For small n this reduces to the Born approximation result (1), but for large n it has the value unity for all angles. We have had occasion to evaluate this numerically for the case appropriate to 14-Mev deuterons on Al, for which $n=0.8$. We find, for example, at 140°, $d\sigma/d\sigma_R=0.67$ while the Born approximation result would be 0.11.

To summarize we would predict that, for deuteron energies below the barrier, the ratio $d\sigma/d\sigma_R$ would show for the cases where n is small a steady decrease as we move towards large angles. As n increases this decrease would become less marked, and finally for $n\gg1$ we would have simply Rutherford scattering. Not much data on Coulomb scattering seem to be available, though there is evidence that when $n\gg1$ the Coulomb scattering is simply Rutherford.[3]

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[1] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford Press, London, 1949) second edition, chap. 6.
[2] A. Sommerfeld, *Wellenmechanik* (Frederic Ungar Publishing Company, New York, 1947), p. 502.
[3] The scattering of 4-Mev deuterons on Au has been measured by L. M. Goldman and is found to be Rutherford, private communication.

The Born Approximation Theory of (d,p) and (d,n) Reactions*

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Butler[1] has given a theory for (d,p) and (d,n) reactions which has had remarkable success in explaining experimental results. The purpose of this note is to demonstrate that in all its essential features Butler's theory is equivalent to a Born approximation calculation. It is indeed clear that it should be, for Butler's theory everywhere ignores the reaction of the elastically and inelastically scattered particles as well as the scattering of the particle that is not captured; also his results are given in terms of the obvious momentum transfers of the problem. The Born approximation theory has already been given by Bhatia *et al.*,[2] but in their paper the connection with Butler's theory has not been made quite clear.

We write the case for (d,p) reactions. To avoid unessential complications we assume at first that the initial nucleus has spin 0 and that the neutron is captured under the influence of a potential $V(r_n)$ (assumed central) into the one-particle state with space dependence $\zeta_l^m(\mathbf{r})=R_l(r)Y_l^m$ with binding energy $\epsilon=\hbar^2/2M$. Let the incident deuteron have wave vector $\mathbf{K}$ and the final proton have wave vector $\mathbf{k}$. Then ($\hbar=1$)

$$\frac{d\sigma}{d\omega}=\frac{M^2}{2\pi^2}\frac{k}{K}\frac{1}{3}\frac{j+\frac{1}{2}}{2l+1}\sum_{m_d,m_p,m_n,m}|\langle V\rangle|^2, \tag{1}$$

where $\langle V\rangle$ is the matrix element between the initial and final states and m_d, m_p, and m_n are the spin magnetic quantum numbers for the deuteron, proton, and neutron; m is the orbital magnetic quantum number for the captured neutron, and j is the final nuclear spin. The summation over spin quantum numbers gives simply a factor 3. The probability amplitude for finding a proton momentum $\mathbf{k}$ in the initial deuteron is

$$P(\mathbf{k}-\tfrac{1}{2}\mathbf{K})=\int\psi_d(\mathbf{s})\exp[i(\mathbf{k}-\tfrac{1}{2}\mathbf{K})\cdot\mathbf{s}]d\mathbf{s}$$
$$=4\pi N\left[\frac{1}{\alpha^2+(\mathbf{k}-\frac{1}{2}\mathbf{K})^2}-\frac{1}{\beta^2+(\mathbf{k}-\frac{1}{2}\mathbf{K})^2}\right], \tag{2}$$

where the last step follows on using for ψ_d the usual Hulthén wave function with constants N, α, β.

The momentum carried by the captured neutron is $\mathbf{q}=\mathbf{K}-\mathbf{k}$. The probability amplitude for capturing this neutron into the state $\zeta_l^m(\mathbf{r})$ is

$$\int\zeta_l^m(\mathbf{r})V(r)e^{i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}=\delta_{m0}[4\pi(2l+1)]^{\frac{1}{2}}i^l\int_0^\infty R_l(r)V(r)j_l(qr)r^2dr$$
$$=-\delta_{m0}[4\pi(2l+1)]^{\frac{1}{2}}\frac{q^2+l^2}{2M}\int_0^\infty R_l(r)j_l(qr)r^2dr. \tag{3}$$

The second form follows by expanding $\exp(i\mathbf{q}\cdot\mathbf{r})$, and we get a nonvanishing result only for $m=0$ by taking the axis of quantization along $\mathbf{q}$. The third form follows by eliminating $V(r)$ by using the Schrödinger equation for $R_l(r)$. The matrix element is now given by the product of factors (2) and (3).

Bhatia *et al.*[2] approximate the radial integral by

$$\int R_l(r)V(r)j_l(qr)r^2dr=j_l(qR)\int R_l(r)V(r)r^2dr, \tag{4}$$

and now the proton angular distribution is given by $|P(\mathbf{k}-\frac{1}{2}\mathbf{K})\times j_l(qR)|^2$. The difficulty here is that there is no reason why R defined by (4) should be independent of q and therefore of angle.

To proceed differently, we use the third form of (3). If we assume, as Butler implicitly does, that we may neglect the contribution to the overlap integral from $r\leqslant r_0$ (where r_0 is greater than the nuclear radius) we can, by using the equations for $R_l(r)$ and $j_l(qr)$ along with Green's theorem, write

$$(q^2+l^2)\int R_l(r)j_l(qr)r^2dr$$
$$=R_l(r_0)r_0^2\left[\frac{\partial j_l(qr)}{\partial r}-\frac{1}{R_l(r)}\frac{\partial R_l(r)}{\partial r}j_l(qr)\right]_{r_0}, \tag{5}$$

where

$$\frac{1}{R_l(r)}\frac{\partial R_l(r)}{\partial r}=\frac{1}{h_l^{(1)}(ilr)}\frac{\partial h_l^{(1)}(ilr)}{\partial r} \tag{6}$$

is a number defined by the l value, binding energy and r_0. Using this, we have precisely Butler's form for the proton angular distribution. The magnitude is given here in terms of the value of the captured neutron wave function on the surface r_0.

If we do not care to omit the contribution to the overlap integral for $r\leqslant r_0$, we can define the quantity

$$\bar{V}(r_0)=-\int_0^{r_0}j_l(qr)R_l(r)V(r)r^2dr\Big/\int_0^{r_0}j_l(qr)R_l(r)r^2dr, \tag{7}$$

and then it is trivial to show that

$$\int_0^\infty R_l(r)j_l(qr)r^2dr=\left(1-\frac{q^2+l^2}{2M\bar{V}}\right)^{-1}\int_{r_0}^\infty R_l(r)j_l(qr)r^2dr. \tag{8}$$

In this case, the proton cross section contains also the angularly dependent factor $[1-(q^2+l^2)/2M\bar{V}]^{-2}$. It should be emphasized that the cross section is invariant to the choice of r_0 provided only that $r_0\geqslant r_{\text{nuclear}}$, but does of course depend on the value of the neutron potential.

Finally, we emphasize that application of the Born approximation in the low energy region is a very crude procedure. For example, the effects of scattering of the proton and deuteron are not at all small. We hope to report later some calculations of these effects.

If we take r_0 to be the nuclear radius and make the reasonable assumption that the neutron potential inside the nucleus is constant, then $\bar{V}$ is simply the potential depth. The extra factor will not disturb the most striking feature of Butler's angular distribution, namely, the angular position of the first maximum. It has a singularity at $q=(2M\bar{V}-l^2)^{\frac{1}{2}}$, but this simply removes one of the zeros of Butler's distribution. These zeros, in fact, occur when the neutron momentum transfer q equals an average wave number which a neutron could have when bound with binding energy $\epsilon=\hbar^2/2M$ and orbital angular momentum l in a well of radius r_0. The extra factor above removes that zero which corresponds to the actual inside wave number of the captured neutron. Thus, for example, the formal Born approximation theory for a $2p$ state would not have the first zero of Butler's theory.

To treat the more general case where we impose no restrictions on the nature of the nuclear wave functions, we can define an "effective neutron wave function" by

$$\zeta_{l'I'^{M'M}}(\mathbf{r}) = \int \Omega_{I'^{M'}}(\mathbf{x}, \mathbf{r})^* \Omega_{I^M}(\mathbf{x}) d\mathbf{x}, \quad (9)$$

where Ω's are the initial and final nuclear wave functions and $\mathbf{x} \equiv$ coordinates of all initial nuclear particles. This can be expanded into a series containing all the l values compatible with the nuclear spins and parities, and the m summation can be easily performed. The only important point (as is clear from references 1 and 2) is that there is no interference between different l's.

* Work performed under the auspices of the AEC.
[1] S. T. Butler, Proc. Roy. Soc. (London) **208**, 559 (1951).
[2] Bhatia, Huang, Huby, and Newns, Phil. Mag. **43**, 485 (1952).

Half-Lives of Eu^{152}, Eu^{154}, and Sm^{151}

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THE half-lives of Eu^{152}, Eu^{154}, and Sm^{151} have been measured directly by mass spectrometric methods. In each case, the half-life was determined by the change with time of the isotopic constitution of samples containing an appreciable percentage of the nuclide under investigation. For Eu^{152} and Eu^{154} the sample used for the determination of the half-lives was a normal europium sample irradiated in a pile. Data on this sample have been used previously for an indirect determination of the half-lives of Eu^{152} and Eu^{154}.[1]

The half-life of Sm^{151} was determined by the change in isotopic abundance of Sm^{151} in a sample of purified fission-produced samarium, used for a previous indirect determination of the half-life by Inghram, Hayden, and Hess.[2]

In every case, isotopic analysis was made on a 12-inch radius of curvature 60°-deflection mass spectrometer.[3] The europium sample was purified from its daughters by elution with pH 3.06 ammonium citrate from an ion exchange column filled with Dowex 50 resin. No further purification was made on the samarium. The data pertinent to the calculations and the results are given in Table I.

The errors quoted for the half-lives consider only the mean deviation of the isotopic determinations.

The value for the half-life of Sm^{151} is in fair agreement with the 122-year value determined indirectly by Inghram, Hayden, and Hess, but disagrees markedly with the 10^3-year half-life obtained by Marinsky.[4]

The serious discrepancy between the half-lives determined for Eu^{152} and Eu^{154} and the half-lives obtained by Hayden, Reynolds, and Inghram may be caused by an experimental error in the determination of the absolute amount of the gadolinium present after bombardment. The presence of a short-lived isomer of Eu^{154} would otherwise be necessary to explain the error in half-life of Eu^{154}. No such isomer is known.

The values for the branching ratios of the 9.2-hour Eu^{152}, the 13-year Eu^{152}, and the proportion of Eu^{151} capture that goes to each of the two Eu^{152} isomers, all measured indirectly by Hayden, Reynolds, and Inghram, are almost certainly in error since they involve the above determined half-lives. No statement about the correct values can be made without a repetition of their experiment.

TABLE I. Decay data.

	% decay	Decay period (yr)	Half-life (yr)
Eu^{152}	16.7 ± 2.4	3.4	13 ± 2
Eu^{154}	13.9 ± 3.3	3.4	16 ± 4
Sm^{151}	3.6 ± 0.9	3.8	73^{+25}_{-14}

[1] Hayden, Reynolds, and Inghram, Phys. Rev. **75**, 1500 (1949).
[2] Inghram, Hayden, and Hess, Phys. Rev. **79**, 271 (1950).
[3] Inghram, Hess, and Hayden (to be published).
[4] Way, Fano, Scott, and Thew, *Nuclear Data*, National Bureau of Standards, Circular 499 (1950).

Fast Coincidences with Čerenkov Counters

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THE duration of the light flash from a Čerenkov radiator is expected to be much shorter than that from a scintillation crystal. This letter reports an experiment performed at the National Bureau of Standards to study the resolving time limitations of Čerenkov counters[1] in preparation for coincidence experiments with high energy particles. The light was produced in Lucite radiators of two different types (A and B of Fig. 1), by secondary

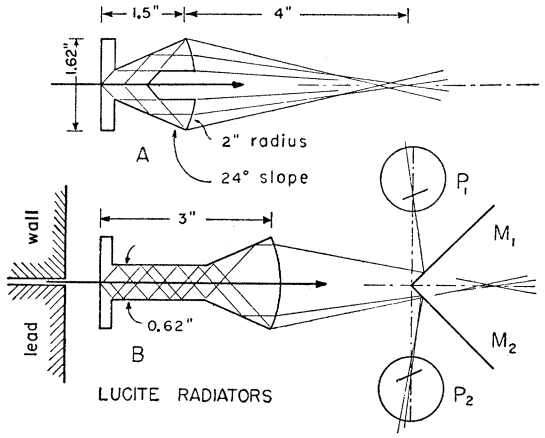

FIG. 1. Čerenkov radiator designs. Heavy arrow indicates direction of high energy electron. Thin lines show paths of optical radiation.

electrons in the x-ray beam of the 50-Mev betatron. The light was brought to an approximate focus by refraction at the spherical end surfaces of the radiators. At the focal point the duration of the light flash from radiator type B is expected to be about 1.5×10^{-10} second due to unequal light collection times. That from type A is expected to be much shorter as the optical path differences are just compensated by the transit time of the electron through the Lucite. However, imperfect focusing can produce path differences of about 3 millimeters and a pulse duration of about 10^{-11} second. The light beam is divided by the mirrors M_1 and M_2 so as to stimulate two 1P21 photomultipliers P_1 and P_2 in coincidence. One can expect to liberate about 6 photoelectrons from each photo-cathode per centimeter penetration of the electron.[2] The phototubes were operated at 1400 volts dc potential. The current pulses from the individual tubes were amplified by 200 megacycle band width distributed amplifiers (Hewlett Packard Model 460A) and brought out of the betatron room through 40 meters of *RG* 59/*U* cable. The coincidences vanish if a piece of black cloth is placed between the Lucite and the mirrors. Thus they are caused by optical radiation coming from the Lucite. The small solid angles (10^{-3} of 4π) subtended by the photo-cathodes inhibit detection of any weak isotropic radiation from the Lucite.