variations of winds and temperatures in the 30–60 km region are evident.

The Alaskan seasonal effect observed by Crary agrees in character with the seasonal effects indicated by meteors. The latitude effects from these propagational studies, if not reversed by temperature changes at higher altitudes than 60 km, appear to confirm qualitatively the meteor results with regard to latitude dependence. It is clear, however, that more data are required to establish definitively the dependences of temperature, pressure, and density in the upper atmosphere with season and with latitude. Possibly some of the observed variations are really more influenced by geographical position with respect to continental masses and atmospheric circulation patterns than with respect to latitude.

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Scattering of Charge Symmetric Pseudoscalar Mesons by Nucleons

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The Pauli theory of extended source interacting with the neutral pseudoscalar field with pseudovector coupling has been extended to the case of the charge symmetric pseudoscalar field, and has been applied to the problem of the scattering of mesons by the nucleons. The results of our calculation show that the scattering cross sections are strongly dependent upon the energy of the incoming mesons, that the cross section for the scattering of π^+ by protons is larger than that for π^- by factor 2.25 at all kinetic energies up to about 150 Mev, and that within the scattering of π^- by protons the cross section for the scattering involving charge exchange interaction is larger than that with ordinary interaction by a factor which varies from 2.1 to 1.5 as one goes from zero kinetic energy to 150 Mev. When the coupling constant $(f\mu)^2$ is taken to be about 0.3, the maximum of the theoretical cross section curve coincides with the experimental maximum if one assigns 1.25μ (μ = the reciprocal Compton wavelength of the meson) for the spin and isotopic spin inertia of the nucleon. The scattering cross-section curves for π^+ and π^- approximately agree with the experimental curves if one assigns a value 1.58 for $|G(K)|$, which appears in the original theory of Pauli.

I. INTRODUCTION

The experimental investigation of the scattering of π^- and π^+ mesons by hydrogen[1] revealed the following three important facts:

a. The total scattering cross sections for both mesons are strongly dependent upon the energy of the mesons. For π^- the total scattering cross section rises rapidly from 20×10^{-27} cm^2 at 60 Mev to a plateau of 60×10^{-27} cm^2, which begins at about 150 Mev kinetic energy.

b. The total scattering cross section of π^+ is approximately three times as large as that of π^- in the range of energies of the experiment.

c. Within the scattering of π^- the total scattering cross section involving charge exchange interaction is approximately twice as large as that arising from ordinary (no charge exchange) interaction.

The quantum theoretical investigations to account for these experimental facts have already been given by Brueckner[2] and Wentzel[3] in their respective papers. These authors have proceeded along the line of strong coupling theory, which provides the possibility of isobaric states of the spin and the isotopic spin of nucleon leading to large scattering cross sections due to the resonance. It was pointed out by Brueckner that the isobaric state characterized by the isotopic spin $\frac{3}{2}$ of the proton would make the resonance scattering of π^+ approximately three times as large as that of π^- in hydrogen, when both ordinary and charge exchange interactions are simultaneously considered for the latter. This point was amply confirmed by the experiment.

When the interaction between the meson and the nucleon is strong, the existence of a dense meson cloud in the close vicinity of the nucleon makes the quantum theoretical investigation very difficult, and no rigorous methods have yet been developed to deal with this problem. Under such circumstances it was thought to be of some interest to restrict ourselves to the classical approximations and investigate the possibility of accounting for experimental facts along the line of the classical theory developed by Pauli.[4] This approach has already been employed by Brueckner and Case in their paper on the production of neutral photomesons.[5]

II. CHARGE SYMMETRIC PSEUDOSCALAR FIELD

We shall employ in our calculation of meson scattering the charge symmetric pseudoscalar field with pseudovector coupling. Then, the procedure is only a slight

[1] Nagle, Anderson, Fermi, Long, and Martin, Phys. Rev. **86**, 603 (1952); **85**, 934, 935, 936 (1952).
[2] K. A. Brueckner, Phys. Rev. **86**, 106 (1952).
[3] G. Wentzel, Phys. Rev. **86**, 437 (1952).
[4] W. Pauli, *Meson Theory of Nuclear Forces* (Insterscience Publishers, Inc., New York, 1946).
[5] K. A. Brueckner and K. M. Case, Phys. Rev. **83**, 1141 (1951).

generalization of the Chapters II and III of Pauli's book,[4] in which the neutral field alone is considered.

In the theory of extended source the total Hamiltonian of the meson field interacting with a nucleon may be written

$$H=\frac{1}{2}\int\{\pi_\alpha^2+(\nabla\varphi_\alpha)^2+\mu^2\varphi_\alpha^2\}d^3x+(4\pi)^{\frac{1}{2}}f\int u(\mathbf{x})(\boldsymbol{\sigma}\cdot\nabla)\tau_\alpha\varphi_\alpha(\mathbf{x})d^3x. \quad (1)$$

$U(x)$ is a real function representing the spatial extension of the nucleon, which must satisfy the condition that the spatial integral is unity, the value that it would have if the nucleon were considered a point particle. μ and f are reciprocal Compton wavelength and the coupling constant of the meson, respectively. We use units $\hbar=c=1$. $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the spin and the isotopic spin vectors of the nucleon, considered in our classical theory as unit vectors in coordinate and charge space, respectively. α runs from 1 to 3, corresponding to the charged and neutral states of the meson.

Following Pauli's methods closely[6] we obtain from Eq. (1)

$$H=\frac{1}{2}\int d^3k\{G(\mathbf{k})\mathfrak{p}_\alpha(\mathbf{k})\mathfrak{p}_\alpha(-\mathbf{k})+(k_0^2/G(\mathbf{k}))\mathfrak{q}_\alpha(\mathbf{k})\mathfrak{q}_\alpha(-\mathbf{k})\}+if(\pi\sqrt{2})^{-1}\int d^3k(\boldsymbol{\sigma}\cdot\mathbf{k})\tau_\alpha\mathfrak{q}_\alpha(\mathbf{k}), \quad (2)$$

where $k_0^2=k^2+\mu^2$ and $G(\mathbf{k})=V(\mathbf{k})V(-\mathbf{k})$. $V(\mathbf{k})$ is the Fourier transform of $U(x)$ in Eq. (1). From this Hamiltonian we obtain the following equation of motion of the meson field in momentum space:

$$d^2\mathfrak{q}_\alpha(\mathbf{k})/dt^2+k_0^2\mathfrak{q}_\alpha(\mathbf{k})=if(\pi\sqrt{2})^{-1}G(\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{k})\tau_\alpha. \quad (3)$$

Assuming $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ obey the classical Poisson bracket equations (l, m, n cyclic),

$$[\sigma_l,\sigma_m]=-2\sigma_n,\quad [\tau_l,\tau_m]=-2\tau_n,$$

we obtain the following equations of motion for $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ vectors:

$$d\boldsymbol{\sigma}/dt=[H,\boldsymbol{\sigma}]=-i\sqrt{2}f\pi^{-1}\left[\boldsymbol{\sigma}\times\int d^3k\mathbf{k}\mathfrak{q}_\alpha(\mathbf{k})\tau_\alpha\right],$$
$$d\boldsymbol{\sigma}/dt=[H,\boldsymbol{\tau}]=-i\sqrt{2}f\pi^{-1}\left[\boldsymbol{\tau}\times\int d^3k\mathfrak{q}(\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{k})\right]. \quad (4)$$

Expanding $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$, and $\mathfrak{q}_\alpha(\mathbf{k})$ into Fourier series individually, we have

$$\boldsymbol{\sigma}=\sum_\nu\boldsymbol{\sigma}_\nu e^{-i\nu t},\quad \boldsymbol{\tau}=\sum_{\nu'}\boldsymbol{\tau}_{\nu'}e^{-i\nu' t},\quad \mathfrak{q}_\alpha(\mathbf{k})=\sum_{\nu''}\mathfrak{q}_{\alpha,\nu''}(\mathbf{k})e^{-i\nu'' t}, \quad (5)$$

[6] We follow Pauli's notation (see reference 4), except that we use $\mathfrak{p}_\alpha$ and $\mathfrak{q}_\alpha$ in place of his $\tilde{p}_\alpha$ and $\tilde{q}_\alpha$.

where frequencies ν, ν', and ν'' take on values O, ω, 2ω, $\cdots$. Substituting Eq. (5) into Eq. (3), one obtains

$$\mathfrak{q}_{\alpha,\nu''}(\mathbf{k})=if(\pi\sqrt{2})^{-1}G(\mathbf{k})(k_0^2-\nu''^2)(\boldsymbol{\sigma}_\nu\cdot\mathbf{k})\tau_{\alpha,\nu'}, \quad (6)$$

where $\nu''=\nu+\nu'$. Rewriting Eq. (4) in the form

$$d\boldsymbol{\sigma}/dt=[\boldsymbol{\sigma}\times\mathbf{F}_\sigma]\quad d\boldsymbol{\tau}/dt=[\boldsymbol{\tau}\times\mathbf{F}_\tau] \quad (7)$$

and following Pauli's procedure closely, one obtains

$$\mathbf{F}_\sigma=\tfrac{2}{3}f^2\sum_{\nu',\nu''}e^{-i(\nu'+\nu'')t}(\boldsymbol{\tau}_{\nu'}\cdot\boldsymbol{\tau}_{\nu'})\boldsymbol{\sigma}_\nu[N-a^{-1}(\mu^2-\omega^2)+\epsilon(\mu^2-\omega^2)^{\frac{3}{2}}],$$
$$\mathbf{F}_\tau=\tfrac{2}{3}f^2\sum_{\nu,\nu''}e^{-i(\nu+\nu'')t}(\boldsymbol{\sigma}_\nu\cdot\boldsymbol{\sigma}_\nu)\boldsymbol{\tau}_{\nu'}[N-a^{-1}(\mu^2-\omega^2)+\epsilon(\mu^2-\omega^2)^{\frac{3}{2}}], \quad (8)$$

where

$$\epsilon=\begin{cases}+1 & \text{for } \nu''^2<\mu^2\\ +1 & \text{for } \nu''^2>\mu^2,\ \nu''>0\\ -1 & \text{for } \nu''^2>\mu^2,\ \nu''<0.\end{cases}$$

N and a^{-1} are new mechanical constants introduced by Pauli, of which the first plays no role in the dynamics of the meson field, while the second is identified as the spin and isotopic spin inertia of the nucleon.

In the Fourier expansions, Eqs. (5), we take at present only terms with O and $\pm\omega$ frequencies, and disregard terms with higher harmonics. Since the higher harmonics contribute higher order effects in the scattering of a meson by a nucleon, we assume that their contributions are small compared with those arising from the zero and the fundamental frequency ω. Then, keeping in mind $\nu''=\nu+\nu'$, one obtains from Eq. (8)

$$\mathbf{F}_\sigma=\tfrac{2}{3}f^2[\boldsymbol{\sigma}_\omega e^{-i\omega t}A_+ +\boldsymbol{\sigma}_{-\omega}e^{i\omega t}A_-],$$
$$\mathbf{F}_\tau=\tfrac{2}{3}f^2[\boldsymbol{\tau}_\omega e^{-i\omega t}A_+ +\boldsymbol{\tau}_{-\omega}e^{i\omega t}A_-], \quad (9)$$

where $A_\pm=\omega^2a^{-1}-\mu^3\pm i(\omega^2-\mu^2)^{\frac{3}{2}}$. For scattering processes the condition $\omega^2>\mu^2$ is required.

III. CALCULATION OF THE SCATTERING OF MESONS BY NUCLEONS

The incident meson field may be written

$$\varphi_\alpha^I(\mathbf{x})=\chi p_{I\alpha}\exp[iK(\mathbf{n}_I\cdot\mathbf{x})-i\omega t]+\text{c.c.},$$

where $\mathbf{n}_I$ is a unit vector in the direction of the incoming field, χ the amplitude, and $\mathbf{p}_I$ a unit vector in charge space representing the charge state of the incoming meson. Here, $K^2=\omega^2-\mu^2$, where ω is the energy of the incoming meson. Introducing the Fourier transform of φ_α^I into $\mathbf{F}_\sigma$ and $\mathbf{F}_\tau$ [see Eq. (4) and Eq. (7)], we obtain for the incoming field

$$\mathbf{F}_\sigma^I=-if4\pi^{\frac{1}{2}}V(K)\mathbf{n}_I(\boldsymbol{\tau}_0\cdot\mathbf{p}_I)(\chi e^{-i\omega t}-\text{c.c.}),$$
$$\mathbf{F}_\tau^I=-if4\pi^{\frac{1}{2}}V(K)\mathbf{p}_I(\boldsymbol{\sigma}_0\cdot\mathbf{n}_I)(\chi e^{-i\omega t}-\text{c.c.}). \quad (10)$$

We now introduce Eqs. (5), (9), and (10) into Eq. (7) and retain terms linear in χ, $\sigma_{\pm\omega}$, and $\tau_{\pm\omega}$ only under the assumption that the motions of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ caused by the incoming field are small in comparison to their stationary values. We then obtain

$$\frac{d}{dt}(\boldsymbol{\sigma}_\omega e^{-i\omega t})+\frac{d}{dt}(\boldsymbol{\sigma}_{-\omega}e^{i\omega t})+\frac{d}{dt}(\boldsymbol{\sigma}_0)\cong[\boldsymbol{\sigma}_0\times\mathbf{F}_\sigma{}^I]$$
$$+\tfrac{2}{3}f^2A_+e^{-i\omega t}[\boldsymbol{\sigma}_0\times\boldsymbol{\sigma}_\omega]+\tfrac{2}{3}f^2A_-e^{i\omega t}[\boldsymbol{\sigma}_0\times\boldsymbol{\sigma}_{-\omega}].$$

Proceeding similarly for the analogous equation for $\boldsymbol{\tau}$, we obtain

$$\begin{aligned}\boldsymbol{\sigma}_\omega&=4\pi^{\frac{1}{2}}fV(K)K\omega^{-1}\chi(\boldsymbol{\tau}_0\cdot\mathbf{p}_I)[\boldsymbol{\sigma}_0\times\mathbf{n}_I]+i\tfrac{2}{3}f^2A_+\omega^{-1}[\boldsymbol{\sigma}_0\times\boldsymbol{\sigma}_\omega],\\ \boldsymbol{\tau}_\omega&=4\pi^{\frac{1}{2}}fV(K)K\omega^{-1}\chi(\boldsymbol{\sigma}_0\cdot\mathbf{n}_I)[\boldsymbol{\tau}_0\times\mathbf{p}_I]+i\tfrac{2}{3}f^2A_+\omega^{-1}[\boldsymbol{\tau}_0\times\boldsymbol{\tau}_\omega].\end{aligned}\tag{11}$$

By solving these equations for $\boldsymbol{\sigma}_\omega$ and $\boldsymbol{\tau}_\omega$ one obtains

$$\begin{aligned}\boldsymbol{\sigma}_\omega&=\beta(1-R^2)^{-1}\{[\boldsymbol{\sigma}_0\times\mathbf{n}_I]+iR[\boldsymbol{\sigma}_0\times(\boldsymbol{\sigma}_0\times\mathbf{n}_I)],\\ \boldsymbol{\tau}_\omega&=\beta'(1-R^2)^{-1}\{[\boldsymbol{\tau}_0\times\mathbf{p}_I]+iR[\boldsymbol{\tau}_0\times(\boldsymbol{\tau}_0\times\mathbf{p}_I)],\end{aligned}\tag{12}$$

where

$$\beta=4\pi^{\frac{1}{2}}fV(K)K\omega^{-1}\chi(\boldsymbol{\tau}_0\cdot\mathbf{p}_I),$$
$$\beta'=4\pi^{\frac{1}{2}}fV(K)K\omega^{-1}\chi(\boldsymbol{\sigma}_0\cdot\mathbf{n}_I),$$
$$R=\tfrac{2}{3}f^2A_+\omega^{-1}.$$

The scattered field can be obtained by following the procedure described in the Appendix in Pauli's book:

$$\varphi_\alpha{}^s(\mathbf{x})=i(2\sqrt{\pi})^{-1}fV(K)K\frac{e^{iKr}}{r}\sum_{\nu'\nu}\tau_{\alpha,\nu'}(\boldsymbol{\sigma}_\nu\cdot\mathbf{n}_s)+\text{c.c.}$$

The complex amplitude of the field with the positive frequency ω scattered into the directions of $\mathbf{n}_s$ and $\mathbf{p}_s$ in the coordinate and charge space, respectively, is then given by

$$(\chi_s\cdot\mathbf{p}_s)=i(2\sqrt{\pi})^{-1}f\,V(K)K\{(\boldsymbol{\tau}_0\cdot\mathbf{p}_s)(\boldsymbol{\sigma}_\omega\cdot\mathbf{n}_s)+(\boldsymbol{\tau}_\omega\cdot\mathbf{p}_s)(\boldsymbol{\sigma}_0\cdot\mathbf{n}_s)\}.\tag{13}$$

Introducing Eqs. (12) into (13), we obtain finally

$$\begin{aligned}(\chi_s\cdot\mathbf{p}_s)=D\chi[&(\boldsymbol{\tau}_0\cdot\mathbf{p}_s)(\boldsymbol{\tau}_0\cdot\mathbf{p}_I)\{([\boldsymbol{\sigma}_0\times\mathbf{n}_I]\cdot\mathbf{n}_s)\\&+i(\xi+i\zeta)([\boldsymbol{\sigma}_0\times[\boldsymbol{\sigma}_0\times\mathbf{n}_I]]\cdot\mathbf{n}_s)\}\\&+(\boldsymbol{\sigma}_0\cdot\mathbf{n}_s)(\boldsymbol{\sigma}_0\cdot\mathbf{n}_I)\{([\boldsymbol{\tau}_0\times\mathbf{p}_I]\cdot\mathbf{p}_s)\\&+i(\xi+i\zeta)([\boldsymbol{\tau}_0\times[\boldsymbol{\tau}_0\times\mathbf{p}_I]]\cdot\mathbf{p}_s)\}],\end{aligned}\tag{14}$$

where

$$\xi=\tfrac{2}{3}(f\mu)^2\{(a\mu)^{-1}\omega^2\mu^{-2}-1\}\mu\omega^{-1},$$
$$\zeta=\tfrac{2}{3}(f\mu)^2(\omega^2\mu^{-2}-1)^{\frac{3}{2}}\mu\omega^{-1},$$
$$D=2if^2K^2\omega^{-1}[V(K)]^2[1-(\xi+i\zeta)^2]^{-1}.$$

The square of the absolute value of Eq. (14) corresponds to the intensity of the scattered field along the unit vectors $\mathbf{n}_s$ and $\mathbf{p}_s$ due to a particular set of orientations of the vectors $\boldsymbol{\sigma}_0$ and $\boldsymbol{\tau}_0$ in the coordinate and charge space, respectively.

IV. SCATTERING CROSS SECTIONS

If the kinetic energy of the incoming meson is not too large, so that for each incoming meson only one meson is created in the scattering, there exists the following ten different reactions between the mesons and the nucleons:

$$\begin{aligned}&(1)\ \pi^++p\to p+\pi^+\\&(2)\ \pi^++n\to n+\pi^+\\&(3)\ \pi^++n\to p+\pi^0\\&(4)\ \pi^-+n\to n+\pi^-\\&(5)\ \pi^-+p\to p+\pi^-\\&(6)\ \pi^-+p\to n+\pi^0\\&(7)\ \pi^0+n\to n+\pi^0\\&(8)\ \pi^0+n\to p+\pi^-\\&(9)\ \pi^0+p\to p+\pi^0\\&(10)\ \pi^0+p\to n+\pi^+.\end{aligned}\tag{15}$$

Of these the reactions (1) and (4) are unique in the sense that they have no alternative, whereas each of the remainder is accompanied by an alternative reaction. It is considered that for the reactions (1) and (4) the charge vector $\boldsymbol{\tau}$ of the nucleon does not play any role at all, and hence the scattering cross section for these can be considered to be the same as the one originally given by Pauli for the neutral pseudoscalar meson. For these reactions we must in our theory first put $\mathbf{p}_I=\mathbf{p}_s$ in Eq. (14), since the charge states of the incoming and out-going particle are the same. Moreover, since $\boldsymbol{\tau}$ plays no role, we must consider it fixed along $\mathbf{p}_I$ or $\mathbf{p}_s$. Thus, we obtain from (14)

$$(\chi_s\cdot\mathbf{p}_s)=D\chi[([\boldsymbol{\sigma}_0\times\mathbf{n}_I]\cdot\mathbf{n}_s)+i(\xi+i\zeta)([\boldsymbol{\sigma}_0\times[\boldsymbol{\sigma}_0\times\mathbf{n}_I]]\cdot\mathbf{n}_s)].\tag{16}$$

This agrees with the corresponding expression given by Pauli (see p. 29 of his book). The differential scattering cross section is obtained by averaging $|(\chi_s\cdot\mathbf{p}_s)|^2/|\chi|^2$ over all directions of $\boldsymbol{\sigma}_0$:

$$dQ_a=|D|^2[\tfrac{1}{3}\sin^2\theta+(\xi^2+\zeta^2)(1+7\cos^2\theta)/15]d\Omega.\tag{17}$$

Here θ is the angle between $\mathbf{n}_s$ and $\mathbf{n}_I$ and $d\Omega$ the differential solid angle. The total scattering cross section is obtained by integrating Eq. (17) over all directions of $\mathbf{n}_s$ in the coordinate space:

$$Q_a=(8\pi/9)|D|^2(1+\xi^2+\zeta^2),\tag{18}$$

where one obtains from Eq. (14)

$$|D|^1=4f^4K^4\omega^{-2}|G(K)|^2[(1-\xi^2-\zeta^2)^2+4\zeta^2]^{-1}.\tag{19}$$

Q_a given by Eq. (18) agrees with that by Pauli.

For the rest of the reactions in Eqs. (15) in which ordinary as well as charge exchange interactions are involved, the square of the absolute value of Eq. (14)

must be properly averaged over all directions of $\boldsymbol{\tau}_0$ and $\boldsymbol{\sigma}_0$ vectors. We then obtain

$$|(\chi_s\cdot \mathbf{p}_s)|^2=|D|^2|\chi|^2[\{(1+2\cos^2\delta)/15\} \times\{\tfrac{1}{3}\sin^2\theta+(\xi^2+\zeta^2)(1+7\cos^2\theta)/15\} +\{(1+2\cos^2\theta)/15\}\{\tfrac{1}{3}\sin^2\delta+(\xi^2+\zeta^2) \times(1+7\cos^2\delta)/15\}+(2/225)(\xi^2+\zeta^2) \times(1-3\cos^2\delta)(1-3\cos^2\theta)], \quad (20)$$

where θ and δ are the angles between $\mathbf{n}_I$ and $\mathbf{n}_s$ and $\mathbf{p}_I$ and $\mathbf{p}_s$, respectively. Let us first consider the reactions (2), (5), (7), and (9) of Eqs. (15) which involve no charge exchange. The vector amplitude of the scattered waves along the charge vector $\mathbf{p}_s$ of the field will be given by $(\chi_s\cdot \mathbf{p}_s)\mathbf{p}_s$, and its projection along the vector $\mathbf{p}_I$ is then given by $(\chi_s\cdot \mathbf{p}_s)(\mathbf{p}_s\cdot \mathbf{p}_I)$. In order to obtain the intensity of the scattered meson waves along the vector $\mathbf{p}_I$ we now average $|(\chi_s\cdot \mathbf{p}_s)|^2(\mathbf{p}_s\cdot \mathbf{p}_I)^2$ over all directions of $\mathbf{p}_s$. The result obtained must further be multiplied by two, because the charge state of the incoming meson designated by the negative of the vector $\mathbf{p}_I$ is the same as that by $\mathbf{p}_I$ itself. For reactions involving no charge exchange we thus obtain from Eq. (20) the following expression for the differential scattering cross section:

$$dQ_b=2\langle|(\chi_s\cdot \mathbf{p}_s)|^2(\mathbf{p}_I\cdot \mathbf{p}_s)^2\rangle_{\text{Av}}/|\chi|^2d\Omega =(2/225)|D|^2[\{(13-7\cos^2\theta)/3\} +(\xi^2+\zeta^2)\{(29+153\cos^2\theta)/15\}]d\Omega. \quad (21)$$

The corresponding total scattering cross section turns out to be

$$Q_b=(8\pi/9)|D|^2[(32/225)+(\xi^2+\zeta^2)(48/225)]. \quad (22)$$

Let us now consider the reactions (3), (6), (8), and (10) in Eq. (15) which involve charge exchange. The projection of the vector amplitude $(\chi_s\cdot \mathbf{p}_s)\mathbf{p}_s$ on the plane perpendicular to the vector $\mathbf{p}_I$ will be given by $(\chi_s\cdot \mathbf{p}_s)[\mathbf{p}_s\times \mathbf{p}_I]$. The average of the square of the absolute value of this over all the directions of $\mathbf{p}_s$, multiplied by two, is the intensity of the scattered field. Then the differential scattering cross section is found to be

$$dQ_c=2\langle|(\chi_s\cdot \mathbf{p}_s)|^2|[\mathbf{p}_s\times \mathbf{p}_I]|^2\rangle_{\text{Av}}/|\chi|^2d\Omega =(2/225)|D|^2[\tfrac{2}{3}(11+\cos^2\theta) +(\xi^2+\zeta^2)\{(46+122\cos^2\theta)/15\}]d\Omega. \quad (23)$$

The corresponding total scattering cross section becomes

$$Q_c=(8\pi/9)|D|^2[(68/225)+(\xi^2+\zeta^2)(52/225)]. \quad (24)$$

When the ordinary interaction and the charge exchange interaction are not distinguished from one another [reactions (2) and (3); (5) and (6); (7) and (8); and (9) and (10)], the resultant differential scattering cross section for each one of these sets would be

$$dQ=dQ_b+dQ_c=(2/45)|D|^2[\tfrac{1}{3}(7-\cos^2\theta) +(\xi^2+\zeta^2)(1+\tfrac{1}{3}\cos^2\theta)]. \quad (25)$$

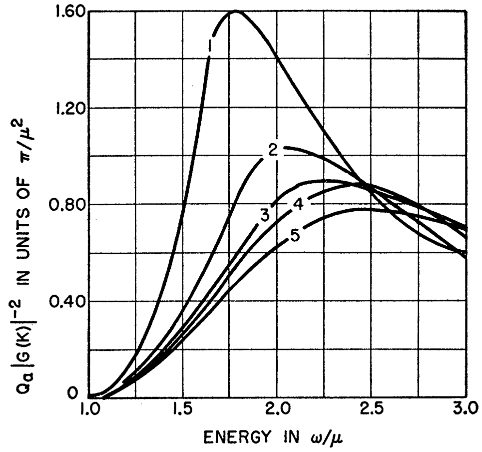

FIG. 1. $Q_a|G(K)|^{-2}$ in units of $\pi\mu^{-2}$ *versus* energy.

The last term in Eq. (20) proportional to $(l-3\cos^2\delta)$ $(l-3\cos^2\theta)$ vanishes in Eq. (25), although it stays in the individual differential cross sections Eqs. (21) and (23). The sum of Q_b and Q_c becomes

$$Q=(8\pi/9)|D|^2(1+\xi^2+\zeta^2)(4/9). \quad (26)$$

V. DISCUSSION

According to the experimental results[1] on the scattering of π^+ and π^- by protons the total scattering cross section of π^+ is larger than that of π^- for the range of energy of the incoming mesons from about 50 Mev to 150 Mev. From Eqs. (18) and (26) we find that the ratio of the scattering cross section of π^+ to that of π^- to be 2.25 at all kinetic energies of the incoming mesons up to about 150 Mev ($\omega\cong 2\mu$). This ratio does not appear to be in disagreement with the experimental ratio in this energy range. However, beyond this range of energy the effects of higher harmonics mentioned at the end of Sec. II are expected to begin to show up, causing deviations from our results. Although the effects of the higher harmonics have not yet been investigated, the experimental observation that at 180 Mev the ratio of the scattering cross section for π^+ over that for π^- is somewhat larger than the ratio at lower energies may perhaps be ascribable to these effects.

From Eq. (18) and Eq. (19) one obtains

$$Q_a=(32/9)(f\mu)^4\mu^2\omega^{-2}(\omega^2\mu^{-2}-1)^2(1+\xi^2+\zeta^2) \times[(1-\xi^2-\zeta^2)^2+4\zeta^2]^{-1}(\pi\mu^{-2})|G(K)|^2. \quad (27)$$

Here, $\pi\mu^{-2}$ is the geometrical area of the nuclear force range, and is equal to 62×10^{-27} cm^2 for π-mesons. Figure 1 shows the curves of $Q_a|G(K)|^{-2}$ in units of $\pi\mu^{-2}$ at various energies of the incoming mesons for different values of the spin inertia a^{-1}; namely, the curve (1) for $a\mu=0.40$, (2) for 0.60, (3) for 0.80, (4) for 1.0, and (5) for 2.0. For the coupling constant we take for convenience the value $(f\mu)^2=0.318$, or $(f\mu)^4=0.100$. Assuming that the factor $|G(K)|$ does not vary appreciably within the range of energy considered, it appears that there is a

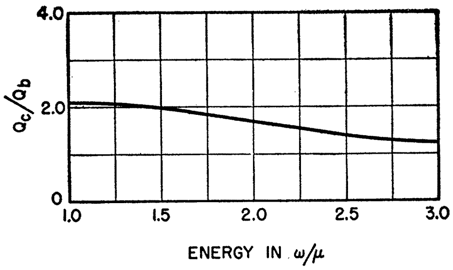

FIG. 2. Q_c/Q *versus* energy ($a\mu=0.8$).

general tendency that the maximum point lowers and shifts to the right in the direction of increasing energy as the values of the parameter $a\mu$ are increased. If one takes $a\mu=0.8$, the theoretical maximum point of the scattering cross-section curve occurs at about 160 Mev ($\omega/\mu=2.17$), which is also the approximate experimental maximum. Hence, we may tentatively take $a\mu=0.8$ or $a^{-1}=1.25\mu$ as the spin and isotopic spin inertia of the nucleon.

The experimental observations also indicate that in the case of the scattering of π^- by protons the total cross section involving the charge exchange interaction is about twice as large as the one involving no charge exchange interaction. From Eqs. (22) and (24) we obtain for this ratio

$$Q_c/Q_b=[17+13(\xi^2+\zeta^2)]/[8+12(\xi^2+\zeta^2)].$$

Figure 2 shows the variation of Q_c/Q_b with the energy of the incoming mesons. Since $(\xi^2+\zeta^2)$ increases with the energy moderately fast, it appears that the two cross sections tend to equalize when the energy of the mesons is sufficiently high.

The parameter $|G(K)|$ is now determined from the experimental observation that the maximum value of the total scattering cross section for π^- equals the geometrical area of the nuclear force range, namely, $\pi\mu^{-2}=62\times10^{-27}$ cm². It turns out from Eqs. (19) and (26) that the maximum of the calculated cross section can be made equal to the above experimental value if one takes $|G(K)|=1.58$, which value may be theoretically not unreasonable. It is easily seen that $|G(K)|=1$, if the nucleon were considered a point particle. In the theory of extended source all that is required is that $|G(K)|\approx1$. Figure 3 gives the curves of the total scattering cross section of π^+ and π^- by protons obtained from Eqs. (18) and (26), respectively, with $|G(K)|=1.58$.

In this manner it appears that the classical theory of meson dynamics involving the Pauli theory of extended source for the nucleon is capable of approximately accounting for the principal experimental facts in the scattering of π^+ and π^- by the protons mentioned in Sec. I.

Another experimental fact[7] that the scattering cross sections of π^+ and π^- for deuterium at about 60 Mev are approximately equal to each other may approximately be discussed from Eqs. (15), (18), and (26); namely, the cross sections for $\pi^++p\rightarrow p+\pi^+$ and $\pi^-+n\rightarrow n+\pi^-$ are given by Q_a, those for $\pi^++n\rightarrow n+\pi^+$ and $\pi^-+p\rightarrow p+\pi^-$ by Q_b, and those for $\pi^++n\rightarrow p+\pi^0$ and $\pi^-+p\rightarrow n+\pi^0$ by Q_c.

As to the resonance in the scattering discussed by Brueckner[2] and Wentzel,[3] we may give its analog in our classical theory in the following manner. Fixing our attention on the denominator of Eq. (27); namely, $\omega^2[(1-\xi^2-\zeta^2)^2+4\zeta^2]$, it is seen that ζ can be regarded as expressing the effects of radiation reaction of the meson field. This is due to the fact that it reduces to ω^2, the radiation reaction term for a Maxwell field, when μ is allowed to vanish (see Bhabha[8]). In the absence of this term the scattering cross section Q_a of Eq. (27) becomes infinite when $\xi=1$, or at the energy of the meson ω given by the equation [see Eq. (14)]

$$\tfrac{2}{3}(f\mu)^2\{(a\mu)^{-1}\omega\mu^{-1}-1\}\mu\omega^{-1}=1.$$

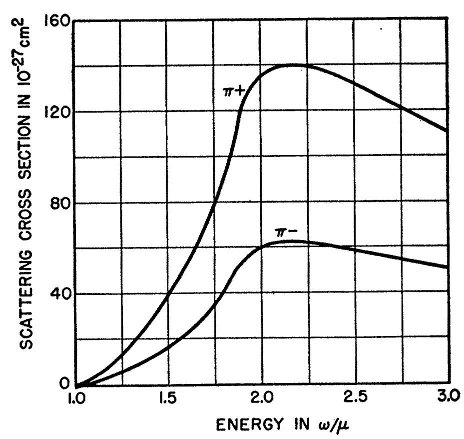

FIG. 3. Scattering for cross-section curves for π^+ and π^- for $a\mu=0.8$, $(f\mu)^4=0.10$ and $|G(K)|=1.58$.

The effect of the radiation reaction term ζ is to make the value of the scattering cross section finite even at this energy. This may be considered roughly as the resonance in our classical theory.

The classical treatment of the interaction of the meson field with nucleons disregards the momentum properties of individual field quanta. However, like the case of the classical electrodynamics, the classical meson dynamics is expected to give approximately correct results for range of energies in which momenta of individual mesons are small compared with the mass of the nucleon.[8] This certainly covers the range of energies up to a few hundred millon electron volts with which we are primarily concerned. Thus, within this range of energies the classical theory is also expected to be applicable to the process of photomesonic production.[5]

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[7] Isaacs, Sachs, and Steinberger, Phys. Rev. 65, 803 (1952).

[8] H. J. Bhabha, Proc. Roy. Soc. (London) A172, 384 (1939).