greater precision in the experimental measurements, it is possible that useful data on nuclear configurations may be derived from μ-meson interactions. The local fluctuations in absorption probability such as those in the vicinity of lead reflect the properties of nuclear shell structure, and since the interaction is not confined to one or two particles near the top of the potential distribution, this reaction gives access to information which is not available from other types of nuclear reaction.

PHYSICAL REVIEW VOLUME 87, NUMBER 6 SEPTEMBER 15, 1952

Theory of Slip-Band Formation

JOHN C. FISHER, EDWARD W. HART, AND ROBERT H. PRY
Research Laboratory, General Electric Company, Schenectady, New York
(Received March 26, 1952)

The fine structure of slip bands on the surfaces of plastically deformed crystals, as described by Heidenreich and Shockley and by Brown, is explained in terms of the model of dislocation-loop generation proposed by Frank and Read. The back-stress at an active Frank-Read source, produced by an expanding avalanche of about 300 dislocation loops, is shown to be sufficient to stop dynamic loop generation at the source. Slip, therefore, occurs in avalanches separated in time.

Avalanches from other slip systems crossing a slip plane containing an active source lead to the observed stepped surface markings, with successive avalanches from the given source displaced relative to each other.

PLASTIC deformation of crystals often occurs by slip, which takes place on only a few of many crystallographically equivalent planes. The translation per active plane is considerable, however, and large amounts of plastic flow can result. Frank and Read,[1] by means of a simple model of dislocation-loop generation, have explained the inactivity of most prospective slip planes, but the concentration of slip into hundred- or thousand-loop avalanches on each active plane has not been accounted for.

We propose that slip avalanches result from the dynamic generation of loops at a Frank-Read source, and are terminated by their own stress fields. The observations of slip-band distribution and fine structure can be interpreted satisfactorily in this way.

OBSERVATION OF SLIP BANDS

A surface marking, or slip band, appears at the intersection of an active slip plane and the free surface of a crystal. Yamaguchi,[2] using a light microscope, has made an extensive study of the relationships between slip-band distribution and work hardening. Heidenreich and Shockley,[3] and Brown,[4,5] using electron microscopes, have examined the fine structure of slip bands. Based largely upon these observations of slip in aluminum, Brown[6] has summarized the current state of knowledge regarding the structure of slip bands and the relationship between slip bands and work hardening. Restated, his summary is as follows:

(1) Plastic deformation in aluminum proceeds by slip on a relatively small number of crystal planes.
(2) Each slip band is composed of one or more avalanches, as sketched in Fig. 1.
(3) The offset per avalanche is nearly independent of stress and temperature.
(4) The number of avalanches per band increases as the temperature or stress increases.
(5) A plot of stress *versus* number of slip bands is nearly independent of temperature.

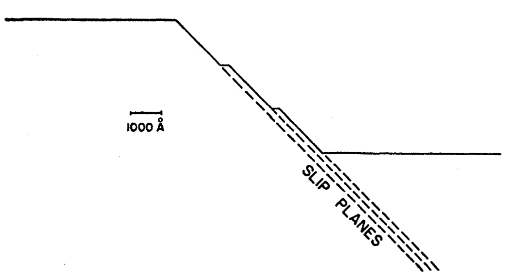

FIG. 1. Slip band fine-structure in aluminum, according to Brown (see reference 6). Cross section of a slip band containing three avalanches.

TWO QUESTIONS RAISED BY THE EXISTENCE OF AVALANCHES

An avalanche of about 10^3 dislocations appears to be the smallest unit of slip in an ordinary slip band. At low stresses and small strains most slip bands contain a single avalanche. The existence of such avalanches poses two questions.

The first is how so many dislocations are generated on a single slip plane. It has been answered convincingly

[1] F. C. Frank and W. T. Read, Phys. Rev. **79**, 722 (1950).
[2] K. Yamaguchi, Sci. Papers Inst. Phys. Chem. Research (Tokyo), **8**, 289 (1928).
[3] R. D. Heidenreich and W. Shockley, J. Appl. Phys. **18**, 1029 (1947).
[4] A. F. Brown, Nature **163**, 961 (1949).
[5] A. F. Brown, Compt. rend. 1er Congr. Internatl. Microscopie Electronique, Paris (1950).
[6] A. F. Brown, J. Inst. Metals **80**, 115 (1951–52).

by Frank and Read.[1] They point out that a dislocation segment, lying in a slip plane and pinned at both ends, is a source of an unlimited number of dislocation loops. Such segments are known as Frank-Read sources, and they react to stress as sketched in Fig. 2. At low stresses the dislocation segment bows in the slip direction and stops at an equilibrium curvature: further glide would increase the energy of the dislocation, by increasing its length, more than the decrease in energy due to glide in the stress field that is present. When a critical resolved shear stress is reached, for which the segment is approximately semicircular, continued growth of the loop is possible with a net decrease in energy, for the energy increase associated with increased length of dislocation is more than offset by a decrease due to glide.

The critical resolved shear stress is approximately

$$\tau^* = Gb/L, \tag{1}$$

where G is the shear modulus, b is the magnitude of the Burgers vector, and L is the length of the Frank-Read source. For a stress greater than τ^* an unlimited number of dislocation loops can be generated, as shown in the figure. After each loop is formed, the segment is regenerated and acts again to form another loop. In this way the grouping of dislocations into an avalanche of a thousand or so loops on a single slip plane can be understood.

The second question raised by the existence of avalanches of dislocations is: why does the generation of dislocation loops in an avalanche stop, and why at a rather definite number of loops?

INTERPRETATION OF CONSTANT OFFSET PER AVALANCHE

In a perfect single crystal and in the absence of any work hardening whatever, it is proposed that the generation of dislocation loops at a Frank-Read source would cease in time because of the back-stress produced by loops already generated, for each dislocation loop has a stress field that opposes the applied stress in the neighborhood of the source. When enough loops have been generated, the stress at the source will fall to a value so low that additional loops cannot form. Only after the original avalanche of loops has moved some distance away can another avalanche occur.

A first estimate of the minimum stress at a Frank-Read source, below which the source ceases to generate loops, might be τ^*, the critical stress for loop generation; but τ^* is the critical stress for loops that must grow from a standing start. After the first, all loops that form at a given source form with a running start. A regenerated segment has kinetic energy as it moves through its position of minimum length, and continues to have work done upon it as it moves toward the position of maximum energy. Under a stress τ^*, therefore, a dislocation loop will pass through the critical semicircular shape with a positive velocity. Even though the stress drops somewhat below τ^*, loops still can be generated.

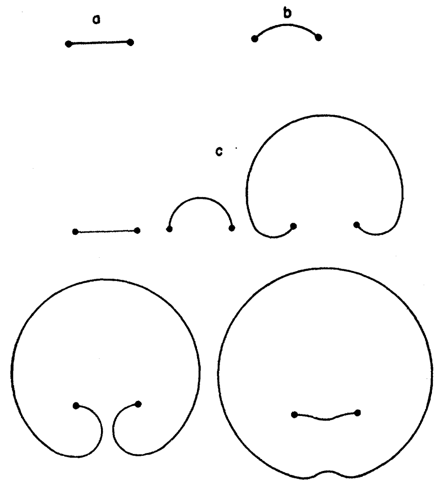

FIG. 2. Response of a Frank-Read source to stress. (a) Equilibrium under no stress. (b) Equilibrium under a subcritical stress. (c) Loop generation under a supercritical stress.

In order to calculate the number of loops that should be present in an avalanche, it is necessary to estimate (a) the back-stress produced by a number of successive loops occurring in one burst, and (b) the minimum stress at which a Frank-Read source, once started, can continue to operate.

ESTIMATION OF BACK-STRESS CAUSED BY AN AVALANCHE

The stress at a distance r_i from a straight screw dislocation is

$$\tau_i = Gb/2\pi r_i. \tag{2}$$

As a first approximation, it is also the back-stress at the center of a single dislocation loop of radius r_i. The back-stress at the center of a number of concentric loops will be

$$\tau_b = \sum \tau_i = (Gb/2\pi) \sum 1/r_i. \tag{3}$$

If the length of the Frank-Read source is L, the spacing of loops will be approximately πL, so that

$$r_i = \pi L i, \tag{4}$$

and the back-stress is

$$\begin{aligned} \tau_b &= (Gb/2\pi) \sum 1/\pi L i \\ &= (Gb/2\pi^2 L) \ln i, \end{aligned} \tag{5}$$

where i is the number of loops. Since the applied stress is just the critical stress to start the first loop, as given by Eq. (1), the ratio of back-stress to applied stress is

$$\tau_b/\tau^* = \ln i/2\pi^2 \tag{6}$$

for an avalanche of i loops, independent of stress or temperature. For a hundred loops, $\tau_b/\tau^* = 0.23$, and for a thousand $\tau_b/\tau^* = 0.35$.

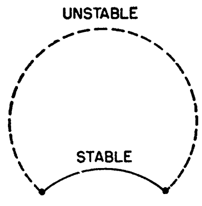

FIG. 3. Stable and unstable equilibrium positions for a Frank-Read source subjected to a subcritical stress.

MINIMUM STRESS FOR CONTINUED OPERATION OF A FRANK-READ SOURCE

In order to estimate the minimum stress at which a Frank-Read source can operate, it is necessary to know the extent to which energy is conserved during the motion of a dislocation. There are two limiting cases, one corresponding to high losses, where the dislocation can have no kinetic energy, and the other to low losses, where the sum of the potential and kinetic energies of the dislocation remains constant. We shall consider the second case, for the first leads to the generation of only a single loop per avalanche.

The potential energy of a free dislocation loop of radius r lying in a slip plane where the resolved shear stress is τ is

$$\Delta E = 2\pi r\gamma - \pi r^2 b\tau, \tag{7}$$

where $\gamma = Gb^2/2$ is the energy per unit length of dislocation. The first term is the self-energy of a dislocation at rest, the second the potential energy decrease associated with the motion of a dislocation with Burgers vector b through area πr^2 in a resolved shear stress field τ. The energy ΔE is a maximum when

$$r = r^* = \gamma/b\tau = Gb/2\tau, \tag{8}$$

for which $\partial\Delta E/\partial r = 0$ and the loop is in unstable equilibrium. A Frank-Read source of length L will have two equilibrium positions, one stable and one unstable, both with radius r^*, as long as $r^* > L/2$. In the unstable position the dislocation line is a longer than semicircular arc, and in the stable position a shorter than semicircular arc, as sketched in Fig. 3. When $r^* < L/2$ there are no equilibrium positions and the Frank-Read source must generate loops. The limiting condition, $r^* = L/2$, together with the approximation $\gamma = Gb^2/2$, leads to Eq. (1) for the critical stress to activate a Frank-Read source.

The Frank-Read source configuration just after the birth of a new loop and the regeneration of the dislocation segment is sketched in Fig. 4a for a stress $\tau < \tau^*$. The position of unstable equilibrium also is sketched. The minimum stress τ for which another loop can be generated is that for which the energy (kinetic plus potential) of the regenerated segment just equals the potential energy of the loop at the unstable equilibrium position, where an infinitesimal disturbance will cause it to grow with continuously decreasing potential energy.

Figure 5 shows the potential energy of a regenerated dislocation segment as a function of the area swept out. Calculated curves are given for several stresses. The top one, for $\tau = \tau^*$, is monotonically decreasing. The others, for $\tau < \tau^*$, have a minimum and a maximum corresponding to the two equilibrium positions. In the absence of energy dissipation, and neglecting the kinetic energy of the regenerated segment, dynamic loop generation can continue as long as the initial energy E_0 of the regenerated segment exceeds the energy E_1 of the unstable equilibrium position. The minimum stress for continued operation of a source is that for which $E_0 = E_1$. According to the figure this stress is about $0.5\tau^*$.

The minimum stress can be calculated more easily and probably more accurately by assuming that the regenerated segment has the shape shown in Fig. 4b or 4c, for it is relatively unlikely that energy is conserved near the sharp apex of the regenerated segment of Fig. 4a. The points E_0' in Fig. 5 correspond to regenerated segments sketched by hand as in Fig. 4b, and to a minimum stress of about $0.7\tau^*$ for dynamic loop generation. Assuming the configuration in Fig. 4c for numerical calculation, the potential energy of the regenerated segment is

$$E_0' = 2\theta r^*\gamma, \tag{9}$$

where θ is the angle shown in Fig. 4c and γ is the energy per unit length of dislocation. The corresponding potential energy of the loop in the unstable position is

$$E_1 = (2\pi - 2\theta) r^*\gamma - \pi r^{*2} b\tau. \tag{10}$$

The single term in E_0' and the first term in E_1 represent the energies of the corresponding dislocation segments. The second term in E_1 represents the energy change associated with the area swept out between the two configurations. The stress in question is that for which $E_0' = E_1$, or for which

$$\theta = (\pi/4)(2 - r^* b\tau/\gamma) = \pi/4. \tag{11}$$

When $\theta = \pi/4$, the minimum stress τ for continued loop generation is seen to be

$$\tau/\tau^* = \tau/(Gb/L) = L/2r^* = \sin\theta = 1/\sqrt{2}. \tag{12}$$

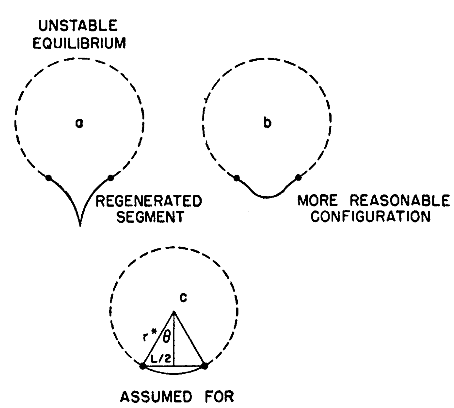

FIG. 4. Alternate configurations of regenerated dislocation segments.

According to Eq. (12), a stress as low as $1/\sqrt{2}=0.71$ of the critical stress required to start a Frank-Read source will suffice to continue it in operation, in the absence of energy dissipation. If the stress reduction from τ^* to $\tau^*/\sqrt{2}$ is caused by a back-stress $\tau_b/\tau^*=1-1/\sqrt{2}=0.29$ due to an avalanche of loops, Eq. (6) states that there are $i=320$ loops in the avalanche. In other words, according to the back-stress model, and with the approximations here employed, Frank-Read sources should generate loops in avalanches of about 300 loops each.

INTERPRETATION OF SLIP-BAND FINE STRUCTURE

Since the number of slip bands observed in a crystal depends only upon the stress, it is reasonable to assume that each band is produced by the action of a single Frank-Read source. Generation of the first avalanche at a source is easily understood. When the stress at the source reaches τ^*, loops are generated, and continue to be generated until the back-stress stops the avalanche. A second avalanche will not occur immediately in polycrystals, for the loops in the first avalanche are stopped or partially stopped at grain boundaries, and at least a portion of the back-stress remains. Even in single crystals there is enough crystal irregularity that not all loops leave the crystal, and some back-stress remains. Only if the external stress is increased substantially will a second avalanche be formed. In this way the formation of additional avalanches with rising stress can be understood.

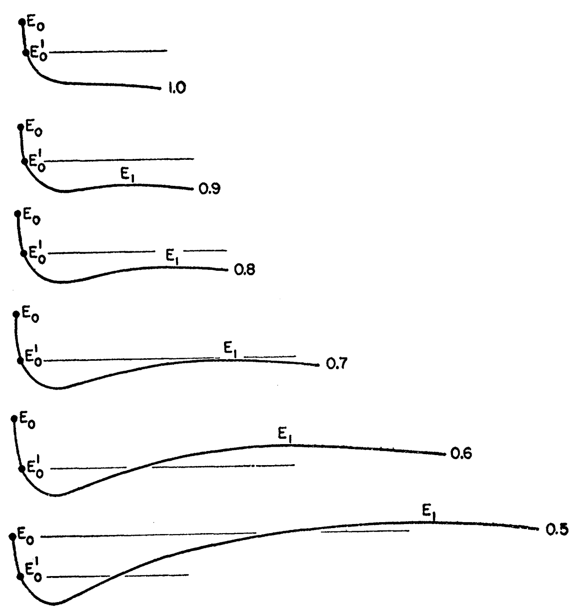

FIG. 5. Potential energy of a dislocation segment (vertical coordinate) *versus* area swept out (horizontal coordinate) for stresses in the range $0.5\tau^*\leqslant\tau\leqslant\tau^*$. Points E_0 correspond to Fig. 4a, points E_0' to Fig. 4b. Numbers beside each curve give values of τ/τ^*.

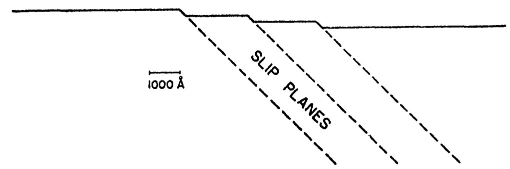

FIG. 6. Expected slip band fine-structure for large strains and cross slip.

At high temperatures recovery processes are known to reduce the back stress. A Frank-Read source that is subjected to a constant stress a little higher than τ^* will produce another avalanche every time recovery brings the back-stress down.

It remains to explain the displacement of successive avalanches by a small amount normal to the slip plane, thereby accounting for the observed fine structure of slip bands. A displacement of this type requires that a Frank-Read source move relative to the surface where slip bands are observed. Avalanches on other dihedrally inclined slip planes that pass between a Frank-Read source and its surface slip markings must produce such relative displacements, and whenever slip is occurring on more than one slip system it would be possible, in an inter-avalanche period, for one or more avalanches to pass between a Frank-Read source and its associated surface marking. These avalanches should all produce relative displacements of the same sign, so that the surface configuration in the neighborhood of a slip band could appear as in Fig. 6, where slips of several hundred atom diameters appear on parallel planes that are several times as widely separated.

Figure 6 is the inverse of Fig. 1 which gives the reported configuration in the neighborhood of a slip band as determined by electron microscopic observation of aluminum oxide replicas. In Fig. 1, slips of a thousand or so atom diameters occur on parallel planes whose relative offset is several times less than the slip per plane. Either configuration could be expected to occur, that in Fig. 1 at low strains where there is less than one intersecting avalanche per inter-avalanche period, and that in Fig. 6 at large strains where there are several.

CONCLUSION

The model of slip based upon (a) Frank-Read sources, and (b) a limitation of the offset per slip by back-stress, appears to explain satisfactorily the observed structure and distribution of slip bands.