# Remarks on Some Questions of Neutron Optics

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Several derivations of the expression for the index of refraction of neutrons are discussed and difficulties recently raised are cleared up. The theory is extended to cover cases involving absorption (through capture or incoherent scattering). Formulas for the scattering due to disorder in crystals are used for the determination of scattering amplitudes of isotopes in agreement with direct experiments. It is shown how the polarization of neutrons scattered under small angles can be used to obtain information about the structure of mixed crystals. The problem of primary and secondary neutron extinction is also discussed in the light of a recent publication.

## I. ON THE INDEX OF REFRACTION OF NEUTRONS IN MAGNETIC MATERIALS

AN expression for the index of refraction of neutrons including the case of magnetic refraction and of rotation of the plane of polarization was first given in the form[1]

$$n_{\pm}-1=\frac{N\lambda^2}{2\pi}(a_n\pm a_m)=\frac{N\lambda^2}{2\pi}a_n\pm\frac{4\pi\mu I}{2E}. \quad (1)$$

In (1) $N$ denotes the number of scattering centers per unit volume, $\lambda$ the neutron wavelength, $a_n$ the coherent scattering amplitude of the nucleus, $\mu$ the magnetic moment of the neutron, $E$ its kinetic energy, and $I$ the density of magnetization of the ferromagnet traversed; $a_m$ stands for the suitably defined (see reference 8) amplitude of magnetic scattering in the forward direction. The formula was later[2] applied to devise a method for the production of polarized neutron beams by total reflection; an experimental approach of this kind was shown to allow a direct measurement of the neutron spin (as distinguished from the neutron's magnetic moment) and a study of the interaction function of the neutron's magnetic moment with the magnetic moment of the atom. It was also pointed out that, in general, $4\pi I$ would have to be replaced by $B$, the magnetic induction, to take into account the influence of a possibly present over-all magnetic field and its dependence on the shape of the magnetized body.

Equation (1) has been confirmed by a large series of experiments. While the well-known investigations by Fermi[3] and his collaborators on total reflection of neutrons dealt mostly with the nonmagnetic case, several papers by Hughes[4] and his associates took up the problem as formulated for the presence of a magnetic field and verified (1) in several cases. By studying polarization effects produced by double refraction, as suggested by the author,[2] Hughes and his collaborators could also obtain, as predicted, information concerning the interaction function.

The derivation of (1) for the general case followed the well-known classical procedure used in optics. This mode of approach was the more justified, since $n-1$ is very small compared to 1, so that no questions of differentiation between the real and exciting field need to be discussed. The wave propagated in the medium was the sum of the incident wave and all scattered wavelets coming from points reached before the point of observation. This procedure could be carried out following the well-known approach of Fresnel. After carrying through the integration, it was found in agreement with optics that the scattering amplitude of the elementary scatterer in the forward direction determined essentially the index of refraction.

Lately, criticism[5] has been raised against our derivation of (1). It was claimed to be in error because a detailed calculation of the magnetic scattering amplitude in the forward direction gave for it the value 0. The conclusion was drawn that the classical method breaks down for long-range forces like magnetic interactions. A modified derivation,[6] on the other hand, led to our results for the index of refraction and also confirmed our predictions,[2] experimentally verified by Hughes, concerning angular effects and the interaction function. No attempt was made to explain the surprising agreement of a supposedly wrong derivation with the "new" theory or the experiments.

This criticism has been taken up in some recent papers by Lax,[7] who obtained our results with an operational method but failed to give any reason for the apparent validity of a wrong proof.

The theoretical importance of this question as well as the various experimental confirmations of our theory may justify perhaps a few remarks intended to clear up these apparent contradictions. Summarizing in advance the results of the reasoning to be presented, we want to state that the criticisms raised are without any basis, that our derivation and the results obtained are correct and are, in fact, identical with the differently worded derivation given by Lax.[7] The origin of this

[1] Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[2] O. Halpern, Phys. Rev. **75**, 343 (1949); **76**, 1130 (1949).
[3] See, e.g., E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).
[4] See, e.g., D. J. Hughes and M. T. Burgy, Phys. Rev. **81**, 498 (1951).
[5] H. Eckstein, Phys. Rev. **78**, 731 (1950).
[6] H. Eckstein, Phys. Rev. **76**, 1328 (1949).
[7] M. Lax, Phys. Rev. **80**, 299 (1950); Revs. Modern Phys. **23**, 287 (1951).

untenable criticism is to be found, in our opinion, in a formalistic misunderstanding[5] of a very simple physical situation.

That the magnetic scattering amplitude for $\vartheta=0$ vanishes as especially proven in reference 5 is correct, but was known to us and is contained in formula (4.05) of an early investigation[8] in which the form of the interaction function has been studied carefully. Formula (4.05) of reference 8 shows that the magnetic scattering amplitude for all values of $\vartheta>0$ is a continuous function of the angles but has a (completely immaterial) discontinuity for $\vartheta=0$ at which it vanishes. This (completely immaterial) discontinuity has its origin in the fact that the vector potential of the neutron's magnetic field was for simplicity's sake permitted to have a singularity of the form $1/r^2$. Forming a matrix element with this vector potential, an ambiguity arises for $\vartheta=0$, since the infinite result of integration over the radius vector is canceled by the factor 0 coming from the integration over the angles.

This discontinuity at $\vartheta=0$ is called by us completely immaterial, since there does not exist a physical statement in which the wave function (or its square) has to be known at *one* point. In all cases of physical interest, one is concerned with integrals over domains (however small). We considered well known, and therefore failed to call attention to, the results of elementary calculus that show no integral is changed by changing the value of the integrand by a finite amount at one point (or, for that matter, at a suitably infinite number of points). Therefore, the expression for the magnetic scattering amplitude was made continuous at the point $\vartheta=0$. (4.06 of reference 8.)

It is, of course, this scattering amplitude made continuous which enters rigorously into the expressions for the index of refraction. As shown in detail in reference 1 and as mentioned in this introduction, the index of refraction arises through an integration over all elementary wavelets contributing to the eigenfunction at a specified point. Integrating over these wavelets and using partial integration, the value of the integrand is, of course, determined by the scattering amplitude made continuous and not by the, in principle, completely arbitrary value of the eigenfunction at $\vartheta=0$.

The procedure by Lax[7] follows in operational language exactly our classical procedure of summing the elementary wavelets contributed by all scatterers. It is not surprising that he arrives at our results. There is, of course, no need to discuss anomalous conditions arising from long-range magnetic forces, since there are no anomalies to be explained and since the classical method applies without any deviations.

## II. THE INFLUENCE OF ABSORPTION PROCESSES ON TOTAL REFLECTION

As known from optics, the presence of an index of absorption affects the refractive processes, particularly the phenomenon of total reflection. Although in the case of neutrons the index of absorption is, as a rule, not only very small by itself but also small compared to $n-1$, it will turn out that absorptive processes have still a very characteristic effect.[9] Precision determinations of the coherent scattering amplitude made with the aid of measurements of the critical angle will be seen to depend in a noticeable amount on the presence of an absorption cross section.

Absorptive processes are, for the present purpose, not only produced by pure capture but equally by any incoherent scattering process that may occur. The incoherently scattered beam constitutes, as far as refractive processes are concerned, simply a loss to the original beam; for the formal treatment, capture cross section and incoherent cross section enter quite symmetrically.[9]

To account for such general absorption processes in the Schrödinger equation, we write it in the form

$$\Delta\psi+k^2\psi+i\gamma k\psi=0. \tag{2}$$

The expression $\gamma$ stands for the sum

$$\gamma=N_c\sigma_c+N_i\sigma_i. \tag{3}$$

In (3), we have denoted by $\sigma_c$ and $\sigma_i$ the cross section of capture or incoherent scattering and by $N_i$ and $N_c$ the number of particles per unit volume possessing these cross sections. Since it is always true that $\gamma\ll k$, the solution

$$\psi=e^{-ikx}e^{-\frac{1}{2}\gamma x} \tag{4}$$

of (2) gives for the density of the neutrons the expression $\sim e^{-\gamma x}$. We see that the exponential damping has the right coefficient which justifies the addition of the imaginary part of the Schrödinger equation.

If we now study the reflection of a neutron wave incident under the glancing angle $\varphi\ll 1$ on a medium with the index of refraction $n$ and the damping term $\gamma$, we find from the boundary condition at $x=0$ the following relations:

$$k_x'^2+k_y'^2=k^2(n^2+i\gamma/k)=k^2(1-2\delta+i\gamma/k), \tag{5}$$

$$1-n=\delta\ll 1, \tag{6}$$

$$k_y'=k_y=k\cos\varphi=k(1-\varphi^2/2), \tag{7}$$

$$k_x'^2\cong k_x^2-k_y^2(2\delta-i\gamma/k)\cong k^2(\varphi^2-2\delta+i\gamma/k)$$
$$\cong(k_x^2/\varphi^2)(\varphi^2-2\delta+i\gamma/k), \tag{8}$$

$$r=\left|\frac{k_x'-k_x}{k_x'+k_x}\right|^2\cong\left|\frac{\varphi-(\varphi^2-2\delta+i\gamma/k)^{\frac{1}{2}}}{\varphi+(\varphi^2-2\delta+i\gamma/k)^{\frac{1}{2}}}\right|^2, \tag{9}$$

where $r=$ coefficient of reflection.

In the absence of absorption, the reflection coefficient for values of $\varphi=\varphi_0(1+p)$, where

$$\varphi_0^2=2\delta,$$

[8] O. Halpern and M. H. Johnson, Phys. Rev. **55**, 898 (1939).

[9] O. Halpern and C. B. Shaw, Phys. Rev. **78**, 88 (1950).

can be shown to be given by

$$r=1-(32p)^{\frac{1}{2}}, \quad (p\ll 1). \tag{10}$$

It is well worth remembering that according to (10) the reflection coefficient differs from unity by about 10 percent if $p$ is as small as $3\times 10^{-4}$.

In the presence of absorption, the reflection coefficient at the angle

$$\varphi^2=2\delta$$

(which would correspond to the critical angle in the absence of absorption) amounts to

$$r_0=1-(2\gamma/k\delta)^{\frac{1}{2}}. \tag{11}$$

Here we have assumed that

$$\gamma/k\delta\ll 1.$$

(11) is always valid except perhaps in the neighborhood of absorptive resonances as they may occur, for example, in Cd and Mn.

If we choose for illustrative purposes the values $10^{-2}$ and $10^{-3}$ for the ratio $2\gamma/k\delta$, we find that $r_0$ becomes, respectively 0.9 and 0.97.

Equation (9) shows that the angle $\varphi=\varphi_0(1-g)$,

$$\gamma/8k\delta\ll g\ll 1,$$

has to be used to obtain a reflection coefficient of about

$$r=1-(\gamma/k\delta)[1/(2g)^{\frac{1}{2}}]. \tag{12}$$

If the ratio $\gamma/k\delta=10^{-2}$ then one obtains for $g=1/30$ the value $r\sim 0.96$.

The case of H and its components deserves special mention. Here the large incoherent scattering cross section $\sigma i\sim 80$b makes the ratio exceptionally favorable for the discovery of the influence of "absorption" processes on total reflection.

## III. SCATTERING DUE TO DISORDER IN CRYSTALS

It has been shown[1] that an otherwise ideal crystal composed of two randomly distributed constituents which have respectively the scattering amplitude, $a_1$ and $a_2$, has a cross section of disorder scattering of the amount

$$\sigma=4\pi c_1c_2(a_1-a_2)^2. \tag{13}$$

Here $c_1$ and $c_2$ denote the percentages of the two constituents. This formula has, among other things, been applied to the study of the scattering amplitudes of two isotopes constituting a single crystal.[1]

Lately,[10] this method has been used to determine the scattering amplitudes of mixed crystals, in particular NiMn. After determining the coherent scattering amplitude of the element Ni, a rather large disorder cross section was found for the mixed crystal, which fact constituted convincing proof that the scattering amplitudes of the elements Ni and Mn have opposite sign.

The authors,[10] in evaluating their data, have not paid attention to some additional information contained in them which does not seem to be without interest. They observed a disorder scattering cross section for pure Ni of the magnitude of $\sim$4b. Keeping in mind that the percentages of the two dominant Ni isotopes are $\sim$70 percent and 30 percent, one obtains for the coherent scattering cross section of $Ni^{58}$ and $Ni^{60}$, respectively, the values

$$\sigma_{58}=25\text{b}; \quad \sigma_{60}=1\text{b}.$$

This result is somewhat surprising, since the two isotopes appear to have probably the largest and smallest coherent scattering cross sections known so far. Still, our result seems to be confirmed by measurements[11] on Ni in which one or the other of the isotopes was enriched, which led to the result of 27b and 1b, respectively.[12]

The study of the neutron polarization effect upon disorder scattering may in some cases lead to a deeper knowledge of the composition of a mixed crystal.

Consider the behavior of a mixed crystal or polycrystal, one component of which shall be, e.g., Fe. The mixed crystal shall be magnetized close to saturation. Denote the nuclear coherent scattering amplitude of Fe by $a_1$, its magnetic scattering amplitude, which, of course, depends on the direction of the spin of the incident neutron, by $a_m$, and the coherent scattering amplitude of the second component by $a_2$. To obtain insight into the effect which this admixture will have upon polarization phenomena we proceed as follows.

Imagine all atoms of the second component replaced by Fe atoms; also add at the places occupied by the atoms of the second component anti-Fe atoms, that means Fe atoms with inverted sign of the nuclear and magnetic scattering amplitude. We then have first a complete crystal (polycrystal) containing saturated Fe and, in addition to it, at random places an equal number of coincident atoms 2 and anti-Fe atoms. Their scattering amplitude will, therefore, be

$$\bar{a}=a_2-a_1-a_m. \tag{14}$$

Since they are distributed randomly, they will scatter isotropically as far as the nuclei are concerned and with the form factor of $a_m$ as far as the magnetic scattering is concerned. They will add to the polarization effect of the Fe crystal (polycrystal) provided that $a_2<a_1$ since then the relative sign of nuclear and magnetic scattering is the same as it is in Fe and will diminish it if $a_2>a_1$.

[10] P. J. Bendt and I. W. Ruderman, Phys. Rev. **77**, 575 (1950).

[11] Koehler, Wollan, and Shull, Phys. Rev. **79**, 395 (1950).

[12] When the author described his result for Ni during a lecture course at Brookhaven National Laboratory in the summer of 1950, Professor M. Goldhaber pointed out that the values theoretically obtained were confirmed by some recent Oak Ridge measurements (reference 11). We want to thank Professor Goldhaber for calling our attention to these results.

The result just now obtained was essentially based on a random distribution of the atoms 2. But the method just discussed permits us, without too great difficulties, to discover experimentally a possible case in which the atoms 2 are not randomly distributed among the Fe atoms but form microcrystalline aggregates among themselves; these in turn may be randomly located among the Fe atoms.

The case just mentioned falls into a category discussed to some extent in Paragraphs VI and VII of an earlier investigation.[13] We proceed again by filling all spaces occupied by atoms 2 simultaneously with Fe and anti-Fe atoms, thus obtaining a complete iron crystal (polycrystal) and in addition randomly distributed microcrystals, the elements of which have again the scattering amplitude $\bar{a}$. We learn now from reference 13 that these randomly distributed microcrystals will essentially scatter into an angle of the order of magnitude $\lambda/dZ^{1/3}$, where $d$ is the lattice distance; their integral cross section is proportional to $\bar{a}^2Z^{4/3}$, where $Z$ denotes the total number of atoms contained in a microcrystal. If now the transmitted beam is analyzed with the aid of a counter subtending different angles, one can separate the small angle contribution arising from the presence of the microcrystals and, in particular, measure, without extreme difficulty, the influence of magnetization. Any polarization effect can be expected to be rather large because the form factor of the magnetic scattering amplitude will be close to 1; it is well known that at the angle at which the first Debye-Scherrer rings occur in Fe, the magnetic form factor is smaller than $\frac{1}{2}$. Experiments of this kind would permit direct study of possible agglomerations of the second component in ferromagnetic mixed crystals.

## IV. NEUTRON EXTINCTION PHENOMENA

In a recent note, R. J. Weiss[14] discusses extinction effects in the transmission of neutrons through polycrystalline material; absence of extinction shows itself in the proportionality of the effective cross section with the cross section of the isolated nucleus. He arrives at the conclusion that the treatment of this question which for neutrons was first given in reference 1 (and later extended to the case of polyatomic lattices by Fermi, Sturm, and Sachs[15]) is insufficient in both papers; he claims that only his investigation shows how information concerning grain sizes and the length of the constitutive microcrystals (mosaic blocks) can be obtained. The position taken by Weiss[14] is not acceptable to us.

What follows is largely a paraphrased version of p. 984, the right column of p. 987, and the left column of p. 989 of reference 1. The discussion there given has apparently not been appreciated, to judge especially by the last paragraph of reference 14.

One talks about primary extinction if the cross section of a microcrystal (mosaic block) calculated by Born approximation is comparable with its geometric cross section. Similarly we say that secondary extinction is present if the cross section of the grain compares with its area.

If one assumes that the microcrystals (due to disorder within the grain) scatter incoherently so that their intensities add, then the grain cross section obviously reaches its maximum if the microcrystals, though disordered, are still so well aligned that the incident radiation can strike all of them approximately under the Bragg angle. For the estimate of the maximum cross section of the grain we had explicitly[1] introduced this assumption.

Denoting by $N_i$ the number of scatterers along one linear dimension of the microcrystal, by $N_k$ the same number for the grain, by $a$ the nuclear scattering amplitude, and by $d$ the length of the unit cell, we found [see Eqs. (2.9) to (2.12) of reference 1] the following conditions for the absence of primary and secondary extinction, respectively:

$$N_i a/d < 1; \tag{14a}$$

$$N_i N_k a^2/d^2 \ll 1. \tag{14b}$$

We showed numerically that for a typical case of Fe primary and secondary extinction are small; we also pointed out that for larger sizes of the constituents extinction may well be present and actually seems to have been observed, for example, with Cu.

After some deliberations which we were not quite able to follow, Weiss[14] also accepts our assumption that primary extinction is generally absent; his condition agrees with our (14a) after allowance is made for the new meaning of $N$ in his Eqs. (6), (7) which now denote the number of scatterers per unit volume. Similarly his condition for the absence of secondary extinction $\sigma T \ll 1$ is equivalent with (14b). This is not surprising since we both are dealing with conventional x-ray theory applied to neutron diffraction. The lengthy and comprehensive formulas quoted by Weiss from the monograph of Zachariasen reduce, of course, for the limiting cases discussed, to the simple expressions underlying our treatment.

Weiss[1] points out that particularly near the cut-off wavelength strong diffraction occurs over a wide angular range in the neighborhood of the Bragg angle. He therefore concludes (rightly) that this case is most favorable for secondary extinction since it is then likely that the microcrystals are better aligned than this angular spread around the Bragg angle; to have overlooked this fact is his principal accusation against the earlier treatments.[1,15] But he has missed the point that for the purpose of making our condition for the absence of secondary extinction most stringent we have

[13] O. Halpern and E. Gerjuoy, Phys. Rev. **76**, 1117 (1949).
[14] R. J. Weiss, Phys. Rev. **86**, 271 (1952).
[15] Fermi, Sturm, and Sachs, Phys. Rev. **71**, 589 (1947).

just assumed[1] such a behavior of the microcrystals within a grain to occur, even far away from cutoff. Very clearly his condition for the absence of secondary extinction in the most unfavorable case is no more restrictive than ours.

But there exists a physical difference between x-ray and neutron diffraction in the case of a ferromagnet which leads to the well-known transmission effects due to polarization. We have laid[1] great emphasis on the fact that marked extinction would show itself in a radical reduction of these transmission effects. Obviously if the cross section of the grain is no longer proportional to the cross section of the elementary scatterer, then a slight change of the latter, produced by magnetization, will not affect the total transmission very much. A study of the transmission effects therefore permits insight into extinction effects even if sharp spectral resolution is not feasible.

In this connection an extinction problem deserves to be mentioned which is not treated in any exposition based on conventional x-ray theory. For wavelengths very near the cutoff it may occur that the angular width of the beam is larger than the deviation of the Bragg angle from $\pi/2$. This is contrary to the assumption underlying all x-ray theories; these x-ray treatments are realistic since one can see by a closer study that the case mentioned before can be established experimentally only with great difficulty. Still, it constitutes at least a mathematical problem which perhaps at some future time will find its experimental counterpart.

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# Angular Correlations in the Reaction $F^{19}(p,\alpha\gamma)O^{16}$

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The angular correlations between alpha-particles and gamma-rays in transitions between states of $Ne^{20}$ and the low excited states of $O^{16}$ have been studied at the 669-, 874-, and 935-kev resonances for proton capture in fluorine. The alpha-particle groups were separated magnetically from each other and from scattered protons. The results lead to definite assignments of spin and parity to the nuclear states involved, and show that the first four excited levels of $O^{16}$ are consistent with a simple alpha-particle model for this nucleus. Where interference between states occurs in the reaction, the phase differences are in accord with the predictions of modern dispersion theory.

## I. INTRODUCTION

THE level structure of $O^{16}$ has been the subject of many experimental and theoretical studies, and particular interest has been directed at those states which can be reached by the $F^{19}(p,\alpha)$ reaction. (See, for example, the review article by Hornyak *et al.*[1]) The resonance levels in $Ne^{20}$ formed in this reaction decay predominantly by alpha-particle emission; besides the transition to the ground state of $O^{16}$, four distinct alpha-particle groups have been observed, associated with transitions to the first four excited states of $O^{16}$. Much of the interest in these levels in $O^{16}$ lies in the attempt to identify them with the low states of excitation of a bound system of four alpha-particles (Wheeler[2] and Dennison[3]). It is known that the first excited state of $O^{16}$ (6.05 Mev) has spin zero and even parity [denoted by $(0,+)$] (Devons and Lindsey[4]) and that the second excited state (6.13 Mev) has the designation $(3,-)$ (Barnes *et al.*[5]). These do in fact correspond (though in reversed order) to the first two excited states predicted by Wheeler and Dennison for a tetrahedral arrangement of alpha-particles, and the experiments described in this paper were carried out with the aim of extending the comparison to higher levels.

The levels in $Ne^{20}$ formed by resonance capture of protons in fluorine seem to be of two kinds, namely, (a) those that can decay by emission of long-range alpha-particles to the ground state of $O^{16}$; (b) those that emit short-range alpha-particles followed by gamma-rays. (From all levels of type (a) one also observes transitions to the well-known pair emitting state of $O^{16}$ at 6.05 Mev.) It is usual to ascribe the absence of long-range alpha-particle emission for type (b) to a strict selection rule arising from the need to conserve total angular momentum ($J$) and parity ($P$) in the transitions; this is achieved by supposing that the $Ne^{20}$ levels concerned have odd $J$ with even $P$, or even $J$ with odd $P$. A full discussion is given by Chao.[6] The present work was confined to levels of this type, and consisted of a study of angular correlations between the various alpha-particle groups and their associated gamma-rays.

[1] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).
[2] J. A. Wheeler, Phys. Rev. **52**, 1083 (1937).
[3] D. M. Dennison, Phys. Rev. **57**, 454 (1940).
[4] S. Devons and G. R. Lindsey, Nature **164**, 539 (1949).
[5] Barnes, French, and Devons, Nature **166**, 145 (1950).
[6] C. Y. Chao, Phys. Rev. **80**, 1035 (1950).