Selection Rules Imposed by Charge Conjugation and Charge Symmetry

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It is found that the invariance with respect to charge conjugation and charge symmetry combined leads to selection rules, additional to those following from covariance and gauge invariance arguments. Besides rigorous selection rules which hold for certain processes involving neutral particles (mesons and photons) only, there exist weak selection rules according to which certain boson-boson transitions are less probable by a factor $e^2/\hbar c$ than the corresponding transitions in which an additional photon is emitted. The rules are stated in Sec. III, and some examples are given.

I. INTRODUCTION

THE existence of at least two bosons heavier than the π-meson ("heavy bosons") seems to be reasonably well established by recent cosmic-ray observations. They are: the τ-meson, a particle with unit charge decaying into three charged π-mesons;[1] and the "light V_0" particle, a neutral object presumably undergoing decay into a pair of oppositely charged π's.[2]

In the present unsatisfactory state of the theory of particles and fields, the safest way of approach to the study of the interrelations of these and other particles is surely to ask how much we can learn about, say, the decay of heavy bosons with as little use as is possible of field theoretical details. This means that in first instance one tries to find some order among various decay schemes by means of the sole use of conservation laws (of energy, momentum, angular momentum, and parity) and of additional invariance considerations, e.g., gauge invariance.

In quantum electrodynamics we have a further symmetry property at our disposal, namely, the one between positive and negative electric charges. It has been shown by Furry[3] that this symmetry can be expressed as an invariance property of the theory with respect to charge conjugation, a concept first introduced by Kramers.[4] Furry's theorem is very useful in two (obviously related) respects: first, it establishes certain absolute selection rules. Secondly, it implies that, in the calculation in some given order of perturbation theory of a matrix element for some particular process, certain "graphs" give no contribution. These are the well-known closed loops consisting of an odd number of electron lines (or charged boson lines).

In their analysis of the τ-meson decay, Fukuda and Miyamoto[5] found that certain graphs in the expansion of the S-matrix in meson theory give vanishing contributions as a result of the application of theorems, rather analogous to the Furry theorem, to a closed nucleon loop which represents the transitory nucleon pairs. Their results can be expressed as follows: Consider a graph consisting exclusively of a closed nucleon loop and of external boson lines.[6] Define the following three numbers characteristic of the graph:

$n(v)$: The total number of vector couplings operative in the vertices of the loop, irrespective of whether they involve scalar or vector mesons. (Note: if one kind of meson is coupled to nucleons in more than one way, like, for example, scalar mesons with scalar and vector coupling, it is here to be understood that one uses one coupling per vertex in one graph). $n(v)$ also comprises all electromagnetic vector couplings of the proton that may occur if external photons are involved.

$n(t)$: The total number of tensor couplings, irrespective of whether they refer to the tensor coupling of vector mesons or to the pseudo (=dual) tensor coupling of pseudovector mesons. $n(t)$ also comprises whatever electromagnetic couplings of nucleons of the Pauli-type that may occur.

$n(\tau_{(3)})$: The total number of neutral meson couplings involving $\tau_{(3)}$, the z component of the isotopic spin vector. (Note: if a certain neutral meson interaction is proportional to $\alpha+\beta\tau_{(3)}$, one has to take for the present purposes either the "α-part" or the "β-part" per vertex in one graph.)

Then the following theorems hold:

(I) If none of the external lines are photons, and if some of them are charged bosons, the graph gives no contribution if $n(v)+n(t)+n(\tau_{(3)})=\text{odd}$.

(II) If none of the external lines are photons or charged mesons, the graph gives no contribution if either $n(\tau_{(3)})=\text{odd}$, or, for even $n(\tau_{(3)})$, $n(v)+n(t)=\text{odd}$.

(III) If the external lines are exclusively photons and neutral mesons, the graph gives no contribution if $n(v)+n(t)=\text{odd}$.

The physical contents of these rules is of course intimately connected with the fact that nucleons with v or t coupling to an external field are subjected to a force equal and opposite to the one for antinucleons,

[1] C. F. Powell *et al.*, Phil. Mag. **42**, 1040 (1951).

[2] See references 2–11 in A. Pais, Phys. Rev. **86**, 663 (1952), for a list of the experimental papers on V-decay. More recently, another particle decaying into a pair of charged π-mesons has tentatively been identified by Danysz, Lock, and Yekutieli, Nature **169**, 364 (1952).

[3] W. H. Furry, Phys. Rev. **51**, 125 (1937).

[4] H. A. Kramers, Proc. K. Ned. Acad. Wet. **40**, 814 (1937).

[5] H. Fukuda and Y. Miyamoto, Progr. Theoret. Phys. **4**, 389 (1950). See also K. Nishijama, Progr. Theoret. Phys. **6**, 614 (1951) and A. Pais, reference 2, footnote 26.

[6] The usual couplings are considered in which one meson occurs per vertex.

while protons are acted on by a force opposite to that for neutrons in the case of a $\tau_{(3)}$ coupling. More formally, these theorems may be considered as consequences of two distinct symmetry properties of the theory:

(a) The symmetry with respect to an interchange of the nucleon and the antinucleon with all their attributes like electric charge and mesic coupling constants. This symmetry can be expressed by means of invariance for general charge conjugation (see Sec. III).

(b) The symmetry, apart from electromagnetic effects, with respect to the interchange of protons and neutrons (charge symmetry). This symmetry[7] can be expressed as the invariance with respect to a certain transformation in isotopic space.

It is the aim of this note to answer the question: under what circumstances of coupling and for what kinds of processes do the rules (I)–(III) represent absolute selection rules? Thus, we will be interested in a rigorous forbiddenness rather than in the vanishing of a matrix element to a given order, which in mesonic phenomena seems to be not a very useful statement anyhow. Following Furry's original procedure[3] we will give direct proofs based on the two mentioned invariance properties. Hence, the arguments will not explicitly refer to a power series expansion of the S-matrix.

In the next section the necessary formal properties of the charge conjugation and charge symmetry transformations are given. In Sec. III the selection rules are stated and a number of examples for specific processes is given. The main practical interest at present lies of course in the heavy boson decays.

II. FORMAL CONSIDERATIONS

A. Charge Conjugation

1. Fermions

Charge conjugation is defined independently of the representation chosen for the γ_μ in the Dirac equation[8]

$$[\gamma_\mu \partial/\partial x_\mu + m]\psi = 0, \tag{1}$$

but to fix ideas we take the γ_μ to be Hermitian. The adjoined equation to Eq. (1) is

$$(\partial/\partial x_\mu)\bar{\psi}\gamma_\mu - m\bar{\psi} = 0, \quad \bar{\psi} = \psi^\dagger \gamma_4. \tag{2}$$

The charge conjugate wave functions are obtained by adjoining in a Lorentz invariant way solutions of Eq. (1) to those of Eq. (2). One uses the following well-known procedure:[9] Define ψ^C and $\bar{\psi}^C$ by

$$\psi^C = C\bar{\psi}^T, \quad \bar{\psi}^C = (C^{-1}\psi)^T, \tag{3}$$

respectively, where the matrix C satisfies

$$C^{-1}\gamma_\mu C = -\gamma_\mu{}^T. \tag{4}$$

The $\gamma_\mu{}^T$ satisfy the same anticommutation relations as do the γ_μ, so C exists. As the γ_μ are Hermitian, C can be taken to be unitary

$$C^\dagger C = 1. \tag{5}$$

By comparing Eq. (4) with its transposed one finds that $C^T C^{-1}$ commutes with all γ_μ. Hence,

$$C^T C^{-1} = \beta \cdot 1,$$

where β is a number. β is independent of the choice of representation. Indeed, a change of representation of the γ_μ is expressed by

$$\gamma_\mu' = S^{-1}\gamma_\mu S, \tag{6}$$

and the corresponding transformation of C is (apart from an irrelevant phase factor)

$$C' = SCS^T, \tag{7}$$

whence $C'^T C'^{-1} = C^T C^{-1}$. Thus, β can be determined by explicitly constructing C in a special representation and one finds $\beta = -1$:

$$C^T C^{-1} = -1. \tag{8}$$

Considering Eq. (7) as the transformation of C under the proper Lorentz group (excluding reflections), for which S is uniquely determined, one has $C' = C$. There is a well-known arbitrariness in the choice of S for space-, time-, and space-time reflections. In the case of space reflections, one has, according to Eq. (7), $C' = C$, provided one chooses $S = i\gamma_4$.

If time reflections are involved, and one requires that, with $\psi' = S\psi$, also $\psi^{C'} = S\psi^C$ one has to replace Eq. (7) by $C' = -SCS^T$, due to $\bar{\psi}' = -S\bar{\psi}$. One can then again choose S such that $C' = C$, so that C may be considered as a scalar for the full Lorentz group. One has $S = \gamma_5\gamma_4$ (time-reflection); $S = i\gamma_5$ (space-time reflection).[10] However, it is not a physical requirement that C be a scalar; in particular the considerations of Sec. III are independent of the behavior of C under reflections.

Now let $\bar{\psi}\Omega\psi$ generally represent any of the five well-known covariant structures that can be formed by taking for Ω appropriate functions of the γ_μ; in obvious shorthand we denote them by s, v, t, pv, ps. According to Eqs. (3) and (8):

$$\bar{\psi}^C \Omega \psi^C = -\psi^T C^{-1}\Omega C \bar{\psi}^T.$$

Hence, using Eq. (4) and the anticommutation properties of ψ and $\bar{\psi}$:

$$\begin{aligned} \bar{\psi}^C\Omega\psi^C &= +\bar{\psi}\Omega\psi \text{ in the cases: } s,\ pv,\ ps, \\ &= -\bar{\psi}\Omega\psi \text{ in the cases: } v,\ t. \end{aligned} \tag{9}$$

(t denotes the tensor and the pseudotensor case as well.)[11] In Eq. (9) an infinite C-number (which can be

[7] The proton-neutron mass difference is neglected. The symmetry considered here should not be confused with charge independence, i.e., invariance with respect to all rotations in isotopic space.

[8] $\hbar = c = 1$; $x_\mu = x, y, z, it$.

[9] W. Pauli, Ann. Inst. H. Poincaré 6, 109 (1936); J. Schwinger, Phys. Rev. **74**, 1439 (1948). T denotes transposition.

[10] This choice is due to G. Racah, Nuovo cimento **14**, 322 (1937). For the relevance of the choices of S in the case of more than one spinor field see C. N. Yang and J. Tiomno, Phys. Rev. **79**, 495 (1950).

[11] It has been suggested by Okayama [Phys. Rev. **75**, 308 (1949)] to use the difference in properties of s, pv, ps on the one

disposed of elegantly) has been dropped. The implications of the connections (9) for selection rules were first noted by Furry[12] in his original paper. They are occasionally rediscovered.[13]

Now let $S(x_\mu)$ represent any one of the S-functions representative for a spinor field:

$$S(x)=[\gamma_\mu\partial/\partial x_\mu-m]\Delta(x),$$

where Δ is either a solution of $(\square-m^2)\Delta=0$ or of $(\square-m^2)\Delta=-\delta(x)$. If $\Delta(-x)=\epsilon\Delta(x)$, $\epsilon=\pm1$, the corresponding S-function satisfies, according to Eq. (4),

$$[C^{-1}S(-x)C]^T=\epsilon S(x).$$

With the help of this relation one proves, in the interaction representation:

(1) The anticommutation relations are invariant under the C-transformation:

$$\{\psi_\alpha{}^C(x_\mu),\bar\psi_\beta{}^C(x_\mu')\}=\{\psi_\alpha(x_\mu),\bar\psi_\beta(x_\mu')\}.$$

(2) The invariance of the vacuum expectation value of any P-bracket. This follows directly from the invariance of the elementary P-bracket:

$$\langle P[\psi_\alpha{}^C(x),\bar\psi_\beta{}^C(x')]\rangle_{\text{vac}}=\langle P[\psi_\alpha(x),\bar\psi_\beta(x')]\rangle_{\text{vac}}.$$

Note that this invariance is true independent of whatever γ-matrices may occur within the general P-bracket.[14]

2. *Bosons.*

For a charged boson field ϕ the C-transformation is given by

$$\phi^C(x)=C\phi(x)=\phi^*(x), \qquad (10a)$$

$$\phi^{C*}(x)=C\phi^*(x)=\phi(x), \qquad (10b)$$

irrespective of the Lorentz transformation properties of ϕ. A neutral boson field remains unaffected. Again one verifies easily that, in the interaction representation, the commutation relations and vacuum expectation values are invariant for the C-transformation.

B. Charge Symmetry

Describing the proton-neutron in the usual way by an 8-component nucleon spinor ψ, the interchange of proton and neutron is expressed by[15]

$$\psi'=T\psi \qquad T\cong\begin{pmatrix}0&1\\1&0\end{pmatrix}.$$

If the nucleons are coupled to charged mesons ϕ, the role of positive and negative mesons has to be interchanged, so the "T-transformation" on ϕ is

$$\phi'=\phi^*, \qquad \phi^{*\prime}=\phi.$$

For neutral meson fields the transformation depends on whether the coupling to nucleons goes via $\tau_{(3)}$ or "1." Call the corresponding fields $\phi_{(3)}$ and $\phi_{(0)}$. In the first case proton and neutron have opposite mesic coupling,

$$\phi_{(3)}'=-\phi_{(3)}, \qquad (11)$$

while

$$\phi_{(0)}'=\phi_{(0)}. \qquad (12)$$

Comprising ϕ and $\phi_{(3)}$ to a vector $\phi_{(i)}=\boldsymbol{\phi}$ in isotopic space we have

$$\boldsymbol{\phi}'=T\boldsymbol{\phi} \qquad T\cong\begin{pmatrix}1&&\\&-1&\\&&-1\end{pmatrix}.$$

The most general interaction invariant under T is of the form $a(\phi\bar\psi\tau^+\psi+\phi^*\bar\psi\tau^-\psi)+b\phi_{(3)}\bar\psi\tau_{(3)}\psi+c\phi_{(0)}\bar\psi\psi$ with real a, b, c. Charge independence requires in addition $a=b$.

Commutation relations and vacuum expectation values are invariant under the T-transformation. We note that

$$CT\phi=\phi,\ CT\phi^*=\phi^*,\ CT\phi_{(3)}=-\phi_{(3)},\ CT\phi_{(0)}=\phi_{(0)}. \qquad (13)$$

In connection with a discussion of V-decay it has been suggested[2] that there may exist fermions with different mass than the nucleon but for the rest with analogous properties. Denoting the 8-component spinor for the ith kind of such particles by ψ_i, one can now consider interactions of the general type

$$G_{ij,k}[\bar\psi_i\tau^+\psi_j\phi_k+\bar\psi_j\tau^-\psi_i\phi_k{}^*],$$

where ϕ_k denotes charged mesons of the kind k. This interaction is charge symmetric if $G_{ij,k}=G_{ji,k}$. Neutral meson couplings of the kind

$$(\bar\psi_i\tau_{(3)}\psi_j+\bar\psi_j\tau_{(3)}\psi_i)\phi_{(3),k} \quad\text{or}\quad (\bar\psi_i\psi_j+\bar\psi_j\psi_i)\phi_{(0),k}$$

are automatically charge symmetric. Provided that the condition on $G_{ij,k}$ is satisfied, all considerations of the next section hold irrespective of the number of nucleon-like fermions.

III. SELECTION RULES

Consider a general system of nucleons, mesons, and photons in mutual interaction. The Lagrangian L of

hand and v, t on the other as a criterion for the exclusion of certain nucleon-meson interactions. This criterion, stated as a "conservation of mesic charge," would at present not seem applicable as it would in particular exclude the existence of any coupling between pseudoscalar mesons and nucleons.

[12] See reference 3, Eq. (18) where the connections are given for c-number spinors. The formal derivation was given earlier by W. Pauli, see, e.g., reference 9.

[13] See S. R. de Groot and H. A. Tolhoek, Physica **16**, 456 (1950); R. H. Good, Jr., Phys. Rev. **86**, 620 (1952).

[14] The conditions (5) and (8) which have to be satisfied by C still leave open the possibility of multiplying C with an arbitrary phase. It is irrelevant for what follows whether one wishes to assign to C a different phase for different spinor fields.

[15] If one wishes to consider this transformation as a representation in spin space of a rotation in a 3-space one would have to replace T by iT.

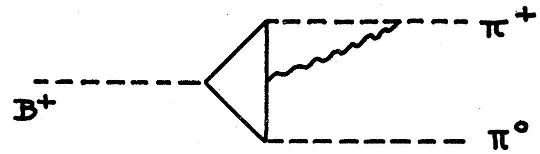

FIG. 1. Example of an allowed graph for transition (14).

this system remains unchanged if one applies C to all relevant operators in L and changes the sign of all coupling constants referring to v- and t-interactions [see Eq. (9)].

It is already clear from Sec. I that only then can we at all have selection rules if we assume: each scalar meson field that may occur is coupled to nucleons by either a scalar or a vector interaction (i.e., not by both simultaneously). Likewise a pv field may only have either pv or pt interaction. With this assumption (and only then) we may replace the prescription for the sign change of v- and t-coupling constants to a change in sign of the fields with v- and (or) t-interaction. We understand by invariance under general charge conjugation the invariance of L with respect to C and this change of sign of fields.

In addition, we shall assume L to be invariant with respect to T. This implies:

(1) Neutral fields transforming like Eqs. (11) and (12) are coupled to nucleons exclusively via $\tau_{(3)}$, 1, respectively.

(2) The electromagnetic field does not satisfy this requirement. Provisorily we omit it from our considerations. It is reincluded in subsection (b) below.

We now investigate various decay processes.

A. Boson-Boson Transitions (photons excluded)

Consider the S-matrix element for a given transition in which the fields representing the initial and the final particles are $\phi_a, \phi_b, \cdots \phi_n$. S is then generally of the form

$$S=\int d^4x_a\cdots d^4x_n\phi_a(x_a)\phi_b(x_b)\cdots\phi_n(x_n)K(x_a,\cdots,x_n),$$

where K is a complicated kernel involving implicitly the occurrence of all possible transitory nucleons and mesons. L is invariant under C and T and, thus, also under CT. S is also invariant under CT but, in general, not under C and T separately. For if in a process charged external bosons are involved, the application of C to S would lead us from the given process to one in which all these bosons now have opposite charge, i.e., we now have in general to do with another process. Thus we have further to distinguish two cases:

(α) *Transitions Involving Charged (External) Bosons.* These are forbidden if

$$n(\tau_{(3)})+n(v)+n(t)=\text{odd}. \qquad \text{(I)}$$

Proof: apply CT to S and denote this by priming all quantities:

$$S'=\int d^4x_a\cdots d^4x_n\phi_a'(x_a)\cdots\phi_n'(x_n)K'(x_a\cdots x_n).$$

Of course $S'=S$. Furthermore, $K'=K$ as all vacuum expectation values are invariant. Thus, according to Eqs. (9) and (13),

$$S'=(-1)^{n(\tau_{(3)})+n(v)+n(t)}S,$$

which proves (I).

Example: consider the process

$$B_v^+\rightarrow\pi^++\pi^0, \qquad (14)$$

where B_v^+ is a positively charged vector meson with vector and/or tensor coupling. π^+, π^0 are the usual pseudoscalar mesons. Let π^0 be described by a $\phi_{(0)}$-field. Then process (14) is forbidden. Likewise

$$B_v^0\rightarrow\pi^++\pi^- \qquad (15)$$

is forbidden if the B^0 field is of the $\phi_{(0)}$ type.

Note: the forbiddenness of processes (14) and (15) is a strict one only as long as the electromagnetic field is excluded. It will be discussed presently what happens if photons are also brought in.

(β) *Transitions Involving no Charged (External) Bosons.* Now besides (I) also

$$n(\tau_{(3)})=\text{odd is forbidden.}^{16} \qquad \text{(II)}$$

Hence from (I) and (II):

$$\text{if } n(\tau_{(3)})=\text{even}, \quad n(v)+n(t)=\text{odd is forbidden.} \qquad \text{(II}')$$

The proof is immediate from the application of T separately to S which is now legitimate.

Examples: the following two processes are forbidden

$$B^0_{ss\tau(3)}\rightarrow 2\pi^0, \qquad (16)$$

$$B^0_{vv\tau(3)}\rightarrow 2\pi^0. \qquad (17)$$

This is true whether π^0 is described by a $\phi_{(3)}$ or by a $\phi_{(0)}$ field.

B. Boson-Boson Transitions, Photons Included

Now only the C-invariance is strictly enforced. By the same argument as mentioned above, one can only expect rigorous selection rules if all external bosons

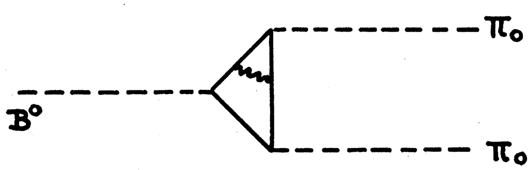

FIG. 2. Example of an allowed graph for transition (16).

[16] The formal generalization of (II) in the case of a charge independent theory is $n(\tau_{(i)})=\text{odd}$ is forbidden. This would apply to cases where the only external lines are $\phi_{(i)}$ and $\phi_{(0)}$, a situation which has no physical significance.

lines denote neutral mesons. One shows directly that, with this restriction,

$$n(v)+n(t)=\text{odd is forbidden.} \qquad \text{(III)}$$

Examples: (1) The 2 γ-decay of a neutral vector meson is rigorously forbidden, as first noted by Sakata and Tanikawa.[17]

(2) The 3γ-decay of a neutral pv meson with pv coupling is rigorously forbidden;[18] (the 2γ-decay is forbidden for parity reasons).

It is now also necessary to review the processes which according to (I) and (II) were strictly forbidden. To see what happens it is enough to consider the examples (14)–(17):

Transitions (14) and (15): These are now allowed as can be seen by inspection of a low order graph for process (14) drawn in Fig. 1. The waved line denotes a photon. When replacing B^+, π^0 by B^0, π^-, Fig. 1 also applies to Eq. (15).

Transition (16): Same situation, a representative graph is given in Fig. 2. Note that the internal nucleon lines here can only be protons.

Transition (17): This is still strictly forbidden as it falls under the jurisdiction of (III).

It should be noted that the internal photon line occurring in Figs. 1 and 2 makes the transition probabilities for processes (14), (15), and (16) proportional to at least the second power of $\alpha=1/137$. This means that

$$B_v^+\rightarrow\pi^++\pi^0+\gamma, \qquad (14')$$

$$B_v^0\rightarrow\pi^++\pi^-+\gamma, \qquad (15')$$

$$B^0_{ssT(3)}\rightarrow 2\pi^0+\gamma \qquad (16')$$

are more probable by a factor α than Eqs. (14), (15), and (16), respectively. It should be emphasized that this circumstance makes it indispensable to include such transitions as Eqs. (14′)–(16′) in the discussion of competing disintegration modes when studying V-decay. An example is given in reference 2, Sec. V.

Thus, for these processes the remarkable inversion occurs that the transition with an additional photon is favored to that without one. Hence, in as far as one can make qualitative statements concerning a theory fraught with divergences, one may say that for processes (14) and (15) a weak selection rule is still operative. This is true for any process to which (I) and (II) apply.

In a recent paper by one of us,[2] use was made of the Fukuda-Miyamoto theorems, mentioned in the introduction for the discussion of certain V-decay processes. It is easily seen that all processes which were found to be forbidden in low order are actually either strictly forbidden or else weakly forbidden in the sense just mentioned.

[17] S. Sakata and Y. Tanikawa, Phys. Rev. **57**, 548 (1940).

[18] In discussing the γ-decay of neutral mesons, R. Oehme, Z. Naturforsch **7a**, 55 (1952), reaches some conclusions which contradict the rigorous selection rule (III).

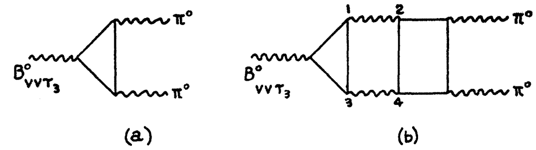

FIG. 3. Graphs referring to transition (17).

C. Boson-Fermion Transitions

We consider here decays in which electron-positron pairs may be created through electromagnetic interaction in addition to bosons. Always following the same reasoning it is readily seen that no rigorous selection rule is possible in this case. In fact, C inverts the charge of bosons *and* fermions, while T inverts the charge of bosons only. Therefore, the application of the CT-transformation to the S-matrix element for a process of the present type leads, in general, to the element for another process.

However, it is easy to see that in one instance a weak selection rule holds. This is for the case of a transition involving neutral bosons only plus a single electron-positron pair. One shows that now (III) is a weak selection rule, with the understanding that the pair counts as one v-interaction. Thus, for example, $\pi^0\rightarrow e^++e^-+\gamma$ ($\sim\alpha$ less probable than $\pi^0\rightarrow 2\gamma$) is more probable by a factor α than $\pi^0\rightarrow e^++e^-$, for a pseudoscalar meson. As was pointed out by Oehme[18] and by Fermi,[19] the latter process is not rigorously forbidden, however.

After this paper had been written, a survey article by L. Michel[20] came to our notice in which the rules (I)–(III) are also given for the restricted coupling scheme in which they are of absolute validity. Nevertheless, we believed it to be useful to give the present more comprehensive account in which no explicit study of graphs is necessary (see Fig. 3).[21]

[19] E. Fermi, Proc. Rochester Conference, 1952. The forbiddenness indicated by J. Steinberger, Phys. Rev. **76**, 1180 (1949), Table I, always refers to the lowest order in e.

[20] L. Michel, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1952), Chapter III. See especially pp. 142–144.

[21] Michel approaches the problem by attempting to locate in each graph a particular closed loop to which the Fukuda-Miyamoto theorems can be applied. This procedure suffices as one only allows *neutral* mesons to occur in intermediary states, and to this case Michel confines his attention. This method is not applicable in general, however, as it may lead to pitfalls of the following kind: Consider the graph in Fig. 3(a) for the process (17). Its contribution vanishes according to (II). Consider next the higher order graph drawn in Fig. 3(b), where 12 and 34 denote charged ps mesons. Now rule (I) has to be applied to the triangle as well as to the square, but neither yield a factor zero. The zero actually comes from the combination of the graph of Fig. 3(b) with one in which the charges along 12 and 34 have been interchanged. It is just for such reasons that we felt the need of a derivation in which no such subtleties are needed.

In reference 20, Tables II and III, some processes stated to be forbidden by Furry's theorem should be reinterpreted using the discussion on weak selection rules given above. In the present paper Fermi interactions have not been considered. A further selection rule pertaining to this case and of interest for processes like $\pi-\mu$ decay has been given by Michel, reference 20, p. 144. This is readily shown also to be a weak selection rule.