High Energy Proton-Proton Scattering and Associated Polarization Effects*

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Three phenomenological descriptions of *p-p* scattering have been proposed to date, two of which, the singular tensor-force model of Christian-Noyes and the **L**·**S** force model of Case-Pais, are characterized by a singular triplet potential. The Jastrow model introduces a hard core in the singlet interaction. The parameters of these various potentials were all determined, in part, by use of Born approximation estimates of triplet scattering cross sections at high energies. Inasmuch as the validity of such estimates is not clear in those cases where the interaction is singular, more exact calculations have been performed at 240 Mev with the singular potentials (and also, for comparative purposes, with the well-behaved triplet interaction of Jastrow) using Schwinger's variational method and/or numerical integration procedures. Such calculations have also been performed at 450 Mev for the singular tensor-force model. To insure the existence of solutions, a zero cutoff has been introduced in the singular potentials at distances of the order of the nucleon Compton wavelength.

It is found that, in all instances where the potential is singular, the Born approximation and the use of Born approximation trial functions in a variational treatment are completely unreliable. More exact calculations of the differential scattering cross sections than those in Born approximation introduce large anisotropies in the case of the singular interactions and, in particular, in the case of the **L**·**S** potential. The hard-core model, on the other hand, is in qualitative agreement with experiment.

Also discussed are the results of calculations of polarization effects in a double *p-p* scattering and their implications for the various potentials considered.

I. INTRODUCTION

WITH tne advent of high energy accelerating machines, the range of energies available in p-p scattering experiments has progressed beyond the classical domain of less than 10 Mev[1] so as to include energies up to 450 Mev. Scattering in the classical domain is consistent with pure *S*-wave scattering in singlet states. It is nonetheless clear[2] that this can only provide knowledge of two parameters, the scattering length and the effective range, and leaves undetermined the shape of the potential. In addition, no knowledge can be obtained of scattering in triplet states. It was accordingly believed that experiments at higher energies which would include contributions from higher angular momentum states would provide a more unique phenomenological description of the proton-proton interaction.

As experiments were pushed to higher energies, it became increasingly evident that the predictions of conventional central-force potential models, adjusted so as to explain the low energy scattering data, would be in complete contradiction with experiment. These predictions are characterized by a preponderance of scattering in the forward direction and relatively negligible scattering at 90°; in contrast to this, experiments give a fairly isotropic distribution after one takes into account the Coulomb scattering at smaller angles. In Fig. 1, we have indicated the general behavior of the differential cross sections as obtained at 32 and 345 Mev at Berkeley[3] and as obtained at 240 Mev at Rochester.[4] The isotropy at the two higher energies persists down to angles[5] less than 15° below which occurs the expected rise due to Coulomb scattering. Other data not shown in the figure give the same characteristic flatness at 105 Mev[6] and at 146 Mev,[7] with the magnitude of the differential cross section ~5 mb corresponding to the cross section reported by the Rochester group. There is still a discrepancy of ~1 mb in magnitude between results quoted by the Berkeley group and those quoted by others; however, there is general agreement as to the angular dependence.

Although the parameters of the singlet potentials are predetermined by the low energy results, there is still freedom in the choice of the triplet potential. Since the anomalous high energy scattering cannot be accounted for in terms of central forces, one is led to consider the only other interactions which may occur in the *p-p* system subject to the restriction that the velocities appear in no higher power than the first, *viz.*, tensor (S_{12}) and **L**·**S** forces.[8] We shall refer to these, collectively, as spin-orbit forces. Various phenomenological models have been suggested using spin-orbit coupling; in particular, Christian and Noyes[9] have employed a tensor force, whereas Case and Pais[10] have proposed an

* This research was supported in part by the AEC.

[1] Energies quoted correspond to the kinetic energy of the incident proton as measured in the laboratory system.

[2] J. Schwinger, Harvard lecture notes (1946–47) (unpublished); H. A. Bethe, Phys. Rev. **76**, 38 (1949); G. Breit, Revs. Modern Phys. **23**, 238 (1951).

[3] Chamberlain, Segrè, and Wiegand, Phys. Rev. **83**, 923 (1951); W. K. H. Panofsky and F. Fillmore, Phys. Rev. **79**, 57 (1950); Cork, Johnston, and Richman, Phys. Rev. **79**, 71 (1950).

[4] C. L. Oxley and R. D. Schamberger, Phys. Rev. **85**, 416 (1952); O. A. Towler, Phys. Rev. **85**, 1024 (1952).

[5] Angles of scattering correspond to those measured in the center-of-mass system.

[6] Birge, Kruse, and Ramsey, Phys. Rev. **83**, 274 (1951).

[7] Cassels, Stafford, and Pickavance, Nature **168**, 468 (1951).

[8] L. Eisenbud and E. Wigner, Proc. Nat. Acad. Sci. **27**, 281 (1941).

[9] R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1950).

[10] K. M. Case and A. Pais, Phys. Rev. **80**, 203 (1950).

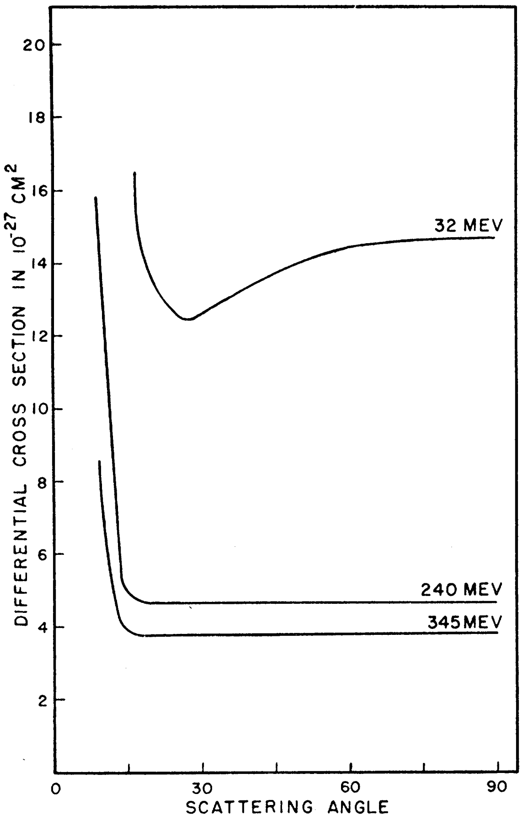

FIG. 1. Experimental differential cross sections for p-p scattering at 32, 240, and 345 Mev.

alternative description in terms of **L**·**S** forces. In both cases the interactions are singular, the former varying near the origin as $1/r^2$ and the latter as $1/r^3$. Jastrow,[11] on the other hand, has proposed a well-behaved triplet tensor potential in conjunction with a repulsive core in the singlet interaction. All of these models were adjusted so as to fit the low energy and 32-Mev data, and Born approximation estimates were made of the scattering at 350 Mev (except for the singlet scattering with the hard-core potential for which exact calculations were performed). The parameters of the different models are shown in Table I.

In view of the singular nature of both the **L**·**S** potential of Case and Pais and of the tensor potential of Christian and Noyes, the validity of the Born approximation, even at high energies, is uncertain, and, for this reason, more exact calculations are highly desirable. Accordingly, calculations with these models have been performed using variational and numerical methods in integrating the differential equations. In order to insure the existence of solutions to the scattering problem, a zero cutoff has been introduced in the singular potentials at distances of the order of the nucleon Compton wavelength. The calculations, which we describe here, are confined to 240 Mev in line with experiments currently in progress at the Rochester 130-inch cyclotron. Calculations have also been performed for the singular tensor force at 450 Mev (the Chicago energy) in order to obtain an estimate of the energy dependence for a singular potential. The results bear out that, at these energies, the Born approximation is, indeed, poor for these potentials. Comparable calculations performed with the Jastrow model which has a well-behaved triplet potential show that the Born approximation is much better, as expected. The resultant cross sections and their implications for the various potential models considered will be discussed in Sec. III following a brief resumé of the methods used in the computation which is given in Sec. II.

A particular consequence of spin-orbit forces is that they lead to a mixing of the triplet spin states so that, after a single p-p scattering, the outgoing particles are polarized.[12] A second scattering of these particles results in a distribution exhibiting an azimuthal asymmetry. Since polarization effects are confined to triplet states, and the relative importance of scattering in these states increases with energy, such effects might very well provide a means of distinguishing among various potential models which can be adjusted to fit the unpolarized cross sections.[13] Calculations of the polarization effects have accordingly been performed for each of the three potential models considered in this paper despite the fact that the agreement with the ordinary scattering cross-section data is, in some cases, poor. The results of these calculations are discussed in Sec. IV.

II. METHODS OF CALCULATION

The analysis which follows is based entirely on the nonrelativistic Schrödinger equation. Actually, relativistic corrections involving both dynamic and kinematic effects begin to assume importance at the energies which we are considering and, at 240 Mev, can amount to ~15 percent.[14] Hence, one must allow for deviations of this order of magnitude in making any comparisons with experiment.

The differential cross section in the center-of-mass system for the scattering of a beam of unpolarized protons by protons for the general case of spin-orbit forces, taking into account the identity of the particles, can be written, in the notation of Ashkin and Wu,[15] as

$$\sigma=\tfrac{1}{4}\sigma_{\text{sing}}+\tfrac{3}{4}\sigma_{\text{trip}}=|S_0(\theta)|^2+\sum_{m_s,m_s'}|S_{m_s'm_s}(\theta,\phi)|^2, \quad (1)$$

[11] R. Jastrow, Phys. Rev. **81**, 165 (1951).

[12] J. Schwinger, Phys. Rev. **69**, 681 (1946); L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

[13] We should like to thank R. E. Marshak for suggesting the use of polarization effects as a test of potential models which suggestion served as the initial motivation for the present investigation.

[14] H. Snyder and R. E. Marshak, Phys. Rev. **72**, 1253 (1947); see also reference 10.

[15] J. Ashkin and T. Y. Wu, Phys. Rev. **73**, 973 (1948).

where the singlet amplitude is given by

$$S_0(\theta)=(1/2ik)\sum_{\text{even }L'}[4\pi(2L'+1)]^{\frac{1}{2}}\times\{\exp(2i\delta_{L'})-1\}Y_{L'}{}^0(\theta),\quad(2)$$

and the triplet amplitude by

$$S_{m_s'm_s}(\theta,\phi)=(1/2ik)\sum_{\text{odd }L}\sum_{J=L-1}^{L+1}[4\pi(2L+1)]^{\frac{1}{2}}\times\{\exp(2i\delta_L{}^{Jm_s})-1\}(SLm_s-m_s'm_s'|SLJm_s)\times(SLJm_s|SL0m_s)Y_L{}^{m_s-m_s'}(\theta,\phi).\quad(3)$$

Here, $\delta_{L'}$ is the singlet phase shift corresponding to the partial wave of orbital angular momentum L', and $\delta_L{}^{Jm_s}$ is the triplet phase shift corresponding to the orbital angular momentum L and total angular momentum J with z-component m_s. The spherical harmonics $Y_L{}^{m_s}$ and the expansion coefficients $(SLm_s-m_s'm_s'|SLJm_s)$ are as defined in Condon and Shortley.[16] The magnitude of the momentum of each proton in the center-of-mass system is given by $\hbar k$.

It is clear from our introductory remarks that our essential problem is to obtain a more accurate estimate of the triplet scattering cross section than is given in Born approximation for the various potential models considered. In discussing methods of computation of triplet phase shifts, we shall for the sake of generality assume that the spin-orbit interaction includes a tensor force so that we have to deal with both coupled and uncoupled radial wave equations.

Three methods were adopted for the calculation of the triplet phase shifts. Generally speaking, phase shifts corresponding to small values of angular momentum were calculated by variational and/or exact procedures. The remaining phase shifts were always included in the scattering amplitude in Born approximation. The computation of phase shifts in Born approximation for spin-orbit forces is well known and will not be discussed here.[10,15]

The variational treatment of coupled and uncoupled states was carried out using the procedure of Schwinger.[17] It will be recalled that the scattering is considered in the parity representation which is designated by the quantum numbers $(-)^L$, S, J, m, where, for a given value of J, there are two modes or scattering eigenstates corresponding to the coupling of the $L=J-1$ and $L=J+1$ states, and one mode corresponding to $L=J$. In the p-p system, the situation is somewhat simplified in that, for even values of J, one can have only $L=J\pm1$, and for odd values of J, only $L=J$. The variational treatment of the uncoupled states is comparatively straightforward and will not be discussed further except to note that the trial functions used were $\bar{v}_L=krj_L(kr)$, where $j_L(kr)=(\pi/2kr)^{\frac{1}{2}}J_{L+\frac{1}{2}}(kr)$.

The coupled modes are characterized by the wave functions $(u_J{}^\alpha F_{J-1}{}^{Jm}+w_J{}^\alpha F_{J+1}{}^{Jm})/r$ and $(u_J{}^\gamma F_{J-1}{}^{Jm}+w_J{}^\gamma F_{J+1}{}^{Jm})/r$, where α and γ label the two modes and where the $F_L{}^{Jm}$ are the normalized spin spherical harmonics.[15] The essential property of the scattering eigenstates is that, asymptotically, for each mode, u_J and w_J involve the same real phase shift. A variational expression can be obtained for $k\cot\delta_J$ in terms of the functions u_J and w_J for which appropriate trial functions can then be chosen, say, $\bar{u}_J$ and $\xi_J\bar{w}_J$. Two values of ξ_J, *viz.*, $\xi_J{}^\alpha$ and $\xi_J{}^\gamma$, are then determined by requiring $k\cot\delta_J$ to be stationary with respect to variations in ξ_J, leading finally to the two phase shifts $\delta_J{}^\alpha$ and $\delta_J{}^\gamma$. In our calculations, $\bar{u}_J$ and $\bar{w}_J$ were chosen to correspond to Born approximation functions, i.e., $\bar{u}_J=krj_{J-1}(kr)$ and $\bar{w}_J=krj_{J+1}(kr)$.

The scattering, for each mode, is determined not only by the phase shift but also by the constant asymptotic ratio of $-w_J/u_J$ which we call the amount of admixture η_J. An integral expression for η_J can be obtained in terms of the functions u_J and w_J which one can again replace with the trial function $\bar{u}_J$ and $\xi_J\bar{w}_J$. The two amounts of admixture $\eta_J{}^\alpha$ and $\eta_J{}^\gamma$ are not independent but satisfy

$$1+\eta_J{}^\alpha\eta_J{}^\gamma=0.\quad(4)$$

It is to be noted that the scattering amplitude involves three independent parameters for a given even value of J, *viz.*, $\delta_J{}^\alpha$, $\delta_J{}^\gamma$ and $\eta_J{}^\alpha$. On the other hand, in the $LSJm$ representation, the phase shifts $\delta_{J\pm1}{}^{J0}$ and $\delta_{J\pm1}{}^{J1}$ are complex so that one apparently has eight parameters but only three are independent.[18] It is to be borne in mind that it is only the phase shifts $\delta_J{}^\alpha$ and $\delta_J{}^\gamma$ which are computed by a variational principle but not the amount of admixture $\eta_J{}^\alpha$.

The Born approximation in the parity representation is obtained from the variational formulation by assuming the potential to be a small perturbation, so that only leading terms in the potential are considered. We again use trial functions corresponding to plane wave solutions. It then follows that the amount of admixture is unaltered from its value assumed in the trial function, resulting in the equality $\eta_J=\xi_J$. This definition of the

TABLE I. Parameters for phenomenological p-p potential models.

	Potential model		
	Singular tensor force	Hard core	L·S force
Singlet potential	$-V_s$, $r<r_s$; 0, $r\geq r_s$	∞, $r<r_0$; $-V_se^{-(r-r_0)/r_s}$, $r\geq r_0$	$-V_s\frac{e^{-r/r_s}}{r/r_s}$
Triplet potential	$V_tS_{12}\frac{e^{-r/r_t}}{(r/r_t)^2}$	$V_tS_{12}e^{-r/r_t}$	$V_t\frac{\mathbf{L}\cdot\mathbf{S}}{(r/r_t)}\frac{d}{d(r/r_t)}\left(\frac{e^{-r/r_t}}{r/r_t}\right)$
	Parameters		
r_s in 10^{-13} cm	2.6	0.40	1.18
r_t in 10^{-13} cm	1.6	0.75	1.07
r_0 in 10^{-13} cm		0.60	
V_s in Mev	13.27	375	45.8
V_t in Mev	±18	−50.8	±29.8

[16] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (The Macmillan Company, New York, 1935), p. 76.
[17] F. Rohrlich and J. Eisenstein, Phys. Rev. **75**, 705 (1949).
[18] W. Rarita and J. Schwinger, Phys. Rev. **59**, 436 (1941).

TABLE II. Singlet phase shifts for the singular tensor-force model at 240 Mev.

L	δ_L (Born)	δ_L (exact)
0	0.232	0.261
2	0.232	0.237
4	0.032	0.035

TABLE III. Triplet phase shifts for the singular tensor-force model at 240 Mev.

	$r_c=0.84\hbar/Mc$ V_t in Mev				$r_c=1.40\hbar/Mc$ $V_t=18$ Mev		
	± 18	-18	18	18			
Phase shifts	Born	Variational	Variational	Exact	Born	Variational	Exact
δ_1^{00}	± 0.897	-0.521	1.119	2.570	0.884	1.067	1.8105
δ_1^{11}	∓ 0.449	0.559	-0.337	-0.3345	-0.442	-0.338	-0.3379
K_1^{20}	∓ 0.035	0.233	0.020	0.1268	-0.037	0.018	0.1108
ζ_1^{20}	0	-0.006	0.005	0.0223	0	0.005	0.0203
K_3^{20}	∓ 0.024	-0.033	-0.029	-0.0399	-0.024	-0.029	-0.0405
ζ_3^{20}	0	0.004	-0.004	-0.0139	0	-0.003	-0.0127
K_1^{21}	± 0.173	0.198	0.202	0.3386	0.172	0.200	0.3237
ζ_1^{21}	0	0.004	-0.004	-0.0139	0	-0.003	-0.0127
K_3^{21}	± 0.184	-0.068	0.153	0.1719	0.184	0.154	0.1725
ζ_3^{21}	0	-0.006	0.005	0.0223	0	0.005	0.0203
δ_3^{31}	∓ 0.074	0.077	-0.071		-0.074	-0.071	
η_2^{α}	-0.862	0.069	-0.627	-0.4093	-0.867	-0.634	-0.4324
δ_2^{α}	± 0.176	0.214	0.186	0.2994	0.177	0.185	0.2864
δ_2^{γ}	∓ 0.029	-0.049	-0.013	-0.0007	-0.029	-0.013	-0.0032

Born approximation in the parity representation is equivalent to that in the $LSJm$ representation provided one consistently neglects terms in δ^2.

By an exact procedure, we mean a numerical integration of the differential equations. The solution of uncoupled equations was carried out using the particular method of Feinstein and Schwarzchild.[19] The extension to coupled equations was carried through as follows.

The differential equations take on the general form

$$\begin{aligned} u''(x)+f(x)u(x)+g(x)w(x)=0, \\ w''(x)+h(x)w(x)+g(x)u(x)=0, \end{aligned} \tag{5}$$

where $u(x)/x$ and $w(x)/x$ are the radial wave functions for $L=J-1$ and $L=J+1$, respectively. Now consider the Taylor series expansions about x of $u(x\pm\Delta)$. Then

$$u(x+\Delta)+u(x-\Delta)=2u(x)+\Delta^2u''(x) +(\Delta^4/12)u^{iv}(x)+\cdots. \tag{6}$$

Neglecting $(\Delta^4/12)u^{iv}(x)$ and denoting $x+n\Delta$ by x_n and $u(x_n)$ by u_n, we have

$$u_{n+1}=B_nu_n-u_{n-1}+C_nw_n, \tag{7}$$

where

$$B_n=2-\Delta^2f(x_n), \tag{8}$$

$$C_n=-\Delta^2g(x_n). \tag{9}$$

We find, similarly,

$$w_{n+1}=D_nw_n-w_{n-1}+C_nu_n, \tag{10}$$

where

$$D_n=2-\Delta^2h(x_n). \tag{11}$$

Clearly, if u_0, w_0 and u_1, w_1 are known, then u and w can be extended into the asymptotic region by simultaneous application of (7) and (10). The choice of Δ is governed by consideration of the error involved in neglecting the fourth and higher order derivatives. In practice, it is convenient to integrate the differential equations as far out from the origin as possible in terms of power series expansions and then to extend these by the numerical procedure.

We can generally find two independent sets of u and w which are regular at the origin.[20] If these solutions are u_a, w_a and u_b, w_b, a Wronskian condition holds,

$$u_au_b'-u_bu_a'+w_aw_b'-w_bw_a'=0, \tag{12}$$

which supplies a useful check on the numerical analysis. The phase shifts themselves are determined by forming the appropriate linear combinations which asymptotically represent the incident plus outgoing spherical waves.

III. UNPOLARIZED SCATTERING CROSS SECTIONS

(a) Singular Tensor-Force Model

Christian and Noyes[9] attempted a fit of both the 32-Mev and 345-Mev p-p data without seeking to maintain the charge-independence hypothesis. On the basis of the lower energy data, they chose for the singlet potential a square well since potentials of other conventional shapes yield too much small angle scattering. Since the singlet potential contributes only to forward scattering at the higher energy, most of the scattering must take place in triplet states. In consequence of the large momentum transfers which are required, i.e., large cross sections at large angles, they were ultimately led to choose a triplet potential which is singular at the origin and of the form $V_{\text{trip}}=V_tS_{12}e^{-x}/x^2$, where $x=r/r_t$ (see Table I for the values of the parameters). Good agreement is obtained with the 32-Mev data, and Born approximation estimates give a fairly flat cross section for angles greater than 30° at 345 Mev. It is to be noted that no choice of sign was made for V_t since the analysis at 32 Mev is still confined to exploring the tail of the potential. This holds even though one might have expected the Coulomb-nuclear interference to be sensitive to the choice of sign.

In actual fact, it is known that solutions to the scattering problem may not exist for potentials which have a $1/r^2$ singularity at the origin.[21] Considering, for the moment, the radial equations corresponding to the

[19] L. Feinstein and M. Schwarzchild, Rev. Sci. Instr. 12, 405 (1941).

[20] If the potentials are so singular that no such solutions can be found, a cutoff can be applied at $x=x_c$ to the potentials, in which case the solutions for $x>x_c$ are obtained by equating values and derivatives at $x=x_c$ to those of the inside solutions.

[21] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, Oxford, 1949), second edition, p. 40.

uncoupled P_0 and P_1 states for either sign of V_t, we have

$$(u_1^{Jm})''+(1-2/x^2-\lambda_J e^{-x/K}/x^2)u_1^{Jm}=0, \quad J=0,1 \quad (13)$$

where $x=kr$, $K=kr_t$, $\lambda_0=\mp 4.444$, $\lambda_1=\pm 2.222$ for $V_t=\pm 18$ Mev. For small x, the solutions of this equation are given by $xj_\nu(x)$, where $\nu(\nu+1)=2+\lambda_J$. Since $xj_\nu(x)\sim x^{\nu+1}$ when $x\sim 0$, well-behaved solutions exist provided $\nu\geqslant 0$.[22] Thus, we must have

$$\nu=\tfrac{1}{2}[-1\pm(9+4\lambda_J)^{\frac{1}{2}}]\geqslant 0, \qquad (14)$$

or $\lambda_J\geqslant -2$. Regardless of the sign of V_t, this relation cannot be satisfied for both $J=0$ and $J=1$. If $-2.5<\lambda_J<-2$, both radial wave functions $u(x)/x$ are unbounded at the origin; however, one is less singular than the other. If the potential is so attractive that $\lambda_J\leqslant -2.5$, the wave functions $u(x)/x$ behave as $x^{-\frac{1}{2}}\exp[\pm\frac{1}{2}i(|9+4\lambda_J|)^{\frac{1}{2}}\log x]$ near the origin, oscillate rapidly and their proper linear combination is unknown. Therefore, in order to insure the existence of well-behaved solutions for the $J=0$ and $J=1$ partial waves, we have introduced a zero cutoff in the triplet potential from the origin to distances $r_c\sim\hbar/Mc$. The introduction of such a cutoff would of course, at the same time, guarantee the existence of well-behaved solutions for coupled or uncoupled states corresponding to higher angular momenta; in actual fact, such solutions do exist in these states in the absence of a cutoff. It is clear that these considerations are independent of energy.

Table II lists the singlet phase shifts for the singular tensor-force model at 240 Mev. Since the shape of the singlet potential is that of a square well, exact analytical solutions are easily found, and comparison with Born approximation phase shifts illustrates the good agreement to be expected at high energies for well-behaved central potentials. The cross section due to singlet scattering alone is contained in Fig. 3 and is seen to be peaked in the forward direction and to fall off rapidly with increasing angle. The concentration in the forward direction is due to the constructive interference of the even Legendre polynomials at 0° and the destructive interference at 90° (the phase shifts are all of the same sign).

In virtue of the coupling of different angular momentum states, the triplet phase shifts for even J (with the exception of the degenerate case $J=0$) are all complex; in the parity representation, they are real. Phase shifts for odd J correspond to uncoupled states and are consequently always real. Table III incorporates the results of Born, variational and exact calculations of phase shifts for $J\leqslant 3$ at 240 Mev for two choices of cut-off radius, $r_c=0.84\hbar/Mc$ and $r_c=1.40\hbar/Mc$. In the case of the coupled phase shifts, we use the notation $\delta_L^{Jm_s}=K_L^{Jm_s}+i\zeta_L^{Jm_s}$. Also listed are the corresponding quantities δ_J^α, δ_J^β and η_J^α in the parity representation.[23]

[22] For a solution to be well-behaved, the radial wave function $u(x)/x$ must be finite when $x=0$.

[23] We adopt the convention that η_J^α is always less than one in absolute value.

The variational calculation for $r_c=0.84\hbar/Mc$ was performed for both signs of V_t. Comparison with the Born approximation shows that considerable changes occur for $V_t=-18$ Mev, and the resultant cross section as shown in Fig. 2 is very anisotropic.[24] A more reasonable cross section is obtained with $V_t=+18$ Mev, and this choice of sign was hence used in all further calculations and is implied throughout the following discussion.[25]

The uncoupled phase shift δ_1^{11} corresponds to a repulsive potential, and there is very good agreement between the variational and the exact values. On the other hand, δ_1^{00} corresponds to a strongly attractive

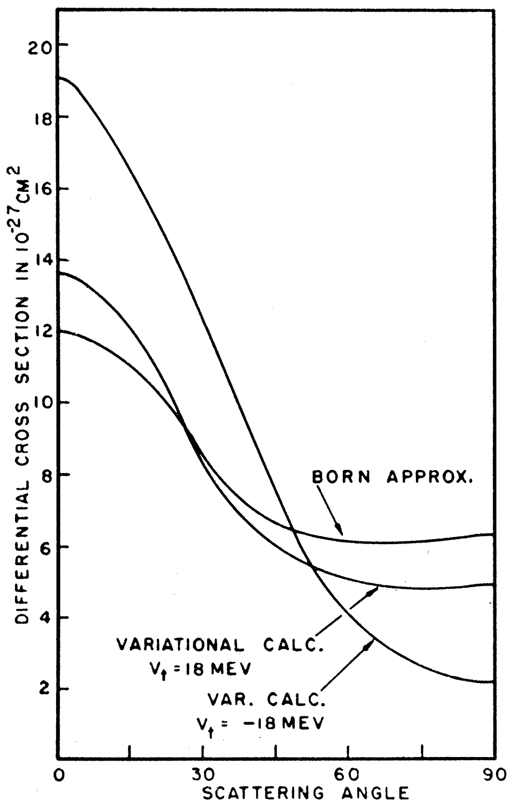

FIG. 2. Differential scattering cross sections for the singular tensor-force model at 240 Mev corresponding to Born approximation and variational treatments of the triplet scattering ($V_t=\pm 18$ Mev, $r_c=0.84\hbar/Mc$). In the latter case, phases with $J\leqslant 3$ were included in variational treatment, higher J phases in Born approximation. The singlet scattering cross section is exact.

[24] Throughout this paper, Born approximation cross sections always correspond to those obtained by replacing $\exp(2i\delta_B)-1$ in the scattering amplitude by $2i\delta_B$, where δ_B represents the Born approximation phase shift.

[25] The fact that there is moderate agreement between the Born approximation cross section and that obtained by the variational method for one sign of the potential, $V_t=+18$ Mev, but not for the other, is to be regarded as quite fortuitous in view of the large differences in the phase shifts as obtained by the two procedures. It is evident from Fig. 3 that more exact calculations for the case $V_t=+18$ Mev maintain the correspondence in cross section with that given in Born approximation, although the phase shifts differ considerably (see Table III). This conclusion has also been reached by Don R. Swanson [Phys. Rev. 87, 208 (1952)].

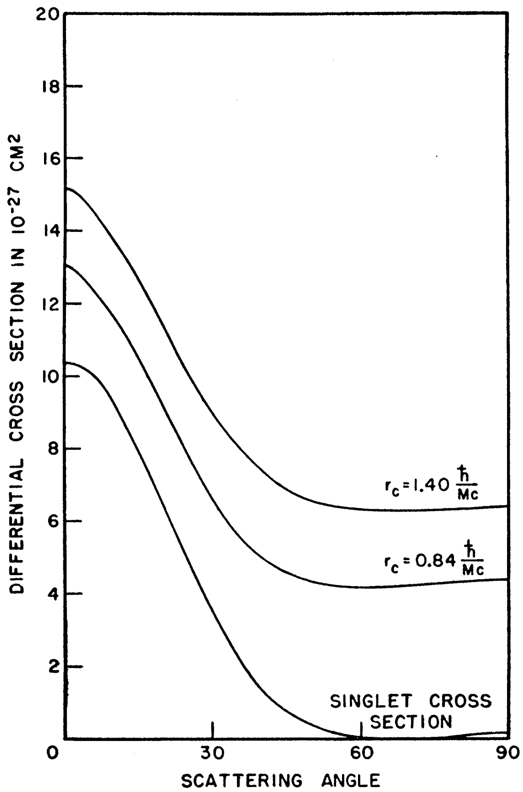

FIG. 3. Differential scattering cross sections for the singular tensor-force model at 240 Mev corresponding to exact treatment of triplet scattering for two values of cut-off radius ($V_t = 18$ Mev). Phases with $J \leqslant 2$ were calculated exactly, those with $J=3$, 4, and 5 were included in variational treatment, higher J phases in Born approximation. Also shown is the exact singlet cross section.

potential, and the variational calculations, while giving results which approach the correct ones, are nevertheless still far from adequate. This particular phase shift depends in a sensitive way on the cut-off radius as is to be expected in view of the fact that no phase shift exists at all in the absence of a cutoff. It is to be noted that the coupled phase shifts are not very sensitive to the change in cut-off radius which is again reasonable since well-behaved solutions exist even without the cutoff.

For purposes of simplicity, it is best to compare the results for the coupled states in the parity representation (see Table III). Here, the inadequacy of the Born approximation trial functions is clearly indicated by the large changes in the amount of admixture η_2^{α} as one proceeds from the Born approximation to the variational calculation, and then to the exact calculation. Note that a comparison of Born approximation with variational estimates for the phase shifts in the more complicated $LSJm$ representation can be misleading, although the agreement with the exact results is, in any case, inadequate.

The cross sections corresponding to the exact phase shifts for $J=0$, 1, 2 and the two choices of cut-off radius, $0.84\hbar/Mc$ and $1.40\hbar/Mc$, are shown in Fig. 3. Phase shifts corresponding to $J=3$, 4, 5 were computed by means of the variational method and found to be generally small ($\gtrsim 0.05$). Higher angular momentum phase shifts were inserted into the scattering amplitude in Born approximation. The cross sections remain essentially flat at $\sim$4.4 and 6.4 mb, respectively, between 45° and 90° and show a rapid rise at smaller angles. Thus, below 25°, the rise of singlet scattering to values greater than 5 mb renders impossible any satisfactory fit in this region. The experimental cross section at 90° corresponds to an intermediate value of r_c.

Calculations have also been made of the scattering cross sections at 450 Mev. Potential models become quite questionable at these energies; nonetheless, it is of interest to study the qualitative predictions of the present nonrelativistic theory in the absence of any better theory.

Table IV includes the 450-Mev phase shifts for $J \leqslant 3$ as obtained in Born approximation, by variational treatment and by exact calculation. At this energy, the Born approximation appears to be even more unreliable. The fact that the cut-off radius used for the Born and variational calculations was $1.02\hbar/Mc$ does not prevent a comparison of the results as obtained by the various methods except for the δ_1^{00} phase shift.

If we compare the results of the exact calculations as the energy is varied from 240 Mev to 450 Mev (see Tables III and IV), we see that δ_1^{00} is altered only slightly, while the other phase shifts show increases of roughly 40 percent. The cross sections as shown in Fig. 4 are similar in shape for both values of cut-off radius, the difference at 90° being 0.7 mb in contrast to 2 mb at 240 Mev. The rise at small angles is again a manifestation of the singlet scattering.

TABLE IV. Triplet phase shifts for the singular tensor-force model at 450 Mev ($V_t = 18$ Mev).

Phase shifts	r_c in $\hbar/Mc$ 1.02 Born	1.02 Variational	0.84 Exact	1.40 Exact
δ_1^{00}	1.093	1.160	2.4135	1.7085
δ_1^{11}	−0.546	−0.409	−0.4406	−0.4373
K_1^{20}	−0.041	0.010	0.1992	0.1629
ζ_1^{20}	0	0.003	0.0364	0.0316
K_3^{20}	−0.009	−0.017	−0.0352	−0.0376
ζ_3^{20}	0	−0.002	−0.0216	−0.0190
K_1^{21}	0.210	0.201	0.4373	0.4065
ζ_1^{21}	0	−0.002	−0.0216	−0.0190
K_3^{21}	0.241	0.175	0.2030	0.2060
ζ_3^{21}	0	0.003	0.0364	0.0316
δ_3^{31}	−0.113	−0.108		
η_2^{α}	−0.928	−0.711	−0.3599	−0.3987
δ_2^{α}	0.223	0.191	0.3996	0.3615
δ_2^{γ}	−0.023	−0.007	0.0125	0.0074

(b) Hard-Core Model

An attempt to give a phenomenological description of p-p scattering within the framework of the charge-independence hypothesis has been made by Jastrow.[11] There is introduced in the singlet state an infinite potential barrier at small distances surrounded by an attractive exponential well. The presence of the repulsive core allows for agreement with the 32-Mev data even though an exponential well is used. At high energies, the main effect of the core is to change the sign of the S-phase shift with respect to the sign of the D, G, $\cdots$ phase shifts. The resultant S-D interference is such as to diminish the singlet scattering at small angles and to enhance it at 90° which is in sharp contrast to the singlet scattering observed for the singular tensor-force model. In triplet states, the hard-core model has an exponential-well tensor force (the parameters of the singlet and triplet potentials are included in Table I). In comparing the predictions of the hard-core model for scattering at high energies with experiment, the singlet cross section was evaluated exactly, while the triplet scattering was calculated in Born approximation only.[11] The latter is peaked at $\sim$45° and combines with the singlet scattering to give reasonably flat cross sections.

In order to test the validity of the Born approximation for the case of a well-behaved spin-orbit potential, a calculation of the triplet scattering has been performed at 240 Mev using the variational procedure. The Born approximation phase shifts together with those obtained by use of the variational method are listed in Table V. It is evident that the plane-wave trial functions are in this case reliable, as is to be expected.

In Fig. 5, we compare the differential scattering cross section obtained in pure Born approximation with an improved cross section based on the variational estimates of the triplet phase shifts for $J\leqslant 3$ (higher angular momentum states, as always, being included in Born approximation). For the singlet scattering, we have used the exact cross section as abstracted from Jastrow's published curve at 250 Mev, the variation in cross section over the energy interval 240–250 Mev being considered inconsequential. Unlike the case of the singular tensor-force model, the corrections introduced by the variational treatment are quite small ($\gtrsim$0.3 mb for $\theta > 30°$).[26] In view of the nonrelativistic nature of the calculations and of the present experimental discrepancy as to the magnitude of the cross section, the quantitative agreement is encouraging.

TABLE V. Triplet phase shifts for hard-core model at 240 Mev.

Phase shifts	Born	Variational
δ_1^{00}	−0.555	−0.546
δ_1^{11}	0.277	0.282
K_1^{20}	0.047	0.042
ζ_1^{20}	0	−0.005
K_3^{20}	0.004	−0.009
ζ_3^{20}	0	0.003
K_1^{21}	−0.124	−0.115
ζ_1^{21}	0	0.003
K_3^{21}	−0.166	−0.166
ζ_3^{21}	0	−0.005
δ_3^{31}	0.080	0.082
η_2^{α}	0.952	0.877
δ_2^{α}	0.024	0.016
δ_2^{γ}	−0.143	−0.137

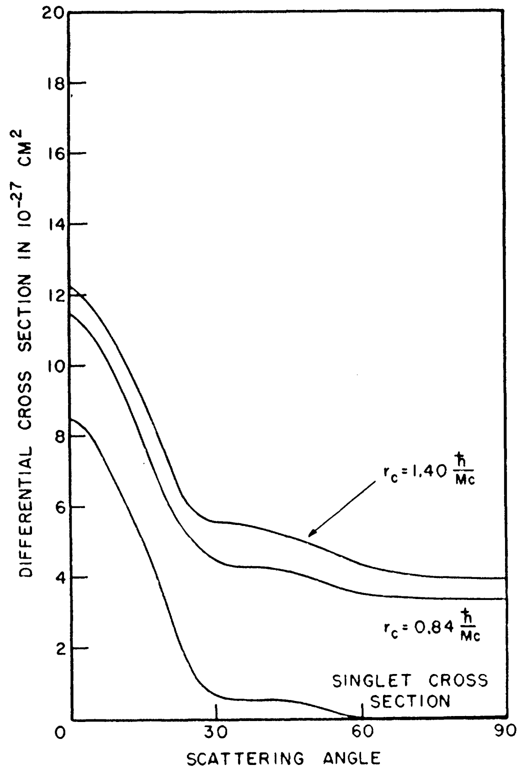

FIG. 4. Differential scattering cross sections for the singular tensor-force model at 450 Mev corresponding to exact treatment of triplet scattering for two values of cut-off radius (V_t= 18 Mev). Phases with $J\leqslant 2$ were calculated exactly, those with J=3, 4, and 5 were included in variational treatment, higher J phases in Born approximation. Also shown is the exact singlet cross section.

(c) L·S Force Model

Case and Pais[10] incorporate in their model the other type of spin-orbit force which can act between two protons, *viz.*, the **L·S** force. They also try to retain the charge-independence hypothesis. The model, as proposed, can only be considered qualitatively since no attempt was made to fit the low energy data with any degree of precision. For example, the singlet potential was chosen to be of Yukawa shape; actually, this would

[26] In the text, we have confined ourselves to one sign of the odd-parity force, *viz.*, $V_t = -50.8$ Mev. For the other choice of sign, V_t= 50.8 Mev, the deviations of the results of the variational calculations from those of the Born approximation are somewhat larger ($\gtrsim$0.7 mb). The resultant differential cross section is smaller than that in Born approximation except at small angles.

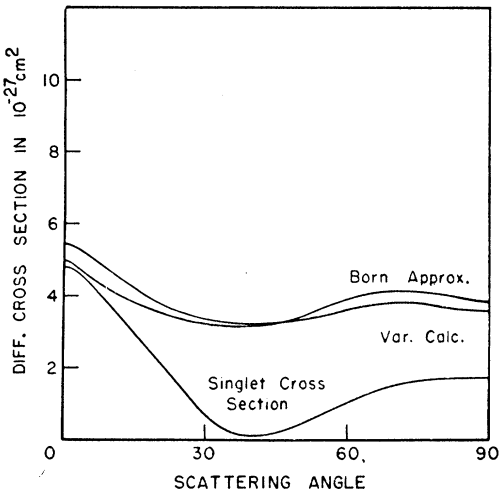

FIG. 5. Differential scattering cross sections for the hard-core model at 240 Mev corresponding to Born approximation and variational treatments of the triplet scattering. In the latter case, phases with $J \leqslant 3$ were included in variational treatment, higher J phases in Born approximation. Also shown is the singlet scattering cross section which is exact.

lead to too much small-angle scattering at 32 Mev. The triplet potential is very singular, varying as $1/r^3$ in the neighborhood of the origin, and only Born approximation estimates were used to determine the parameters of the model. The extreme singularity cannot, of course, be taken literally[27] and, in making more exact calculations, we have accordingly introduced a zero cutoff at $r_c = \hbar/Mc$.

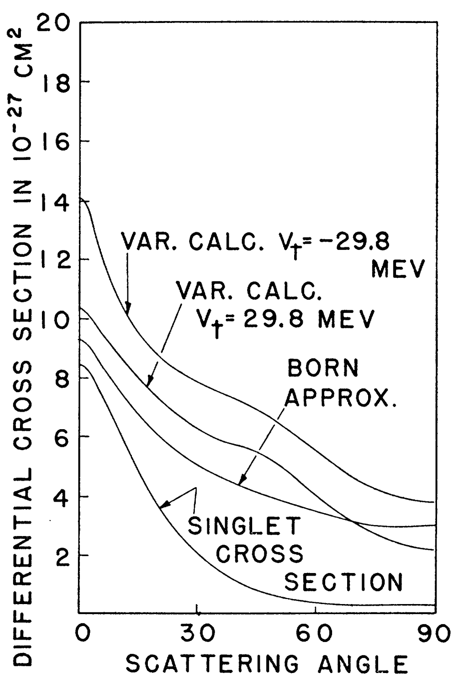

FIG. 6. Differential scattering cross sections for the $\mathbf{L}\cdot\mathbf{S}$ force model at 240 Mev corresponding to Born approximation and variational treatments of the triplet scattering ($V_t = \pm 29.8$ Mev, $r_c = \hbar/Mc$). In the latter case, phases with $L \leqslant 3$ were included in variational treatment, higher L phases in Born approximation. Also shown is the singlet scattering cross section calculated in Born approximation.

In the $LSJm$ representation, the operator $\mathbf{L}\cdot\mathbf{S}$ has definite eigenvalues, so that states corresponding to different values of L are unmixed in the presence of the interaction. Hence, the phase shifts are all real.

Table VI shows the results of variational calculations of the triplet phase shifts for both signs of V_t[28] (the actual parameters employed with the model are listed in Table I). We note that the P-phase shifts change rather considerably in going from the Born approximation to the variational estimates. In view of the singular nature of the potential, one can expect further large changes in performing an exact calculation. For example, for $V_t = 29.8$ Mev an exact calculation of the P_2-phase shift was performed giving $\delta_1^2 = 0.539$. Although the results of the variational calculation cannot be taken too seriously, the large deviations from the Born approximation are significant.

The corresponding cross sections which are plotted in Fig. 6 illustrate the complete inadequacy of the Born approximation for this type of potential. In fact, the qualitative fit with experiment obtained in Born approximation is almost wholly destroyed by the variational calculations. Inasmuch as the parameters of the present model were originally based on a Born approximation analysis, there appeared to be no point in carrying out more exact calculations.

TABLE VI. Triplet phase shifts for $\mathbf{L}\cdot\mathbf{S}$ force model at 240 Mev.

Phase shifts	V_t in Mev ±29.8 Born	29.8 Variational	−29.8 Variational
δ_1^0	∓0.524	−0.301	1.039
δ_1^1	∓0.262	−0.193	0.381
δ_1^2	±0.262	0.381	−0.193
δ_3^2	∓0.080	−0.074	0.088
δ_3^3	∓0.020	−0.020	0.020
δ_3^4	±0.060	0.064	−0.056

IV. POLARIZATION EFFECTS

Consider the scattering, in their center-of-mass system, of unpolarized protons by unpolarized protons. In the presence of spin-orbit forces, it can be shown that the outgoing particles will be polarized in a direction normal to the plane of scattering.[29] Thus, if we consider that the incoming particles move in the z-direction, then, particles scattered through an angle θ_1, ϕ_1 will generally have a nonzero expectation value

[27] This was clearly recognized by Case and Pais who note that their Born approximation results are only qualitative (see reference 10).

[28] If we put aside the question of charge independence and do not consider the n-p interaction, then the analysis of Case and Pais does not uniquely fix the sign of the triplet p-p force.

[29] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949); **76**, 541 (1949); **82**, 308 (1951).

for the y-component of the spin which is given by

$$\bar{s}_y(k_1,\theta_1,\phi_1)=\frac{1}{(2)^{\frac{1}{2}}}\frac{\operatorname{Im}\sum_{m_s} S_{0m_s}(S_{1m_s}{}^*-S_{-1m_s}{}^*)}{|S_0|^2+\sum_{m_s,m_s'}|S_{m_s'm_s}|^2}, \quad (15)$$

where k_1, the momentum of either particle, and $\bar{s}_y$ are in units of $\hbar$. For convenience, we consider particles scattered through θ_1, $\phi_1=0°$, since $\bar{s}_y$ will then represent the only nonvanishing component of the polarization. If the polarized particles are now scattered again by an unpolarized proton target, then a consideration of the scattering in their center-of-mass system shows that the intensity of outgoing particles will have an azimuthal asymmetry given by

$$I=I_0(k_1,k_2,\theta_1,\theta_2)[1+2\bar{s}_y(k_1,\theta_1,\phi_1=0°)\times P(k_2,\theta_2,\phi_2)], \quad (16)$$

where the y_2-axis is parallel to the y_1-axis of the first scattering and the z_2-axis is in the direction of incidence of the polarized beam. I_0 represents the unpolarized intensity. According to Wolfenstein and Ashkin[30] and Dalitz,[31]

$$P(k,\theta,\phi)=2\bar{s}_y(k,\theta,\phi), \quad (17)$$

so that

$$I=I_0(k_1,k_2,\theta_1,\theta_2)[1+P(k_1,\theta_1,\phi_1=0°)\times P(k_2,\theta_2,\phi_2)]. \quad (18)$$

This last expression can be further simplified if one neglects the degradation of energy of the proton in the first scattering. This approximation is a reasonable one for the cases of interest to us since the largest polarization effects occur in the neighborhood of 45° in the center-of-mass system (on general grounds,[29] $P\sim\sin\theta\cos\theta\cos\phi\sum_n a_n\cos^{2n}\theta$), and, corresponding to this angle of scattering, a particle initially having a kinetic energy of 240 Mev in the laboratory system will come off with 205 Mev.

Hence, assuming $\theta_1=\theta_2=\theta$, and $k_1=k_2=k$, Eq. (18) becomes

$$I=I_0(k,\theta)[1+\delta(k,\theta)\cos\phi], \quad (19)$$

where

$$\delta(k,\theta)=[P(k,\theta,\phi)]_{\phi=0°}{}^2. \quad (20)$$

Perhaps the more interesting quantity from an experimental point of view is 2δ, which we shall refer to as the asymmetry in a double-scattering experiment:

$$2\delta=\frac{I(\phi=0°)-I(\phi=180°)}{I(\phi=90°)}. \quad (21)$$

It is a straightforward matter to use the variational and/or exact triplet phase shifts computed in Sec. III in conjunction with the above formulas to evaluate the polarization effects in a double p-p scattering for each

[30] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).
[31] R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952).

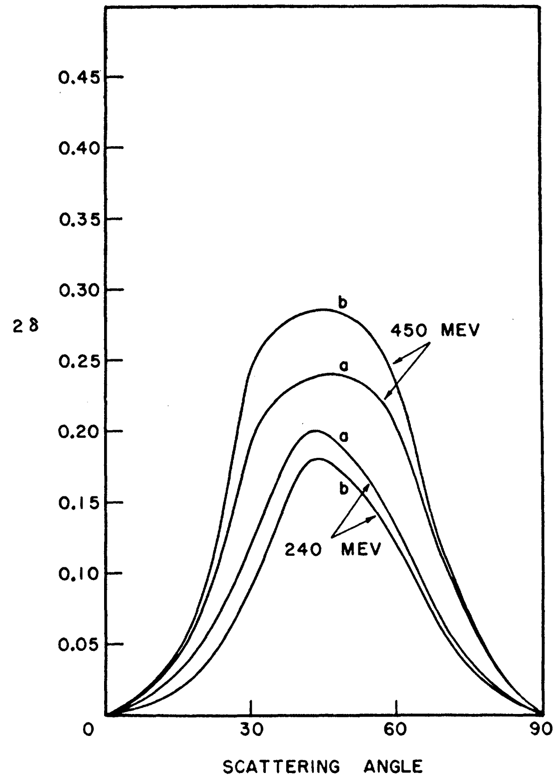

FIG. 7. Asymmetries in a double p-p scattering for singular tensor-force model at 240 and 450 Mev for (a) $r_c=1.40\hbar/Mc$, (b) $r_c=0.84\hbar/Mc$. The phases correspond to those used in Figs. 3 and 4.

of the potential models which we have considered. The results are shown in Figs. 7–9. As expected, the maximum in 2δ tends to occur at $\theta\sim45°$. In the case of the singular tensor-force model, $2\delta_{\max}$ is 0.18 and 0.28 for the energies 240 and 450 Mev, respectively, if we choose the cut-off radius $r_c=0.84\hbar/Mc$; the polarization effects

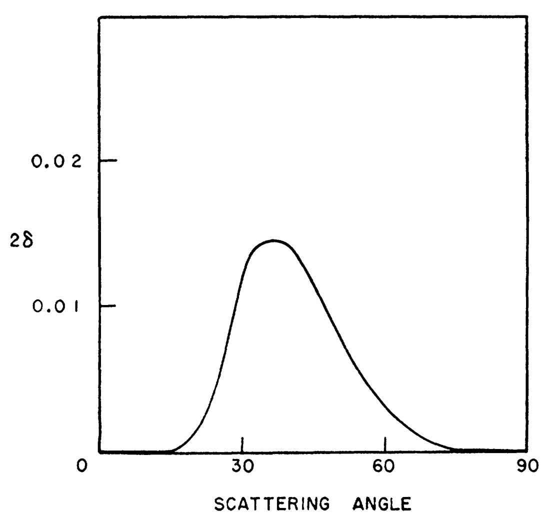

FIG. 8. Asymmetry in a double p-p scattering for the hard-core model at 240 Mev. The phases correspond to those used in Fig. 5.

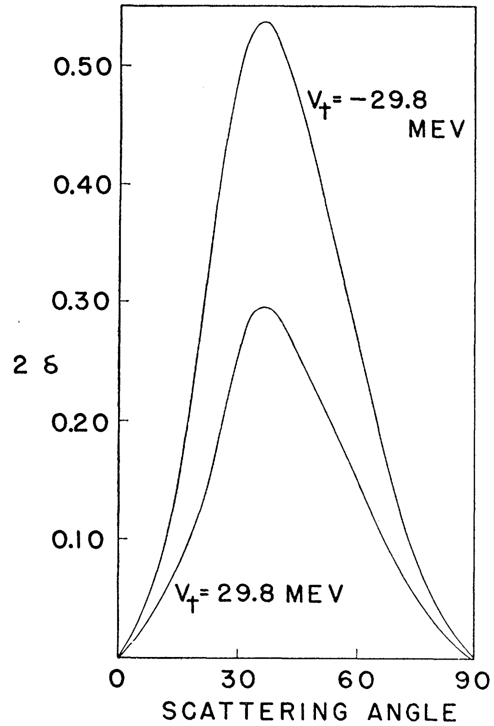

FIG. 9. Asymmetries in a double p-p scattering for the $\mathbf{L}\cdot\mathbf{S}$ force model at 240 Mev with $V_t = \pm 29.8$ Mev. The phases correspond to those used in Fig. 6.

are not very different for the larger cut-off radius $r_c = 1.40\hbar/Mc$. The $\mathbf{L}\cdot\mathbf{S}$ force model also yields large asymmetries, *viz.*, $2\delta_{\max}\sim 0.3$ and 0.5 for $V_t = 29.8$ Mev and -29.8 Mev, respectively. The hard-core model, on the other hand, predicts rather small asymmetries with $2\delta_{\max}\sim 0.015$.[32]

One can understand these results qualitatively by expressing $P(k,\theta,\phi)$ in terms of the phase shifts. For simplicity, let us consider only the real parts of the phase shifts with $J<3$. Then

$$P(k,\theta,\phi) = \frac{2\sin\theta\cos\theta\cos\phi}{k^2\sigma(k,\theta)}(\alpha+\beta\cos^2\theta), \quad (22)$$

where α is given by

$$\alpha = 3[\delta_3^{21}\delta_1^{00}] - 3[\delta_1^{21}\delta_1^{00}] + 4.5[\delta_1^{11}\delta_1^{20}] + 4.5[\delta_3^{20}\delta_1^{11}] + 9[\delta_1^{21}\delta_3^{20}] - 1.5[\delta_3^{21}\delta_3^{20}] - 1.5[\delta_1^{21}\delta_1^{20}] - 6[\delta_3^{21}\delta_1^{20}], \quad (23)$$

with

$$[xy] = \sin x \sin y \sin(x-y). \quad (24)$$

The quantity β is generally smaller than α and need not be given explicitly here; $\sigma(k,\theta)$ is the ordinary unpolarized differential scattering cross section.

In the case of the hard-core model, where the Born approximation is quite good for the triplet scattering, the first two terms on the right-hand side of Eq. (23) are large compared to the remaining terms. Since δ_1^{00} is a comparatively large phase shift, the quantity

$$3[\delta_3^{21}\delta_1^{00}] - 3[\delta_1^{21}\delta_1^{00}]$$

depends in an important way on the difference between δ_1^{21} and δ_3^{21} which are of the same order of magnitude and of the same sign. Thus, the polarization effects tend to be small.[33] For this particular model, tensor scattering is predominant only in the neighborhood of 45°. The singlet scattering gives no polarization so that the asymmetry falls off very rapidly with angle on either side of the maximum.[34]

In the case of the singular tensor-force model, the Born approximation phase shifts are unreliable. If one uses the exact values for δ_1^{00} and δ_1^{11}, and the $J=2$ phase shifts obtained by the variational treatment (which do not differ greatly from Born approximation phase shifts), the asymmetry is, indeed, small. Thus, at 240 Mev, $2\delta_{\max}$ is calculated to be ~ 0.02 for $r_c = 0.84\hbar/Mc$. On performing an exact calculation, however, the coupled P-phase shifts are changed considerably. This enhances the difference between δ_1^{21} and δ_3^{21} and, in addition, increases the importance of terms in Eq. (23) involving δ_1^{20}. The result is an increase in the asymmetry for a double p-p scattering by a factor of 10.[35]

The largest polarizations of all are predicted by the $\mathbf{L}\cdot\mathbf{S}$ force model. In this case, the main contribution to α comes from

$$3[\delta_3^2\delta_1^0] - 3[\delta_1^2\delta_1^0] + 4.5[\delta_1^1\delta_1^2].$$

We note first that δ_1^2 and δ_3^2 are now of opposite sign, since, for $J=2$, the eigenvalues of $\mathbf{L}\cdot\mathbf{S}$ are 1 and -4 for $L=1$ and 3, respectively. Furthermore, the P-phases are large in comparison with the higher angular momentum phase shifts. The net result is that one obtains very large asymmetries. However, it must be borne in mind that these calculations are not exact and that, in any case, the agreement with experiment of the unpolarized cross sections is poor, so that these results should not be taken too seriously.

V. CONCLUSIONS

It appears clear that the use of the Born approximation to provide estimates of scattering for singular potentials is unjustified, even at high energies. Thus we have found that the result of more exact calculations, when performed for the three potentials which

[32] We should like to remark that $P(k,\theta,\phi=0°)$ turns out to be always positive for all the potentials considered for $0<\theta<\pi/2$ except for the case of the $\mathbf{L}\cdot\mathbf{S}$ force model with $V_t = -29.8$ Mev.

[33] This is not unexpected since the Born approximation is reliable in this case for the triplet phase shifts and it is known that polarization effects vanish identically in Born approximation. (See reference 29.)

[34] For the other choice of sign of the odd-parity force, $V_t = 50.8$ Mev (see reference 26), the asymmetry in a double scattering is of the order of several tenths of a percent. For this case, $P(k,\theta,\phi=0°)<0$ for $\theta\sim 45°$.

[35] We should like to emphasize that the results for the singular tensor-force model previously reported [L. J. B. Goldfarb and D. Feldman, Phys. Rev. 87, 208 (1952)] were based on variational calculations of the $J=2$ phase shifts and accordingly the polarization effect was considerably underestimated. We are indebted to Don R. Swanson for pointing out to us that large polarization effects actually do obtain with this particular model.

have been proposed to describe the p-p interaction, has been to modify strongly the previous predictions for those potentials which are singular. Indeed, the use of Born approximation trial functions in the variational procedure is, in itself, inadequate for these cases.

The singular tensor-force model of Christian and Noyes, which was treated exactly, now presents only moderate agreement with experiment for the choice of sign $V_t=+18$ Mev, but only for angles greater than 45°. The predominance of singlet scattering at lower angles introduces too much anisotropy, so that deviations from experiment become considerable. The large corrections, introduced by the variational treatment of the $\mathbf{L}\cdot\mathbf{S}$ force model of Case and Pais, lead to an even less encouraging picture. While it is conceivable that more exact calculations would remove the large anisotropies, there is no *a priori* reason to expect this. On the other hand, the predictions of Jastrow's hard-core model are essentially unchanged on performing a variational calculation for the triplet scattering (which is reliable for this case). The cross section is reasonably flat, although its magnitude is somewhat low.

It must be emphasized that the preceding remarks in regard to the singular potentials are based on calculations performed with a zero cutoff at distances of the order of the nucleon Compton wavelength. It is always possible that a better fit with experiment might be obtained with a different choice of cutoff. In any case, the exact nature of the cutoff must be taken seriously.

The asymmetry in a double p-p scattering experiment was calculated in order to provide an additional means for distinguishing among the potential models. On this basis, the hard-core model which is found to yield small asymmetries is quite different from the singular tensor-force potential which predicts large asymmetries. The $\mathbf{L}\cdot\mathbf{S}$ potential also predicts large asymmetries, but it must be emphasized that this model, particularly, gives a poor fit with experiment. Finally, we should like to reiterate that the foregoing analysis is completely nonrelativistic, and it is quite possible that polarization effects, in particular, would be modified in a more consistent relativistic theory.

We should like to thank Professor R. E. Marshak for useful discussions.

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An Analysis of the Energy Levels of the Mirror Nuclei, C^{13} and N^{13}

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An analysis employing the recent nuclear reaction theories of Wigner and others is given of the experimental data on the low energy interactions of s, p, d orbital neutrons and protons with C^{12} and s neutrons and protons with O^{16}. Assuming the equality of nn and pp nuclear interactions, it is possible to account for the data on the s interactions if the level spacing is considered in addition to the customary two resonance parameters: reduced width and level position; in particular, the displacement of conjugate levels can be attributed to the difference of the external wave functions for the odd particle, although with an uncertainty of about 25 percent which is due primarily to the lack of precise knowledge of the internal Coulomb energy of the excited states. The large magnitudes of the reduced width and level spacing indicate that two-body potential interactions exist between the odd particle and the C^{12} and O^{16} cores, and the values of the respective logarithmic derivatives indicate that these interactions are of about equal strengths. The energy dependence of the radiative capture cross section of s neutrons and protons with C^{12} can be understood if an additional quantity, the final-state reduced width, is included in the theory to take into account the energy-dependent external contribution to the transition moment. The experimental data are only sufficient to treat the p and d interactions in the one-level approximation; a reasonable explanation can be given of the observed displacements of conjugate levels in terms of the differences of the electromagnetic properties of the odd particle such as: external wave functions, spin-orbit interactions, and variations of the internal Coulomb energy. There is some indication from the data on radiative transitions that the independent-particle model also prevails in the p states; on the other hand, the small reduced widths of these states suggest a many-body description. Derivations based on the recent theories are given of the one-channel formulas and of the general one-level formulas which include the negative-energy alternatives. The radial dependences of the resonance parameters are discussed.

I. INTRODUCTION

SINCE there is considerable experimental material on the low levels of the mirror nuclei, N^{13} and C^{13}, it seems worth while to attempt a detailed investigation of such matters as the extent of the validity of the independent-particle model, the assumption of equality of nn and pp nuclear forces, and the applicability of the recent theories of nuclear reactions. The analysis is carried out by means of the theories due to Wigner and others,[1–9] and we are therefore concerned with the de-

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