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Intermediate Coupling as Encountered in Some of the p-Shell Nuclei

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The manner in which a transition into intermediate coupling, between the extremes of (jj) coupling and (LS) coupling, accounts for much of the failure of the (jj) coupling shell model in the light nuclei of the p-shell is illustrated by some typical examples. The set of secular equations for the configurations p^2 and p^{-2} applying to Li^6+He^6 and to $N^{14}+C^{14}$ is sufficiently simple to be worked out in detail. In the configuration p^8 which applies to $C^{12}+B^{12}$ the states are much more numerous and the equations are too complex to be solved in detail, but the solutions for the low states of interest may be treated by approximate methods, relying on some knowledge of the asymptotic behavior in the extremes. The general features of observed energy spectra for p^{-2} and for p^8 are both compatible with the intermediate coupling scheme with the same ratio $a/K\approx 5$ of the spin-orbit coupling parameter a to the "exchange integral" K, which separates multiplets. The criterion for whether the spectrum of the low states slightly resembles (LS) coupling, as it does in $C^{12}+B^{12}$, rather than (jj) coupling, as in $N^{14}+C^{14}$, is not just the value of a relative to K but the magnitude of a relative to the multiplet separations provided by K in (LS) coupling, which are exceptionally large among the low states of C^{12}.

THE impressive success of the (jj) coupling shell model in the heavy nuclei[1] and its almost as spectacular failure in the light nuclei of the p-shell from helium to oxygen has left the question whether these light nuclei might be interpreted in terms of intermediate coupling[2] between the (jj) and (LS) extremes, or whether some other phenomenon such as nascent alpha-clustering[3] or (other) excessive complication of configuration interaction must be called upon to explain the irregularities. Experimental knowledge of the spectra of these nuclei is now becoming so sufficiently extensive that it seems worth exploring the possibility of an interpretation in terms of intermediate coupling in some detail, especially to see if the general features of level density of the low states, elevation of the first state of higher isotopic, spin, etc., may be accounted for. As more identifications of nuclear spin and parity of the excited states become known, further tests of the validity of the intermediate coupling interpretation will be possible.

Nuclei of three atomic weights are here treated as examples of the sort of interpretation that may be encountered, $A=6$ and 14 because of the facility with which the complete analysis of the p-shell states may be carried out, and $A=12$ as an example of another type of spectrum with widely spaced low states and high excitation of the higher isotopic spin. A more complete survey of the trends in all of the nuclei in the p-shell is being prepared to be submitted for publication in the *Reviews of Modern Physics*.

CRITERION FOR INTERMEDIATE COUPLING

In the spectra of the p-shell nuclei, as far as they are known,[4] one notes some striking contrasts of appearance. For example, B^{10} has a rather high density of low states, as might seem compatible with the expectation of four low states from the low (jj) configuration $p_{\frac{3}{2}}^{-2}$ as an example of (jj) coupling; but only one of the four low states, rather than two as would thus be expected, seems to have isotopic spin $T=1$, indicating that there has been some cross-over, perhaps in a transition a little way into intermediate coupling but leaving the ground state with nuclear spin $I=3$ as expected in (jj)

[1] M. G. Mayer, Phys. Rev. **75**, 1969 (1949); **78**, 16 (1950); Haxel, Jensen, and Suess, Phys. Rev. **75**, 1766 (1949); Naturwiss. **36**, 155 (1949); D. Kurath, Phys. Rev. **80**, 98 (1950).
[2] H. H. Hummel and D. R. Inglis, Phys. Rev. **81**, 910 (1951).
[3] D. R. Inglis, Phys. Rev. **85**, 492 (1952).

[4] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950); T. Lauritsen, *Annual Reviews of Nuclear Science* (Stanford, California, 1952), Vol. I. Professor T. Lauritsen has very kindly supplied preliminary compilations and charts for a forthcoming revision in advance of publication. Experimental references available in the published review are omitted here, with apologies to the observers.

coupling.[5] On the other hand, C^{12} has very widely spaced low states, the first excited state being at 4.4 Mev and the next probably at 9.6 Mev (though there is a possibility of one at 7.3 Mev.) In (jj) coupling one would expect the first excited state to be one of a group of four corresponding to the (jj) configuration $p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}$. In (LS) coupling[6] there are three low singlets which are widely spaced as a result of the high degree of symmetry attainable in an even-even nucleus, and the first $T=1$ state is high because of the necessity of breaking up the four-structure to make B^{12}. The observed excitation of this state is indeed about 15 Mev. Thus, in C^{12} it seems that if there is actually any sort of intermediate coupling it must represent a transition only a little way out from the (LS) coupling extreme.

Ordinarily the criterion for the expectation of one or the other coupling scheme is expressed in terms of the relative size of two integrals which appear as competing parameters: the spin-orbit coupling parameter a which appears in the expression for the single-nucleon spin-orbit coupling energy,

$$H'=a(\mathbf{l}\cdot\mathbf{s}), \tag{1}$$

and the "exchange integral" K containing the specific interaction,[6] assumed central, between the particles involved. In atoms K contains the repulsive Coulomb potential e^2/r_{ij}, and in nuclei the attractive specific nuclear force. (In the p-shell nuclei, the direct integral L contributes as well as K, but they are related; so the contributions of the specific nuclear interactions may be expressed in terms of K, and beyond the p-shell still more integrals are involved.) One says that one expects an approximation to (jj) coupling when $a\gg K$, to (LS) coupling when $K\gg a$. Since K is expected to vary roughly as nuclear density, or as R^{-3}, and a about as R^{-2} for a given angular momentum l, we may expect the ratio a/K to vary slowly and smoothly across the p-shell. If the value of a/K were the sole criterion, one might thus expect a similar degree of intermediate coupling in various nearby nuclei.

TABLE I. Quantities referring to the multiplet energies for the configurations p^2 and p^{-2} taken from Feenberg and Phillips. The exchange operators P, 1, PQ, and Q refer to space exchange, nonexchange, space-spin exchange, and spin exchange. b and c apply to p^2, b' and c' to p^{-2}.

	P	1	PQ	Q
$b={}^3P$	$-L+3K$	$L-3K$	$-L+3K$	$L-3K$
$c={}^3D$	$L-K$	$L-K$	$L-K$	$L-K$
$d={}^1D-{}^3P$	$2L-4K$	$2K$	$-2K$	$-2L+4K$
$e={}^1S-{}^3P$	$2L-K$	$5K$	$-5K$	$-2L+K$
$f={}^1P-{}^3D$	$-2L+4K$	$-2K$	$-2K$	$-2L+4K$
$g={}^3S-{}^3D$	$3K$	$3K$	$3K$	$3K$
$b'={}^3P$	$3L+51K$	$45L-75K$	$-17L+51K$	$17L-51K$
$c'={}^3D$	$5L+47K$	$45L-73K$	$-15L+47K$	$17L-49K$

[5] Dieter Kurath, Phys. Rev. (to be published). B. H. Flowers, Phys. Rev. **86**, 254 (1952)

[6] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937); E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

In studying light nuclei we focus our attention on the low states because their separations are apt to be more meaningful and better known; when we do this a more significant criterion for the degree of intermediate coupling to be expected is the magnitude of a, not relative to K itself, but *relative to the intervals between the low states caused by K*, as are most apparent in (LS) coupling.[7] Thus, while a/K may have about the same value in C^{12} and B^{10} (it seems to be about -5 in both cases), it causes much less confusion of the pattern of (LS) coupling in C^{12} than it does in B^{10} because the multiplets are much more closely spaced (and are not all singlets) in the latter. This seems to be the main reason for the lack of consistency of appearance of either coupling scheme in the p-shell and contributes also to the problem of understanding of the four-structure of the binding energies.

INTERMEDIATE COUPLING FOR THE CONFIGURATIONS p^2 AND p^{-2}

Since the p-shell for nucleons holds twice as many nucleons in a nucleus as electrons in an atom, most of the p-shell nuclei have ground configurations containing too many states to be calculable in their entirety. The configurations p and p^{-1} of He^5 and N^{15} at the two ends of the shell are trivial as examples of intermediate coupling, but the next configurations p^2 and p^{-2} of Li^6 and N^{14} are just complicated enough to be interesting and simple enough to be easily treated. In keeping with the theory of holes, the configurations p^2 and p^{-2} differ from one another in the formal treatment only in the sign of the spin-orbit coupling parameter a, which is negative for a nucleon (having "inverted doublets," relative to an electron) and positive for a hole. Their energy states can thus be treated as solutions of the same secular equations.

The configuration p^2 contains the (jj) configurations $p_{\frac{3}{2}}^2$, $p_{\frac{3}{2}}p_{\frac{1}{2}}$, and $p_{\frac{1}{2}}^2$ in order of ascending energy. (In p^{-2} the order is reversed, with negative exponents.) The configuration p^2 belongs not merely to the nucleus Li^6 but to the polyad $A=6$ (which we may denote by Py^6), that is, to the collection of isobars He^6, Li^6, Be^6 (etc.), a system of nucleons of which isotopic spin T is a dynamical variable. By listing, in the usual tabular form while treating isotopic spin variables analogously to ordinary spin variables, the fact that there is only one state of two $p_{\frac{3}{2}}$ nucleons with $M_I=3$, $T_z=0$, and only one with $M_I=2$, $T_z=1$, etc., compatible with the exclusion principle, one sees that the (jj) configuration $p_{\frac{3}{2}}^2$ contains the four states having the quantum numbers $(I,T)=(3,0)$, $(2,1)$, $(1,0)$, and $(0,1)$, respectively. Similarly the first excited (jj) configuration $p_{\frac{3}{2}}p_{\frac{1}{2}}$ contains four states, $(2,0)$, $(1,0)$, $(2,1)$, and $(1,1)$, and the next (jj) configuration consists of the two states $(1,0)$ and $(0,1)$. Of the ten states there is only one each

[7] The criterion is stated in this unsymmetrical way because the contributions of a, being made by nucleons singly, are less subject to the vagaries of nuclear symmetry than are those of K.

of (3,0), (2,0), and (1,1), while there are two each of (2,1) and (0,1), and there are three states (1,0). In the intermediate-coupling transition a/K is varied from zero, where L, S, I, M, and T are good quantum numbers, to infinity, where j_1, j_2, I, M, and T are good quantum numbers, under the assumption of a charge-independent Hamiltonian. In the intermediate situation I, M, and T remain good quantum numbers, M being trivial except for detailed methods of evaluation because the energy does not depend on it, and it has not been included in the above counting of the states. Thus, states with the same (I,T) get mixed up in the scalar problem, and we have two quadratic secular equations and one cubic.

The secular equations for the energies may be written with almost equal ease either by constructing the complete matrix including the nondiagonal elements of H' in the (LS) representation as described completely in Condon and Shortley's book,[8] or by Goudsmit's method[9] of determining the coefficients of the secular equation from knowledge of the diagonal elements alone in both extremes, the relative convenience of the methods depending upon what information is at hand. For the quadratics we use the latter method. The information at hand includes the separation of the multiplets, as dependent on the assumed exchange nature of the nuclear interactions, which are obtained by simple subtraction from the tables of Feenberg and Phillips,[6] and are listed here for convenience in Table I. We shall arrange to obtain the energies directly in a form for plotting in Fig. 1, in which the (jj) asymptotes are arranged symmetrically with slopes 0, $\pm\frac{3}{2}$, by putting $\epsilon=E+a/2+constant$, E being the energy of a state and the constant being a convenient zero of energy for each secular equation. The two states (2,1) are in (LS) coupling a $^3P_2$ and a $^1D_2$ with the asymptotes $\epsilon\rightarrow a$ and $\epsilon\rightarrow d+a/2$, respectively, where in this case $\epsilon=E+a/2-b$. In (jj) coupling these states belong to the (jj) configurations $p_{\frac{3}{2}}{}^2$ and $p_{\frac{3}{2}}p_{\frac{1}{2}}$, having asymptotic slopes given by $\epsilon\rightarrow(3/2)a$ and $\epsilon\rightarrow 0$, respectively. The quadratic having these asymptotic values is

$$\epsilon^2-(d+3a/2)\epsilon+ad=0, \tag{2}$$

as is easily seen by noting that the coefficient of $-\epsilon$ is the sum of the roots, and the term without ϵ the product of the roots, remembering that the asymptotic expressions are only the leading terms of expansions. Similarly for the two states (0,1) one obtains the quadratic secular equation

$$\epsilon^2-e\epsilon-ae/2-(3a/2)^2=0, \tag{3}$$

where again $\epsilon=E+a/2-b$.

[8] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, London, 1935).
[9] S. A. Goudsmit, Phys. Rev. **35**, 1325 (1930).

TABLE II. (jj) asymptotes of $E+a/2$ for configuration p^2 (or p^{-2}).

	(I,T)		For $0.8P+0.2Q$
$p_{\frac{1}{2}}{}^2$	(0,1)	$-3a/2+b+a/3$	$-1.5a+0.4K-3.23K^2/a$
	(1,0)	$-3a/2+c+2f/9+g/27$	$-1.5a+3.33K-4.52K^2/a$
$p_{\frac{3}{2}}p_{\frac{1}{2}}$	(1,1)	b	$-1.8K$
	(2,1)	$b+2d/3$	$1.4K+3.41K^2/a$
	(1,0)	$c+2f/9+16g/27$	$5K-1.07K^2/a$
	(2,0)	c	$5K$
$p_{\frac{3}{2}}{}^2$	(2,1)	$3a/2+b+d/3$	$1.5a-0.2K-1.71K^2/a$
	(1,0)	$3a/2+c+5f/9+10g/27$	$1.5a+1.67K+15.19K^2/a$
	(0,1)	$3a/2+b+2e/3$	$1.5a+2.6K+3.23K^2/a$
	(3,0)	$3a/2+c$	$1.5a+5K$

For the three states (1,0) the matrix for $\epsilon=E+a/2-c$ is

	$^3D_1$	$^1P_1$	$^3S_1$
$^3D_1$	$-a$	$-(5/6)^{\frac{1}{2}}a$	0
$^1P_1$	$-(5/6)^{\frac{1}{2}}a$	$f+a/2$	$(3/2)^{\frac{1}{2}}a$
$^3S_1$	0	$(2/3)^{\frac{1}{2}}a$	$g+a/2.$

Here the terms in a are taken from the matrix of H' (given in reference 8, page 268). By setting the corresponding secular determinant equal to zero one obtains the cubic secular equation

$$\epsilon^3-(f+g)\epsilon^2+[fg-(f+g)a/2-(3a/2)^2]\epsilon+fga+(f/2+4g/3)a^2=0. \tag{4}$$

The (jj) asymptotes given by Eqs. (2), (3), and (4) plus the three linear energies are listed in Table II, along with one additional term in the expansion for the special exchange interaction assumed in plotting Fig. 1.

With our present ignorance of the exact nature of nuclear interactions (and with the likelihood of at least a small amount of configuration interaction) we cannot hope to reproduce the details of nuclear spectra. We can, however, try to match the general features of rough level spacing, but to do this we must assume an interaction at least good enough to account for the excitation of the singlet state of the deuteron and roughly for the saturation properties of nuclear forces. Beyond these requirements, we wish to keep it as simple as possible, and therefore select a linear combination of of space-exchange and spin-exchange (Majorana and Bartlett) interactions:

$$O_{ij}=0.8P_{ij}+0.2Q_{ij}. \tag{5}$$

The ratio of the direct integral L to the exchange integral K depends on the size of the nucleus relative to the range of the nuclear forces, and a reasonable value seems to be[2]

$$L/K=6. \tag{6}$$

Though it may vary slightly, we assume this same value throughout the p-shell.

With these specializations, the solutions of the secular equations are plotted in Fig. 1. The short straight lines in the middle are the asymptotes of the energies in the form of the multiplets of pure (LS)

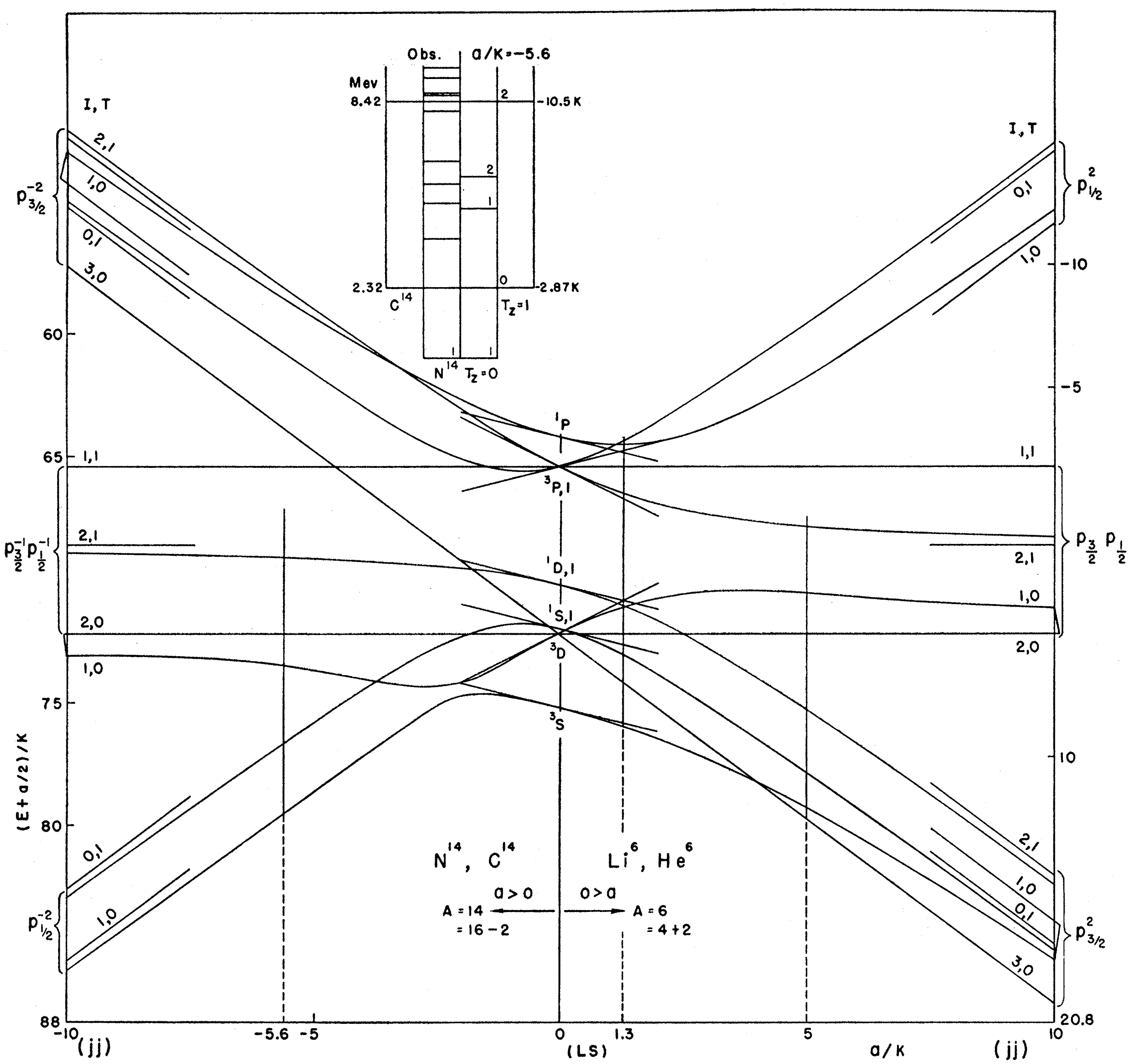

FIG. 1. Intermediate coupling transition of the energy levels of the configurations p^2 and p^{-2}. The exchange interaction specified by (5) and (6) has been assumed. K is negative, the interaction being attractive. The sign of the spin-orbit coupling parameter a refers to a nucleon on the right, to a "hole" on the left.

coupling. The straight lines near the right and left side are the (jj)-coupling asymptotes. It will be noted that the linear energy of the lone state with $(I,T)=(2,0)$ is the (jj) asymptote for one of the (1,0) states, and that in the (jj) configuration $p_{\frac{3}{2}}^2$ the state (1,0) remains rather far from its asymptote, in fact beyond the (0,1) state (compare Table II). This corresponds to an abnormally large matrix element between this and the nearer of the other two (1,0) states in the (jj) representation. The right half of the figure corresponds to positive a/K, or negative a since K is negative for attractive interactions; that is, to Py^6, the left side of the figure to Py^{14}. The (jj) configuration $p_{\frac{3}{2}}^2$ is the low one on the right side, the corresponding $p_{\frac{3}{2}}^{-2}$, with identical asymptotic spacings, is the high one on the left side, and only one state, (3,0), crosses over between the two.

In the comparison with experiment, the most valuable identifications of excited states for the nuclei here considered come through the identification of isotopic spin by comparison of the energy states of the pair of isobars. This is accomplished from the known disintegration energies[10] on the assumption that the Coulomb energy difference $N^{14}-C^{14}$ is the same as $N^{13}-C^{13}$, for example, which gives very good agreement (a good check on the charge-independence of the specific nuclear Hamiltonian), better than good enough for identification of the first $T=1$ state, after which exact equality for this state is assumed between the isobars. (For details of such comparisons, the reader is referred to reference 4, particularly the forthcoming revision.)

For Py^{14}, the value of a/K may be determined by

[10] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. 83, 512 (1951).

matching the energies of the ground state and the first two $T=1$ states (the only two known). This match between theory and experiment occurs at $a/K=-5.6$. The vertical line drawn at this value intersects the energy lines given by the secular equations at the levels indicated in the insert at the top of the figure, in the column $T_z=0$ for the $T=0$ states and in the columns $T_z=0$ and $T_z=1$ for the $T=1$ states. These should correspond to the p-shell states of N^{14} and C^{14}, respectively, and the observed states of these nuclei are shown in the other columns of the insert. This empirical match determines $K=-2.32$ Mev$/2.87=-0.81$ Mev, from which we obtain the spin-orbit coupling parameter $a=4.5$ Mev.

It is seen that only two $T=0$ states from the configuration p^{-2} are expected between about 3 and 7 Mev in N^{14}, whereas more states have been observed in this region.[11] The appearance of extraneous levels to be ascribed to excited configurations such as $p_{\frac{1}{2}}^{-3}d$ and $p_{\frac{1}{2}}^{-3}s$ beginning at about 4 or 5 Mev is not inconsistent with, for example, the levels in N^{15} beginning at 5.3 Mev which also appear to arise from excited configurations. It does, however, complicate the experimental verification of the present analysis of the ground configuration, and means that this is at best only part of the story. The ground-state nuclear spin $I=1$ of N^{14} is of course correctly given, being the same in both extremes. The nuclear spins of the first two states of C^{14} are predicted to be 0 and 2, respectively, as is familiar in other even-even nuclei. All these are essentially the results of (jj) coupling; there have been no cross-overs but the spacing of the levels has been markedly affected by the transition into intermediate coupling.

The puzzle of the long life[12] of C^{14} is not satisfactorily solved by this interpretation, since the beta-transition remains $I=0$ to $I=1$, with no parity change. The fact that one wave function arises from a quadratic secular equation and the other from a cubic means that there is ample opportunity for cancellation to occur in the matrix element, but an anomalous factor 10^{-2} to 10^{-3} is needed in the matrix element; and it is not really satisfactory to be forced to attribute so small a factor to fortuitous cancellation.

For $A=6$, the possibility of any successful comparison of the curves in Fig. 1 with experiment, as attempted in Fig. 2(a), depends on making a/K so small as to give the ground-state nuclear spin $I=1$, characteristic of (LS) coupling, rather than the value $I=3$, given by (jj) coupling[5] for both Li^6 and B^{10} and observed in B^{10}. Matching the lowest three clearly identifiable states as before, in this case the first two states with $T=0$ and the first state[13] with $T=1$, then gives the surprisingly low value $a/K=1.3$, rather near (LS) coupling. The only prediction that may be compared with presently available experimental results is then that the first excited state of He^6 should lie about 2.5 Mev above its ground state rather than 2 Mev as observed. This interpretation requires the implausible values of the parameters $K=-1.23$ Mev, $a=-1.6$ Mev. If we should compare with N^{14} alone, so small a value of a might be attributed to a larger size of the loosely-bound p-shell in Li^6, but this assumption would lead us to expect that K also should be smaller in Li^6 than in N^{14},

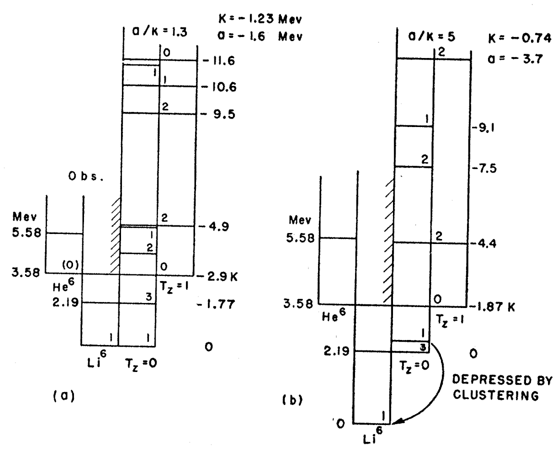

FIG. 2. Alternative interpretations of the energy spectrum of the polyad Py^6 consisting of Li^6, He^6, etc. In scheme (b) it is assumed that the ground state, which has the same (I,T) as the ground state of the alpha-model, is depressed by nascent alpha + deuteron clustering.

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[11] The N^{14} states shown in the insert at 3.9, 5.1, 5.7, and 6.45 Mev are taken from the recent revision mentioned in reference 4, the result mainly of previous work with $N^{14}(p,p')$ and $C^{13}(d,n)$. Very recent work with the latter reaction [R. E. Benenson, Phys. Rev. **87**, 207 (1952), and private communication] suggests additional levels at 3.7, 4.8 (odd), 7.05, (7.5, odd), and 7.7 Mev, and perhaps others, and that the 5.7-Mev level is even, with $l_n=2$ as would be required for the (2,0) state with which it roughly agrees in energy. The configurations $p_{\frac{1}{2}}^{-3}d$ and $p_{\frac{1}{2}}^{-3}s$ give states 3^-, 2^-, 1^-, 0^-, 2^-, 1^- both for $T=0$ and $T=1$. (Still higher odd states arise from $p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-2}d$, etc.) Since no breaking of four groups is involved in the transition $T=0$ to $T=1$ among these states, this additional excitation would be expected to have the same order of magnitude as the 2.32 Mev in $N^{14}\rightarrow C^{14}$ (or 3.56 and 1.74 Mev for the corresponding excitations in the polyads Py^6 and Py^{10}, which contrast strongly with 12.5 to 16.7 Mev in Py^8, Py^{12} and Py^{16}). The N^{14} state at 3.9 Mev could be the (1,0) state of p^{-2}, but if there is really a state with $T=0$ also at 3.7 Mev, either it or the one at 3.9 Mev arises from an excited configuration (with odd parity) and it may seem slightly surprising that it does not also give a $T=1$ state below 8.4 Mev. Between about 3 and 7 Mev in N^{14} the two even levels from p^{-2} are expected and several of the odd levels mentioned seem to appear. In addition to these, there might appear another even state O^+ near 6 Mev corresponding to the first excited state of O^{16}. It has been suggested by M. G. Mayer (private communication), as an alternative to the alpha-model explanation of D. M. Dennison, that the O^+ state at 6.05 Mev in O^{16} might be interpreted as $p_{\frac{1}{2}}^{-2}d_{\frac{5}{2}}^{2}$ arising from two-nucleon excitation and might lie low because of the large $d_{\frac{5}{2}}^{2}$ pairing energy (as does also the probable $P_{\frac{1}{2}}$ at 3.03 Mev in O^{17}). A O^+ state in N^{14} could similarly arise from $p^{-4}d^2$, and the other even states from this configuration would presumably lie considerably higher.

[12] E. Gerjuoy, Phys. Rev. **81**, 62 (1951); S. D. Warshaw, Phys. Rev. **80**, 111 (1950). Experimentally, however, the ground state of N^{14} is even relative to N^{15} assumed odd: Gibson and Thomas, quoted by S. T. Butler, Proc. Roy. Soc. (London) **208A**, 559 (1951); and both C^{14} and N^{14} ground states are even relative to C^{13} assumed odd, so one cannot assume opposite parity for them: reference 11 and D. A. Bromley and L. M. Goldman, Phys. Rev. **86**, 790 (1952).

[13] R. B. Day and R. L. Walker, Phys. Rev. **85**, 582 (1952).

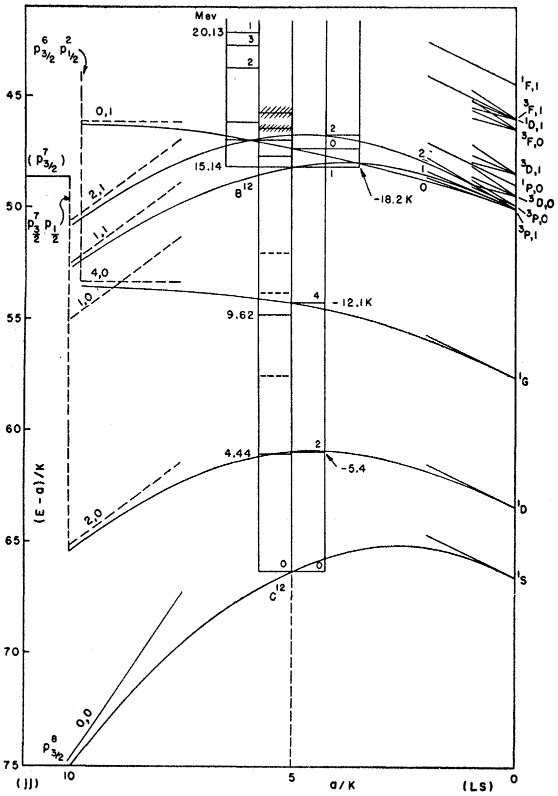

FIG. 3. Approximate intermediate coupling transition for the configuration p^8, and comparison with the observed levels in Py^{12}. The sign of a refers to a nucleon, not a "hole."

whereas it is in this interpretation about 50 percent larger. The comparison of the small value of a for Li^6 with the values dubiously derived from the virtual states of He^5+Li^5 is equivocal: it agrees with the value $(-3a/2)\approx 2.5$ Mev derived from reaction data,[14] but not with $(-3a/2)\approx 5$ Mev inferred from scattering data;[15] and the expected effect of the instability of He^5 on a has not been calculated.

As another more plausible but more complex interpretation of the situation one may assume that the values of a, K, and a/K do not vary so drastically across the p-shell, and thus that Li^6 is not so near (LS) coupling, but that the reason for the appearance of $I=1$ is that this is characteristic of the ground state of the alpha-model: that is, we may assume that the central model in intermediate coupling gives a good approximation to most states of most of the p-shell nuclei, but that under a certain condition there may be strong influence of alpha-clustering on the energy and other characteristics of a state. The (necessary but perhaps not sufficient) condition is plausibly that the internal binding energy of the constituent parts (alpha and deuteron) is large compared to their mutual binding. The internal binding of the deuteron in its ground state is 2.2 Mev, compared to 1.56-Mev mutual binding with the alpha. Correspondingly we assume that the $(I,T)=(1,0)$ state is considerably depressed by "resonating" with this alpha-model configuration, or, we might say, by nascent alpha-clustering. The excited state of the deuteron is unbound, that is, there is no internal binding, so clustering is not expected in the (0,1) state. With this assumption, arbitrarily taking the value $a/K=5$, for example, we have the comparison with experiment shown in Fig. 2(b), which appears to be satisfactory as far as it goes. On the basis of this sort of interpretation it may be predicted that the states (2,0) and (1,0) of Li^6 will be found considerably higher than (2,1) and a search for them should provide an interesting test.

INTERMEDIATE COUPLING FOR THE CONFIGURATION p^8

Some of the energy spectra encountered in the p-shell differ so drastically from one another that a very rough indication of the intermediate coupling schemes can contribute very significantly to an understanding of the general nature of their differences. As an example of a polyad with a spectrum very different from that of the polyad Py^{14}, we select Py^{12} with the ground configuration p^8. It has so many states that we cannot hope to give an exact treatment of the p-shell intermediate coupling scheme as we have done for p^2 and p^{-2}.

A practical procedure is suggested by the method[9] used in the above derivation of the quadratic secular equations. We list as many of the low states as we may need to consider in (LS) coupling, determine the lowest (jj) configuration to which each must correspond, plot as much information as we may have either exactly or approximately about the elevations and slopes of the (LS) and (jj) asymptotes, and then sketch in curves with the proper asymptotic behavior and with very gradual curvature, except where two curves of the same (I,T) come fairly close together and "repel" one another strongly.

The result of this procedure applied to p^8 is shown in Fig. 3. The spacings of the multiplets[6] on the (LS) coupling side are given for the special case (5). The multiplet splittings arise from an expression $A(\mathbf{L}\cdot\mathbf{S})$, where A is a constant times a, depending on details of the wave functions which have not been calculated in this many-nucleon configuration. As a guide by which to guess at reasonable values of A we may recall (a) that in the ground state of Li^7, $A=a/3$, corresponding intuitively to the fact that only one nucleon contributes to S while three participate equally in L so that $\cos(lL)=\frac{1}{3}$, and (b) that the hole theory leads to the expectation that all multiplet splittings reduce to zero as in surveying the elements of the p-shell we pass through B^{10} at the middle of the shell. For the nuclei lighter than B^{10}, A is in general a positive constant

[14] W. T. Leland and H. M. Agnew, Phys. Rev. **82**, 559 (1951); E. W. Titterton and T. A. Brinkley, Proc. Phys. Soc. (London) **64***A*, 212 (1951).
[15] R. K. Adair, Phys. Rev. **83**, 873 (1951); C. L. Critchfield and D. C. Dodder, Phys. Rev. **76**, 602 (1949).

times a, whereas in the heavier half of the p-shell, the constant is generally negative, though there may in complicated cases be exceptions to this rule. For each of the multiplets with which we are concerned in Py^{12}, we accordingly assume that the constant relating A to a is negative and relatively small. We thus obtain the set of (LS) asymptotes shown on the right side of Fig. 3.

In Fig. 3, the ordinate is chosen as $(E-a)/K$, which gives a slope to the singlet asymptotes, in order to keep the figure compact and still leave one set of asymptotes [that for the (jj) configuration $p_{\frac{3}{2}}{}^6 p_{\frac{1}{2}}{}^2$] horizontal. In (LS) coupling, the large spacing between the low singlets and the dense clustering thereafter are noteworthy for the contrast to Py^{14}. Aside from the ground state, the only bits of information used about the (jj) asymptotes are their slopes, and to indicate this they are drawn with broken lines. The spacings between them are calculable [on the basis of assumed interactions such as (5)] but have not been calculated. It is interesting to note that the slopes alone are helpful in inferring the general nature of the spectrum, and by doing so we see for which (jj) states and configurations it may be most significant to calculate the energies. In Fig. 3 the lines for the (jj) asymptotes for the excited states were drawn with fixed slope at such a height as to make possible gentle curves from the known (LS) asymptotes; and the comparison with experiment shown superposed in the middle of the figure was subsequently undertaken (only for the ground state is the height of the (jj) asymptote analytically known, being taken from reference 5). The value $a/K=-5$ is chosen for the comparison with experiment as a rough empirical compromise. Values smaller in absolute magnitude than this would bring the first two excited states of B^{12} closer to its ground state, larger values would raise the (2,0) and (4,0) states higher above the dependably observed 4.44-Mev and 9.62-Mev states of C^{12}, the energy scale having been determined by fitting the ground states of C^{12} and B^{12}. The decision to indicate the other possible, but less dependably observed, excited states of C^{12} with broken lines was not made as a result of this study, but was taken from charts independently prepared and kindly supplied in advance of publication by Professor T. Lauritsen.[4] This comparison does not suggest that those states do not exist, but that if they exist they have odd parity arising from excitation of one of the p-nucleons to the d (or s) shell; and there may well be a connection between the odd parity and the furtive ways both of these states and of those only recently observed[11] in N^{14}. The first odd state in N^{15} apparently lies at 5.3 Mev; it should be higher in C^{12} by roughly the single-nucleon doublet splitting, which makes the uncertain 7.3-Mev state seem a little low for the first odd state, but this rough sort of comparison is unreliable because energies expressible in K are also involved.[16]

CONCLUSION

From the examples here given it begins to be clear that much of the complexity of the relations between energy spectra of the various light nuclei may be accounted for in terms of intermediate coupling. The variety of spacings of the low states provided by the symmetry properties in (LS) coupling accounts for the way in which some nuclei, though in intermediate coupling, resemble (LS) coupling more than others.

The four-structure of the stability curve has long seemed to be accounted for in a general way by these symmetry properties in (LS) coupling,[6] or alternatively by the alpha-model.[17] It has recently been shown that the four-structure does not show up so strongly in a similar treatment of (jj) coupling,[5] where the possibility of attaining high symmetry is suppressed by the demand for the quantum numbers j. If one takes into account not only the specific nuclear interactions but also the spin-orbit interaction (1), one obtains in (jj) coupling a sharp dip in the stability curve at C^{12} because of the negative-energy contribution $a/2$ per nucleon up to the closing of the $p_{\frac{3}{2}}$ sub-shell at $A=12$ and the positive-energy contribution a per nucleon beyond this point. It is apparent from Fig. 3 that in the intermediate coupling there indicated, both the symmetry of (LS) coupling and the spin-orbit contribution to the stability of (jj) coupling play important roles in contributing to the exceptional stability of C^{12}. Thus the interpretation of the p-shell nuclei in terms of intermediate coupling somewhat complicates the question of the four structure, but seems to leave a qualitative explanation at least of the dips in the binding-energy curve at $A=12$ and 16. In the neighborhood of $A=8$ it still appears likely that nascent alpha-clustering plays an important role, and there is no very good evidence for intermediate coupling in that region, as in the examples of Li^6 discussed above and Li^7 for which one possible type[18] of influence of the alpha-model was previously suggested.[3] However, it might instead be that the full complexity of the Feingold-Wigner mechanism of spin-orbit coupling[19] applies in Li, making its behavior anomalous, and is somehow averaged out to approximate (1) in slightly heavier nuclei.

[16] Compare the conclusions of L. J. Koester, Jr., Phys. Rev. **85**, 643 (1952) where in estimates of $(3a/2)$ insufficient allowance is made for energies expressible in K of the general nature of "pairing energies."

[17] J. A. Wheeler, Phys. Rev. **52**, 1083 (1937); L. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938); E. Teller and J. A. Wheeler, Phys. Rev. **53**, 778 (1938).

[18] Another possibility for Li^7 is that the states arising in intermediate coupling from the ^2P are strongly intermixed with alpha-model states, which also include essentially a low and relatively stable ^2P, while the states arising from the ^2F remain unmodified by clustering (with 7/2 at 4.8 Mev and 5/2 at 7.4 Mev). The recently reported broad state at 6.4 Mev [S. Bashkin and H. T. Richards, Phys. Rev. **84**, 1124 (1951)] could then consist largely of the $^2\Sigma_g$ of the alpha-model.

[19] A. M. Feingold and E. P. Wigner, Phys. Rev. **79**, 221(A) (1950), and valued private communications.