetic dipole transition between the $p_{\frac{3}{2}}$ excited and the $p_{\frac{1}{2}}$ ground states of N^{13} indicates an almost complete overlap of the radial wave functions, suggesting that this type of interaction may also prevail in the $p_{\frac{3}{2}}$ state with $J=l+\frac{1}{2}$. The radiative width, $\omega\Gamma_\gamma$[66] of the $p_{\frac{3}{2}}$ level, as measured by Van Patter[29] as well as by Seagrave,[61] is 1.3 ev. Assuming that a two-body interaction is involved between the odd proton and the core and neglecting possible interaction effects,[67] the width for a $p_{\frac{3}{2}}\rightarrow p_{\frac{1}{2}}$ transition is given by[68]

$$\omega\Gamma_\gamma=(2/9)(h\nu)^3(e^2/\hbar c)(Mc^2)^{-2}(g_s-g_l)^2x^2, \quad (44)$$

where g_s, g_l are the spin and orbital g factors, and $x(\leqslant 1)$ measures the extent of overlap of the radial wave functions. With $g_s=5.58$, $g_l=0.96$, $h\nu=3.52$, we find $x=0.90$. Such a slight difference of the wave functions is to be expected, even with similar interactions, because of the difference of the binding in the external region. Further evidence for the similarity of these two states is provided by the fact that the intensities of the $N^{15}+d$ alpha-particle groups associated with them are very nearly equal.[40]

From a consideration of the shell model with strong spin-orbit coupling and of level assignments in other nuclei, it has been proposed that the odd proton of the $p_{\frac{3}{2}}$ state of N^{13} is an excited $2p_{\frac{1}{2}}$ configurational state whereas in the ground state it is in a $1p_{\frac{1}{2}}$ configurational state.[69] If this hypothesis is correct, we would expect a much smaller value of the overlap, x. On the other hand, it would seem consistent with the evidence for the large overlap to consider the $p_{\frac{3}{2}}$ state as a $p_{\frac{3}{2}}$ hole in the $1p$ proton shell and the $p_{\frac{1}{2}}$ ground state as a $p_{\frac{1}{2}}$ hole in the $1p$ proton shell.

According to the analysis of Jackson and Galonsky,[32] the resonance parameters ascribed to the $p_{\frac{3}{2}}$ component of the doublet at $\epsilon_p=1.7$, as observed in the elastic scattering measurement, are: $E_{res}=3.501$, $E_\lambda=3.508$ (with the boundary condition $\bar{b}=a^{-1}$), $\Gamma^\dagger$ (c.m.)$=0.042$ Mev, $\gamma_\lambda{}^2=0.377\times10^{-13}$ Mev·cm ($\theta^2=0.08$). The observed width is smaller than the value 74 ± 9 kev obtained by Van Patter or 70 ± 10 kev obtained by Seagrave from $C^{12}(p,\gamma)$. As mentioned above, the capture data fit a one-level formula and it is unlikely that the $d_{\frac{5}{2}}$ component of the doublet contributes significantly; therefore, we use the 70-kev value, which corresponds to $\theta^2=0.12$, for computing the boundary-condition displacement of the corresponding $p_{\frac{3}{2}}$ levels. In the one-channel, one-level approximation the contribution to this displacement from the $p_{\frac{3}{2}}$ levels is then

$$\Delta_{p_{\frac{3}{2}}}(C^{13})-\Delta_{p_{\frac{3}{2}}}(N^{13})=\gamma^2(g_p{}^{Re}-g_n{}^{Re})=90 \text{ kev},$$

from (30b) with $a=4.9$. In order to compute the effect on the net displacement from the ground-state boundary condition difference, it is necessary to know the ground-state reduced widths. Assuming them to be equal, this information is provided by Fig. 6, there being, however, the two possible values $\theta_f{}^2=0.07$ or 0.3, from regions I and II, respectively. With these values and $l=1$, the ground-state boundary-condition displacement is $\Delta_{p_{\frac{1}{2}}}(C^{13})-\Delta_{p_{\frac{1}{2}}}(N^{13})=27$ or 100 kev, respectively.

Inglis[70] and Mottelson[71] have considered the difference of the electromagnetic spin-orbit interactions in their treatments of the displacements of the first excited states of Be^7 and Li^7; a part of the displacements of the $N^{13}-C^{13}$ levels may be due to this effect. Assuming again a two-body interaction and a uniform distribution of charge throughout a radius R_c, the contribution to $\langle V\rangle_\tau$ from a particular level is[70,72]

$$Ze^2(T^- - g_s)\mathbf{l}\cdot\mathbf{s}/2M^2c^2R_c{}^3, \quad (45)$$

where Z is the core charge, $T^-=1$, 0 for an odd proton or neutron respectively, and g is the spin g-factor. Table II lists the shifts for various states using the Coulomb radius, $R_c=1.46A^{\frac{1}{3}}$. If some of the orbital motion is shared by the core, the shifts are expected to be smaller in absolute value.

From Table II, the contribution to the differences of the E_λ are then

$$E_{p_{\frac{3}{2}}}(C^{13})-E_{p_{\frac{3}{2}}}(N^{13})=\ \ 20 \text{ kev},$$
$$E_{p_{\frac{1}{2}}}(C^{13})-E_{p_{\frac{1}{2}}}(N^{13})=-39 \text{ kev}.$$

The net displacement of the excited $p_{\frac{3}{2}}$ states, as would appear in Fig. 1, is the difference of the total displacements of the excited and ground states: 90−(27 or 100) +20+39=122 or 49 kev, the observed value being 160 kev. Although neither of the calculated values can be considered as in good agreement with the observed value, the ground state reduced width $\theta^2=0.07$ from

[66] The difference between Γ_γ and $\Gamma_\gamma{}^\dagger$ at this resonance is only a few percent and may therefore be ignored. This difference is small because the proton reduced width is small.

[67] R. G. Sachs and M. Ross, Phys. Rev. 84, 379 (1951).

[68] E. U. Condon and G. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1935).

[69] Koester, Jackson, and Adair, Phys. Rev. 83, 1250 (1951).

[70] D. R. Inglis, Phys. Rev. 82, 181 (1951), Eq. (15). This equation should read $E_m=(2/A)(137/1837)^2Zgmc^2=0.0032Z(g/2)mc^2$. The magnetic contributions there attributed to the droplet model should thus be reduced by a factor of two, bringing them much more nearly in agreement with the results of the more refined oscillator treatment on which the main conclusions of the paper were based [D. R. Inglis (private communication)].

[71] Ben R. Mottelson, Phys. Rev. 82, 287 (1951).

[72] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, New York, 1948), Sec. 15.22, Eq. (11).

region I of Fig. 6 is preferred; the 38-kev discrepancy may be due to the diminution of the internal Coulomb energy of the excited state from that of the ground state. Although it was noted that this energy may be uncertain by as much as 200 kev, the overlap of the $p_{\frac{3}{2}}$ and $p_{\frac{1}{2}}$ radial wave functions indicates that the diminution is probably small. By fitting square-well wave functions to the logarithmic derivatives of these states, a Coulomb energy of the excited state is obtained which is 50 kev less than that of the ground state. This observation, however, is only intended to be suggestive, because square well wave functions are not compatible with the observed reduced widths of these levels.

It is noteworthy that the ground-state reduced width $\theta^2=0.07$ is the same order magnitude, although somewhat smaller, as that of the $p_{\frac{3}{2}}$ excited state. If the $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ states are indeed similar, we would in fact expect the reduced width of the more tightly bound ground state to be smaller.

It is possible to give some justification to the use of the one-channel approximation in the boundary-condition level displacement calculations by giving an upper-limit estimate of the displacements from other possible (negative-energy) alternatives. Because the reduced widths of the p levels are considerable smaller than $3\hbar^2/2Ma$, the sum over reduced widths,[54] other alternatives may have large reduced widths and contribute significantly to the displacements. As an example, we consider as alternatives for the $p_{\frac{3}{2}}$ states,

$$C^{13*}: Be^9+\alpha, \quad \epsilon=-7.0 \text{ Mev},$$
$$N^{13*}: B^9+\alpha, \quad \epsilon=-6.0 \text{ Mev};$$

and for the $p_{\frac{1}{2}}$ states,

$$C^{13}: Be^{9*}+\alpha, \quad \epsilon=-13.1 \text{ Mev},$$
$$N^{13}: B^{9*}+\alpha, \quad \epsilon=-11.9 \text{ Mev},$$

the excited state of Be^9 being the one at 2.4 Mev which we assume to be $P_{\frac{1}{2}}$ and B^{9*} being its mirror; the alpha-particle is assigned zero orbital angular momentum. If the reduced widths for both pairs of alternatives are taken to be the upper estimate, $3\hbar^2/2Ma$, the displacements are found to be 120 kev in both states. The net displacement of the excited levels would be the difference and thus zero. (If the reduced widths for these alternatives were not assumed to be equal, any net displacement between ± 120 kev could be obtained.) Other possible alternatives for these levels would have larger binding energies (smaller $|g_n^{\text{Re}}-g_p^{\text{Re}}|$) and thus give rise to smaller shifts. As this is an extreme example, we have some assurance that these alternatives do not contribute significantly to the net level displacement.

We return to the discussion of the radiative capture of thermal neutrons by C^{12}. The cross section for transition to the 3.68 level is predicted in Fig. 6 as a function of $\mathfrak{M}$ and θ_f. Although the factor $q'(h\nu)^3$ of (40a) is 30 times smaller for a transition to this level than to the ground state, the external contribution to the moment may predominate because the binding energy of the final state is small ($B=1.27$ Mev so that $J=-4.6$). From the width of the mirror level of N^{13}, θ_f is expected to be about 0.35 and from the evidence for the similarity of the $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ radial wave functions it is plasuible that $\mathfrak{M}\approx 0.3$. With these values we obtain from Fig. 6 a cross section of about 1.5 mb, which is not inconsistent with the measurements by Kinsey *et al.*[73] of the carbon neutron capture spectrum. Although these investigators do not report a gamma-ray of this energy, a rise in the counting rate with an end point corresponding to a 3.7-Mev transition is visible in Fig. 3 of their paper; considering the rapid diminution of the pair spectrometer sensitivity with decreasing gamma-ray energy, we estimate the upper limit for the intensity of this gamma-ray to be about 75 percent of that of the 4.95 ground-state transition, which puts an upper limit of 1.5 mb on the cross section. Additional evidence favoring such a transition is given by ionization-chamber measurements of photoprotons from the disintegration of deuterons by the capture gamma-rays;[74] a 3.65-Mev transition having an intensity comparable to that of the 4.95-Mev gamma-ray was observed. However, gamma-rays of 3.05, 3.40, and 4.1 Mev were also observed which we cannot account for in terms of the proposed level assignments; a cascade through the 3.10 level should not occur since a single nucleon $s_{\frac{1}{2}}\rightarrow s_{\frac{1}{2}}$ (forbidden) transition would be required. The possibility of impurity contributions in this measurement is indicated.

d Interaction

According to the analysis of Jackson and Galonsky, the resonance parameters ascribed to the $d_{\frac{5}{2}}$ level of N^{13} are: $E_{\text{res}}=3.549$ Mev, $E_\lambda(\bar{b}=2/a)=3.593$, $\Gamma^\dagger$ (c.m.) $=0.040$ Mev, $\gamma_\lambda^2=2.36\times10^{-13}$ Mev$\cdot$cm ($\theta^2=0.50$). In the one-channel, one-level approximation the calculated displacement of the corresponding $d_{\frac{5}{2}}$ levels due to the boundary-condition difference is $\Delta_{d\frac{5}{2}}(C^{13})-\Delta_{d\frac{5}{2}}(N^{13})=190$ kev. To this amount, 78 kev is added for the differences of the electromagnetic spin-orbit interactions (from Table II) and 27 kev subtracted for the ground-state boundary-condition displacement, obtaining a net displacement of 240 kev, which is 100-kev smaller than observed. This discrepancy may be due to the diminution of the internal Coulomb energy of the excited state as a consequence of its higher angular momentum and rather large reduced width. For instance, a square well, d orbital wave function has 80 kev less internal Coulomb energy in a uniformly charged sphere than a corresponding p function; but again, this observation is intended only to be suggestive as a square well wave function with the proper f value has a reduced width four-times larger than observed.

As the reduced width for this interaction is large and as there are no data permitting consideration of the level spacing, the boundary-condition shift computation is rather sensitive to the assumed channel radius. For example, with $a=4.4$ the boundary-condition displacement is 240 kev rather than 190 kev; with $a=3.9$ it is 320 kev.

[73] Kinsey, Bartholomew, and Walker, Can. J. Phys. **29**, 1 (1951).
[74] Richard Wilson, Phys. Rev. **80**, 90 (1950).

No evidence has been found below 6 Mev for a $d_{\frac{3}{2}}$ interaction in either nucleus.

IV. COMPARISON OF THE $F^{17}-O^{17}$ WITH THE $N^{13}-C^{13}$ *s* LEVELS

The analysis of the angular distribution of protons and neutrons from the $O^{16}+d$ reaction indicates that the first excited states of O^{17} at 0.871 Mev and of F^{17} at 0.536 Mev are formed by the capture of an *s* neutron and proton, respectively, and that the ground states are formed by the capture of a *d* neutron and proton, respectively.[75–77] The spin of O^{17} in the ground state is 5/2 making the assignment $J=l+\frac{1}{2}$ in agreement with the observed magnitude of the magnetic moment on the independent-particle model.[78] The measured quadrupole moment is also consistent with this assignment if the O^{16} core is considered to be undistorted but its center displaced from the center of mass due to the presence of the odd neutron.[79] These assignments are consistent with the measured internal conversion coefficient of the O^{17*} radiation, which indicates that the transition is electric quadrupole or a mixture of this and magnetic dipole;[80] the $s_{\frac{1}{2}}$ assignment to O^{17*} is also compatible with the observed isotropy of the gamma-radiation accompanying the deuteron reaction.[81] Koester, Jackson, and Adair[69] have noted that these two levels of $F^{17}-O^{17}$ are the analogs of the first and third excited states of N^{13}, though in reverse order, and fit in a shell model scheme. Here we show that there is indeed a similarity in the energy dependence of the interactions of *s* nucleons with O^{16} and C^{12}.

Figure 2(a) also shows $Z(E)$ for the $O^{16}+p$, *n* interactions with $Ka=3.0$, 4.0 using a channel radius of 5.27. The points at $E=0.536$ and 0.871 are obtained from the values of f_e for the bound levels. The point at $E=4.145$ Mev is from the O^{16} epithermal neutron scattering cross section, 3.73 ± 0.04 barns, as measured by Melkonian.[82] The points determine a straight line which is parallel to the $C^{12}+p$, *n* line, indicating that the respective reduced widths and level spacings are about the same. Since the C^{13} neutron binding energy is 0.8-Mev greater than that of O^{17}, the $2\frac{1}{2}$-Mev displacement of the lines indicates that the O^{16} interaction is about $1\frac{1}{2}$-Mev stronger despite the use of a larger channel radius. The low energy neutron scattering cross section for O^{16} can be calculated from the extrapolation of $Z(E)$ beyond 4 Mev; the values obtained are consistent with the observed "nonresonant" scattering up to $\epsilon_n=2$ Mev. In this range Z is near $\pi/2$ so that f is large and the scattering essentially potential for the channel radius, 5.27, used in the calculation.

Heitler, May, and Powell[60] also measured the *s*-wave phase shift from the scattering of 4.2-Mev protons by O^{16} ($E=4.6$). From a phase shift analysis they conclude that $\delta_0=140°$; for this shift we find $Z=2.2$, a value lying considerably off the dashed lines of Fig. 2(a) through the low energy points. As with their $C^{12}+p$ measurement, the discrepancy may be due to the omission of the higher partial waves in the analysis or to an erratic behavior of Z. In connection with the latter possibility, Baldinger, Huber, and Proctor[83] find from a phase shift analysis of the scattering of neutrons by O^{16} an inverted *s* resonance at $\epsilon_n=2.4$ Mev ($E=6.4$) having a width of 180 kev; this level is also identified as *s*-wave by Bockelman *et al.*[58] The presence of such a narrow level implies that Z increases by π in a narrow energy interval. A similar phenomenon has been observed by Laubenstein from the scattering of protons by O^{16};[84] at $\epsilon_p=2.66$ ($E=3.1$) there is an inverted resonance which is only 20-kev wide and attributed to *s* protons, implying that Z also increases by π in a narrow region about $E=3.1$. According to the sum-over-levels,[7,9] the width of this resonance corresponds to a level spacing of about 150 kev and the neutron resonance at $E=6.3$ to a level spacing of about 1 Mev. Such level spacings are not compatible with the observed level density. However, it is not known to what extent the sum-over-levels is valid or, equivalently, to what extent a smooth energy dependence for Z is to be expected. At any rate, it appears that there are narrow *s*-levels in addition to the broad levels, the latter being depicted by Fig. 2(a).†

The energy difference of the ground states of F^{17} and O^{17}, as obtained from recent Q determinations,[46] is about 140 kev less than the value obtained from the assumption of a uniform distribution of charge throughout a volume with a radius equal to $1.46A^{\frac{1}{3}}$. This deviation may be partly due to the boundary-condition level displacement. If the reduced widths of the ground states of F^{17} and O^{17} are assumed to be the same as that of the $d_{\frac{5}{2}}$ state of N^{13}, we obtain a boundary-condition displacement of $\Delta d_{\frac{5}{2}}(O^{17})-\Delta d_{\frac{5}{2}}(F^{17})=190$ kev and an additional displacement of 40 kev due to the electromagnetic spin-orbit interaction (from Table II). Therefore, the nuclear excitation energy of F^{17}, referred to the ground state of O^{17}, may be about 200 kev less than its value referred to the ground state of F^{17}; if this is the case, the point in Fig. 2(a) at $E=0.536$ should actually be placed somewhere between $E=0.3$ and 0.4 when comparing with the O^{17} data. The possibility of this shift does not affect the above conclusions concerning the $O^{16}+p$, *n* *s* interaction.

V. CONCLUSIONS

An analysis using recent nuclear resonance theories has been made of the available data on the interaction

[75] Burrows, Gibson, and Rotblat, Phys. Rev. **80**, 1095 (1950).
[76] Fay Ajzenberg, Phys. Rev. **83**, 693 (1951).
[77] El-Bedewi, Middleton, and Tai, Proc. Phys. Soc. (London) **A64**, 756 (1951).
[78] F. Alder and F. C. Yu, Phys. Rev. **81**, 1067 (1951).
[79] Geschwind, Gunther-Mohr, and Silvey, Phys. Rev. **85**, 474 (1952).
[80] R. G. Thomas and T. Lauritsen (to be published).
[81] Jacques Thirion, Compt. rend. (1951).
[82] E. Melkonian, Phys. Rev. **76**, 1750 (1949).
[83] Baldinger, Huber, and Proctor, Phys. Rev. **84**, 1058 (1951).
[84] Laubenstein, Laubenstein, Koester, and Mobley, Phys. Rev. **84**, 12 (1951); R. A. Laubenstein and M. J. W. Laubenstein, Phys. Rev. **84**, 18 (1951).

† *Note added in proof:* According to a private communication from F. J. Eppling of the University of Wisconsin, angular distributions of protons scattered by O^{16} indicate that the $E=3.1$ Mev level is $p_{\frac{1}{2}}$ rather than $s_{\frac{1}{2}}$.

of s, p, d nucleons with C^{12}, and of s nucleons with O^{16}. The large reduced widths and level spacings characterizing the s interactions at low excitation energies are indications that two-body types of potentials predominate between the odd nucleon and the C^{12} and O^{16} cores; the near equality of the resonance parameters for the s-nucleon interaction with C^{12} and O^{16} is an indication that the respective potentials are similar. The displacement of the corresponding s states of N^{13} and C^{13} is accounted for within an uncertainty of about 25 percent which is due to the lack of precise knowledge concerning the internal Coulomb energy of the excited states. Within this uncertainty, there is no evidence for the inequality of nn and pp nuclear forces. In the $O^{16}+p$, n s interactions there appear to be narrow levels in addition to the broad level. By utilizing radiative capture data between various states, it is possible to compute the level displacement of the conjugate $p_{\frac{3}{2}}$ states, again with an uncertainty of about 25 percent, the agreement with observation being satisfactory. The reduced widths of the $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ states are about 15-times smaller than the value expected for a simple two-body potential, such as a square well, between the odd particle and core, and it is inferred from the sum rule for processes that other alternatives are involved. In contrast, the magnetic moment of the $p_{\frac{1}{2}}$ state of C^{13} and the large value of the internal transition moment $\mathfrak{M}$ connecting the $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ states suggests that the single alternative, odd nucleon plus C^{12} core, occurs a major fraction of the time in the p states. Thus, there is evidence for both the independent- and many-particle models; this paradox is not new, and we refer to interesting discussions by Weisskopf.[85]

The position of the $d_{\frac{5}{2}}$ state in C^{13} is somewhat uncertain and the computation of the displacement sensitive to the assumed channel radius. At any rate, the splitting in C^{13} of the $p_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ levels, which are almost unresolved in N^{13}, can be qualitatively understood as due to the differences of the electromagnetic properties of the odd nucleon.

[85] V. F. Weisskopf, Helv. Phys. Acta **23**, 187 (1950); Science **113**, 101 (1951).

Some concluding remarks concerning the channel radius may be appropriate. In the present investigation a rather large radius, $a=4.9\times10^{-13}$ cm in the case of the C^{12} interactions, was used to insure that the nuclear forces do not extend significantly into the external region. It was found that a 10 percent smaller radius gave a better account of the level displacements associated with the s and d interactions with C^{12}; however this observation is not particularly significant in view of the other uncertainties in the computation, such as in the internal Coulomb energy and in the representation used for the energy dependence of the logarithmic derivatives. It does not appear possible to solve for a from the given data,[86] although a lower limit of 3.6×10^{-13} cm could be set in the case of the $C^{12}+p$ s-wave interaction. The value $a=4.9$ is from 20- to 50-percent larger than the values obtained in various measurements (indicated on Fig. 3), but this is not unreasonable because these measurements do not necessarily indicate the limit of the nuclear interactions. The resonance parameters are found to be sensitively dependent upon a, which is to be expected in the barrier region where these parameters are, by definition, functions of a. On the other hand, in applications involving approximations the cross-section formulas are found to be rather insensitive to reasonable variations of a, provided that the same value is consistently used.

The author wishes to express his appreciation to Professors T. Lauritsen, R. F. Christy, W. A. Fowler, and Dr. Claude Bloch for their helpful discussions and suggestions. He is grateful to the AEC for a predoctoral fellowship which enabled him to carry out this investigation.

[86] In a recent paper by D. C. Peaslee, Phys. Rev. **85**, 555 (1952), a "phenomenological" radius for C^{13} is determined from the same data as treated here. The significance of this determination is questionable because the critical radial dependence of the N^{13} s proton reduced width was apparently neglected. Taking into account this radial dependence, the radius so determined would actually be that radius which would enable the one-level approximation to fit the interaction data from $E=2.37$ to 4.95 Mev; we could not determine such a radius.