TABLE I. Summary of experimental results.

	Bandwidth (ev)	M_2 emission edge (ev)	M_{23} separation (ev)	Ratio of M_3 to M_2 intensities
Copper	7.1 ± 0.5	75.9 ± 0.2	1.2 ± 0.1	0.51 ± 0.03
Chromium	7.2 ± 1.0	42.1 ± 0.2	0.45 ± 0.1	0.52 ± 0.04

is emphasized by the E^4 factor. Therefore, the M_2 and M_3 bands were completed by the dotted lines at the low energy end, showing the probable position of the bottom of the Brillouin zone.

In order to minimize the effect of fluctuations, the average of experimental chromium curves is plotted modified by the E^4 factor in Fig. 4. The curve shown in Fig. 2 is shown (dotted) for comparison. M_2 and M_3 bands are completed by extrapolation.

The results are summarized in Table I.

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Elastic π-Carbon Scattering*

D. C. PEASLEE
Columbia University, New York, New York
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Elastic scattering of 60-Mev π-mesons on carbon is analyzed to determine the following qualitative features of elementary π-nucleon scattering at this energy: p-wave scattering is predominant, with some s-wave interference at a relative phase that favors backscattering. The rather crude interpretation does not indicate d-wave scattering. Spin flip scattering is essentially absent from the C^{12} scattering. The Coulomb interference provides an absolute calibration, from which it is inferred that the p-wave "resonance" energy lies above 60 Mev.

1. INTRODUCTION AND SUMMARY

THE elastic scattering of 60-Mev π-mesons by carbon is analyzed to determine some qualitative features of the elementary π-nucleon scattering process. A second-order perturbation treatment is used, which takes into account the possibly "resonant" nature of the elementary scattering process. The final expressions are similar to a first-order scattering from C^{12} with the addition of a "form factor" arising from the asymmetry of the elementary scattering. A crude estimate of this asymmetry is obtained from comparison with the measured angular distribution from C^{12}. Because of the neglect of multiple scattering, the estimated asymmetry is actually a minimum estimate, and only qualitative conclusions can be drawn.

The qualitative conclusions are that for π-nucleon encounters at 60 Mev in the c.m. system (a) p-wave scattering is predominant, (b) some s-wave scattering is present, and (c) the relative phases of s- and p-wave scattering are such as to favor backscattering at the expense of forward scattering. The rather crude interpretation does not indicate the presence of d-wave scattering.

Spin flip of the nucleon in elastic scattering from C^{12} is forbidden by the exclusion principle. Its absence, within 20 percent limits, is indicated by the experimental data. This absence makes it impossible to determine from elastic C^{12} measurements the relative amounts of $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ scattering.

The pronounced Coulomb interference dip in the $C-\pi^+$ scattering at about 20° makes it possible to form some estimate of the absolute phases of the nuclear scattering amplitudes, since that of the Coulomb scattering is known. It is found that the p-wave scattering is compatible with a "resonance" at an energy higher than 60 Mev and with a half-width of the same order of magnitude. The s-wave scattering has rather little absorption and can be most satisfactorily represented in terms of a repulsive potential or possibly in terms of a far-away bound level of the meson.

2. FORMULATION

We wish to consider the scattering of π-mesons by an aggregation of nucleons, in particular C^{12}. The scattering is assumed to be the sum of single-nucleon scatterings alone; any scattering by multibody potentials is neglected. An important effect in the C^{12} scattering is the possibility that the individual nucleons do not scatter isotropically, as is generally the case if a quasi-compound state of an excited nucleon is formed.[1] A straightforward way to exhibit this anisotropy is by means of a second-order perturbation calculation. Suppose a nucleon fixed in space with internal wave function U_0 in the ground state and U_c in an unstable "compound" state of excitation energy $E_c-i\Gamma_c/2$. The cross section for elastic scattering of spinless mesons by a nucleon with initial spin $s_0=\frac{1}{2}$ is[2]

$$\sigma=\frac{\pi}{k^2}\frac{1}{(2s_0+1)}\sum_{m_i,m_f}\left|\left\langle m_i\left|\sum_c\frac{M_i^cM_f^{c*}}{E-E_c+i\Gamma_c/2}\right|m_f\right\rangle\right|^2, \quad (1)$$

* This work was performed under the research program of the AEC.

[1] K. A. Brueckner, Phys. Rev. 86, 106 (1952).
[2] H. A. Bethe and G. Placzek, Phys. Rev. 51, 450 (1937).

where m_i, m_f are the initial and final nucleon spin projections, and E is the kinetic plus rest mass energy of the meson. The matrix elements can be written symbolically as

$$M_{i,f}{}^c = N_E \int\int U_0(r') V(r-r') e^{i\mathbf{k}_{i,f}\cdot\mathbf{r}} U_c{}^*(r') dV' dV, \quad (2)$$

where $N_E = [(1/\pi)k^2(dk)/(dE)]^{\frac{1}{2}}$ is a normalization factor per unit energy for the meson plane wave.

The angular dependence of the scattering is associated with the l values of the incident and scattered mesons; because $s_0=\frac{1}{2}$, spin and parity conservation require that $l_i = l_f$. The differential scattering cross section is

$$\frac{d\sigma}{d\omega} = \frac{\pi}{k^2}\frac{1}{(2s_0+1)} \sum_{m_1, m_f=\pm\frac{1}{2}} |\langle m_i | \sum_j a_j \sum_{m_j} C_{\frac{1}{2}lj}{}^{m_i 0 m_j} \times C_{jl\frac{1}{2}}{}^{m_j m m_f} (2l+1)^{\frac{1}{2}} Y_l{}^m(\mathbf{k}_i, \mathbf{k}_f) | m_f\rangle|^2, \quad (3)$$

where

$$a_j{}^l = \sum_{c(j,l)} \frac{M_i{}^c M_f{}^{c*}}{E - E_c + \frac{1}{2}i\Gamma_c}$$

is the sum over all levels with a given j and given parity, specified in this case by l. In view of the provisory nature of both measurement and calculation, we consider only three possibilities for l and j: $l=0$ and 1, $j=\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$. Then, designating $m_i = m_f$ as "non-spin-flip" (nf) and $m_i = -m_f$ as "spin flip"(f), one has

$$\begin{aligned} d\sigma_{nf} &= (\pi/k^2)|a_{\frac{1}{2}}{}^0 + (a_{\frac{1}{2}}{}^1 + 2a_{\frac{3}{2}}{}^1)\cos\theta|^2 d\omega/4\pi, \\ d\sigma_f &= (\pi/k^2)|(a_{\frac{1}{2}}{}^1 - a_{\frac{3}{2}}{}^1)\sin\theta|^2 d\omega/4\pi. \end{aligned} \quad (4)$$

The use of perturbation theory to obtain the Breit-Wigner formula is, of course, unjustified for strong coupling. It has the remarkable feature, however, of leading to phenomenological results correct in every detail, as is known from nuclear reaction theory. The present calculation simply attempts to exploit this fortunate circumstance and obtain a first-order estimate of the π-carbon scattering. The perturbation treatment should be at least as valid for the motion of the struck nucleon in the C^{12} nucleus as for the π-nucleon collision, which occurs much more rapidly than the transfer of energy among nucleons. Crudely speaking, the "half-width" of the excited nucleon is on the order of a few hundred Mev, while the half-width corresponding to nucleon energy transfer in a nucleus is apparently on the order of a few Mev. Accordingly, the C^{12} nucleus is considered as a number of nucleons occupying independent orbits in a fictitious central potential. If there is one nucleon bound in a potential with wave functions ϕ_0, ϕ_n for the center-of-gravity motion, the argument of (1) becomes

$$\begin{aligned} &\sum_c \sum_n \frac{M_i{}^{c,n} M_f{}^{c,n*}}{E - E_c + \frac{1}{2}i\Gamma_c - E_n + \frac{1}{2}i\Gamma_n}, \\ &M_{i,f}{}^{c,n} = M_{i,f}{}^c \int \phi_0(r) e^{i\mathbf{k}_{i,f}\cdot\mathbf{r}} \phi_n{}^*(r) dV. \end{aligned} \quad (5)$$

Now for all the lowest excited nuclear states E_n, $\Gamma_n \ll E_c$, Γ_c, so that the last two terms in the denominator of (5) may be neglected, and the sum over n evaluated by closure. Then (5) becomes, with $\Delta\mathbf{k} = \mathbf{k}_i - \mathbf{k}_f$,

$$\sum\nolimits_{l,j} a_j{}^l \phi_0(r) e^{i\Delta\mathbf{k}\cdot\mathbf{r}} \phi_0{}^*(r) dV = \sum\nolimits_{l,j} a_j{}^l \rho(\Delta k), \quad (6)$$

For the C^{12} nucleus with ground state wave function Φ_0 the corresponding expression for the cross section is

$$d\sigma = \frac{\pi}{k^2} \left| \int \Phi_0 \sum\nolimits_q a_q e^{i\Delta\mathbf{k}\cdot\mathbf{r}_q} \Phi_0{}^* \right|^2 \frac{d\omega}{4\pi}, \quad (7)$$

where the sum extends over all nucleons. The amplitudes a_q have the forms given in (4) for (nf) and (f) scattering. The subscript q allows for the fact that the amplitudes for neutrons and protons have different magnitudes. In the single-nucleon approximation, the matrix element in (7) becomes $\sum_q a_q \rho_q(\Delta k)$, where ρ_q is the Fourier transform of the single-particle density as given in (6) and $\Delta k = 2k\sin\frac{1}{2}\theta$ is the magnitude of $\Delta\mathbf{k}$. The expression (7) is equivalent to the first-order perturbation formula for x-ray scattering in which a_q is an intrinsic form factor for the elementary scattering process.

3. COMPARISON WITH EXPERIMENT

For elastic impacts on C^{12}, f scattering is forbidden by the exclusion principle. The nf scattering contains a coherent part of amplitude $b = \frac{1}{2}(a_q + a_n)$, and an incoherent part of amplitude $b' = \frac{1}{2}(a_p - a_n)$. Then

$$d\sigma_{nf} = \frac{\pi}{k^2}[|b^2||\sum\nolimits_q \rho(\Delta k)|^2 + |b'|^2 \sum\nolimits_q |\rho_q(\Delta k)|^2] \frac{d\omega}{4\pi}. \quad (8)$$

The quantities $|\sum_q \rho_q|^2$ and $\sum_q|\rho_q|^2$ are little known, but it seems reasonable to expect the first to be of order $A=12$ times as large as the second. If we furthermore assume[3] that all scattering occurs through a nucleon intermediate state of isotopic spin $T=\frac{3}{2}$, $|b'| = \frac{1}{2}|b|$. Hence to the crude order of accuracy maintained here, the incoherent term can be dropped from (8) in first approximation. Then only the average component, $\bar{\rho}(\Delta k) = (1/A)\sum_q \rho_q(\Delta k)$, is necessary. For $|\bar{\rho}(\Delta k)|^2$ we take the distributions inferred[4] from the production of π-mesons by protons on C^{12}, noting that the quantity written here as $|\bar{\rho}(\Delta k)|^2$ is written as $\rho(\mathbf{k})$ in reference 4 and is plotted in Fig. 1 of that reference.

From (8) and (4) the elastic scattering cross section is

$$d\sigma_{\mathrm{el}} = (\pi/k^2)|b_{\frac{1}{2}}{}^0|^2 (12)^2 |\bar{\rho}(\Delta k)|^2 \times [1 + 2\alpha\cos\delta\cos\theta + \alpha^2\cos^2\theta] d\omega/4\pi, \quad (9)$$

where $\alpha = |b_{\frac{1}{2}}{}^1 + 2b_{\frac{3}{2}}{}^1| / |b_{\frac{1}{2}}{}^0|$ and $\delta = (\delta_1 - \delta_0)$ is the cor-

[3] Anderson, Fermi, Long, and Nagle, Phys. Rev. **85**, 936 (1952). If appreciable $T=\frac{1}{2}$ scattering occurs, the quantities b below will be replaced by more complicated averages of $T=\frac{3}{2}$ and $T=\frac{1}{2}$ scattering amplitudes for neutrons and protons. Qualitative conclusions should remain valid, however, as long as $T=\frac{3}{2}$ is the dominant scattering mode.

[4] E. M. Henley, Phys. Rev. **85**, 204 (1952).

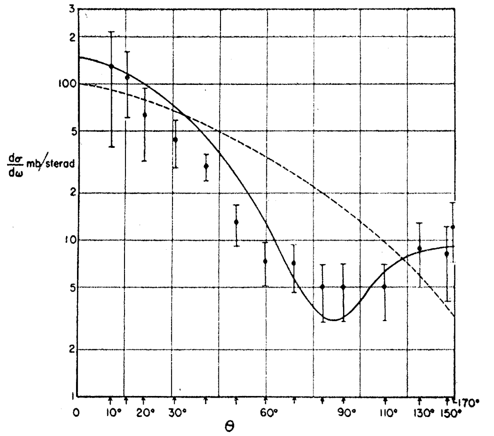

FIG. 1. Angular distribution of elastic π-carbon scattering. The solid circles give the average π^+ and π^- data of reference 5. The solid curve shows the fit of (9) with $\alpha=3$, $\cos\delta=-\frac{1}{2}$. The dashed curve is $|\bar{\rho}(\Delta k)|^2$, relative scale only. The abscissa is linear in $\Delta k=2k\sin\frac{1}{2}\theta$.

responding relative phase of the two terms. The quantities α and $\cos\delta$ are fitted by comparison with the measurements[5] of $d\sigma_{\text{el}}/d\omega$ for 60-Mev π-mesons on C^{12}. To obtain a best fit, the experimental points are multiplied by the reciprocal of the angular factor in (9) and the resulting points compared with the smooth curve of the assumed $|\bar{\rho}(\Delta k)|^2$. The choices indicated by this procedure are

$$\alpha\sim 3,\ \cos\delta\sim -\tfrac{1}{2}. \tag{10}$$

Figure 1 shows Eq. (9) with these parameters in comparison with the experimental points. The assumed shape of $|\bar{\rho}(\Delta k)|^2$ is also shown. The values (10) are little better than order-of-magnitude precision, but it is very probable that α considerably exceeds unity and the $\cos\delta<0$. This implies that for the individual nucleon-meson scattering process p-wave scattering is predominant, but that s-wave scattering is also present with a phase such as to enhance backward scattering at the expense of forward scattering.

4. DISCUSSION

Major sources of error in the present treatment are neglect of absorption and of multiple scattering. The absorption may be specified[6] by a mean free path $\lambda\sim 3.5\times 10^{-13}$ cm. A crude estimate of absorption effects is the following: for the incident meson wave $\mathbf{k}_i$ put $\mathbf{k}_i'=\mathbf{k}_i[1+i/2\lambda k]$, and for the final meson wave $\mathbf{k}_f$ put $\mathbf{k}_f'=\mathbf{k}_f[1-i/2\lambda k]$, so that $\Delta k'=\Delta k[1+i/\lambda k]$. Then $\Delta k'$ is substituted for Δk as the argument of $\bar{\rho}$ in (9). The curve in Fig. 1 corresponds approximately to the choice $\bar{\rho}(\Delta k)=\exp[-\alpha(\Delta k)^2]$ with $\alpha=0.77\times 10^{-26}$ cm^2. Then

$$|\bar{\rho}(\Delta k')|^2=|\bar{\rho}(\Delta k)|^2\exp[0.25\sin^2\tfrac{1}{2}\theta]. \tag{11}$$

The inclusion of absorption therefore tends in first approximation to increase backscattering relative to forward scattering. In the present case this increase is a maximum of about 1.3, which is small in comparison with the uncertainty in the assumed shape of $\bar{\rho}(\Delta k)$ and in the $\pi-C$ measurements. This suggests that the effects of absorption may safely be neglected here.

The effect of multiple scattering is always to smooth out the angular distribution. Hence neglect of multiple scattering means that the true π-nucleon angular distribution is somewhat more asymmetrical than here determined, and we are limited to qualitative conclusions on the basis of a minimum asymmetry. The assumed curve of $\bar{\rho}(\Delta k)$ is based on experiments that also involve a certain amount of neglected multiple scattering. This curve therefore represents a sort of effective density component that contains some contribution from multiple scattering, and it does not seem worth while to attempt further corrections. The most important effect of multiple scattering is to make $\bar{\rho}(0)<1$, so that only a minimum estimate of the absolute amplitude $|b_{\frac{1}{2}}{}^0|$ can be obtained. Inserting $1/k^2\approx 18$ mb and $d\sigma(0)/d\omega=150$ mb/sterad in Eq. (9), one has $|b_{\frac{1}{2}}{}^0|>0.18$. If all scattering is assumed to pass through a nucleon state of isotopic spin $T=\frac{3}{2}$, the amplitude for $p-\pi^+$ scattering is $a_{\frac{1}{2}}{}^0=3/2b_{\frac{1}{2}}{}^0$. The corresponding cross sections for scattering of 70-Mev (60-Mev in c.m. system) π^+ mesons by protons without spin flip is $\sigma_{nf}>17$ mb. The measured value[3] of the total $p-\pi^+$ cross section at this energy is $\sigma\approx 37$ mb, which suggests that $\sigma_f/\sigma_{nf}\lesssim 1$. If the p-wave scattering is entirely $j=\frac{3}{2}$, the ratio expected with $\alpha=3$ is $\sigma_f/\sigma_{nf}=\frac{3}{8}$.

It is possible that the experimental points do not correspond to perfect elastic scattering but contain some admixture of slightly inelastic scattering. This scattering can involve spin flip, with

$$d\sigma_f=(\pi/k^2)|b_{\frac{1}{2}}{}^0|(12)^2|\bar{\rho}(\Delta k)|^2[\gamma^2-\gamma^2\cos^2\theta]d\omega/4\pi, \tag{12}$$

where

$$\gamma=[|b_{\frac{1}{2}}{}^1-b_{\frac{3}{2}}{}^1|/|b_{\frac{1}{2}}{}^0|]|\rho'(\Delta k)/\bar{\rho}(\Delta k)|, \tag{13}$$

and $|\rho'(\Delta k)|^2$ is an effective density for inelastic scattering, summed over the final states involved. As a crude approximation, we take $|\rho'(\Delta k)/\bar{\rho}(\Delta k)|$ to be independent of Δk. Then if we add (12) and (9), condition (10) becomes

$$(\alpha^2-\gamma^2)/(1+\gamma^2)\approx 9,\quad \gamma^2\leqslant \alpha^2/10. \tag{14}$$

Since $\int\sin^2\theta d\omega=2\int\cos^2\theta d\omega$, relation (14) implies that in the experimental points the amount of scattering with spin flip is less than $2(1/10)=\frac{1}{5}$ the amount of scattering without spin flip.

Because of the essential absence of spin-flip scattering, it is not possible to determine the relation between $b_{\frac{1}{2}}{}^1$ and $b_{\frac{3}{2}}{}^1$ separately from measurements on C^{12}. Only for the lightest targets like the proton or possibly the deuteron can elastic or almost elastic spin-flip scattering occur with appreciable probability. In π-proton scat-

[5] Byfield, Kessler, and Lederman, Phys. Rev. 86, 17 (1952).
[6] Brueckner, Serber, and Watson, Phys. Rev. 84, 258 (1951).

tering the measurement of maximum to minimum $d\sigma/d\omega$ at 180° and $\lesssim 90°$, respectively, is a sensitive index of the relation between $a_{\frac{1}{2}}^1$ and $a_{\frac{3}{2}}^1$. For example, with the parameters of (10) the ratio $d\sigma_{\max}/d\sigma_{\min}\approx 17$ or ≈ 4 for the choices $a_{\frac{1}{2}}^1/a_{\frac{3}{2}}^1=1$ or 0.

The asymmetry of the form factor, which leads to a minimum in Fig. 1 around 80°, arises from the interference of an even-l wave with the dominant p-scattering. It is assumed above that the even-l interference is predominantly s-wave. The assumption of d-wave interference instead could only worsen the agreement with observation. The nf d-wave amplitude is $[\frac{3}{2}b_{\frac{3}{2}}+b_{\frac{1}{2}}]\times(3\cos^2\theta-1)$. A destructive interference of this factor with $\cos\theta$ in the 80° region implies considerably larger destructive interference at 180° and a corresponding constructive interference at 0°. This is just the opposite of the behavior of the p–s interference at 0° and 180°, which is in better agreement with experiment. Therefore the measurements in Fig. 1 suggest that the interference is mainly s-wave.

5. COULOMB SCATTERING INTERFERENCE

Coulomb scattering by the protons in C^{12} can be treated quite analogously to the nuclear scattering, except that it is no longer necessary to go so far as second-order perturbations. The first-order elastic scattering amplitude from a bound proton is proportional to

$$\int \phi_0(r')e^{i\Delta\mathbf{k}\cdot\mathbf{r}}\frac{e^2}{|r-r'|}\phi_0(r')dVdV'=\frac{4\pi e^2}{(\Delta k)^2}\rho(\Delta k). \quad (15)$$

To write the elementary cross section in a form corresponding to (4), we must include a Coulomb amplitude $a^c=2\eta/(1-\cos\theta)$ in the matrix element of (4), where $\eta=e^2/\hbar v=0.01$ for 60-Mev π-mesons. Now since $a_p{}^c=a^c$, $a_n{}^c=0$, one has $|b^c|=|b'^c|=|1/2a^c|$. Dropping the incoherent scattering as before, we have

$$d\sigma_{\mathrm{el}}{}^c=\frac{\pi}{k^2}(12)^2|\bar{\rho}(\Delta k)|^2 \times\left|(1+\alpha e^{i\delta}\cos\theta)Be^{i\beta}+\frac{\eta}{(1-\cos\theta)}\right|^2\frac{d\omega}{4\pi}, \quad (16)$$

where $b_{\frac{1}{2}}{}^0=Be^{i\beta}$.

The scattering of π^+ mesons shows a pronounced interference minimum in the neighborhood of $\theta=20°$. Since the Coulomb term in (16) is in this approximation entirely real, the minimum means that the real part of the matrix element of (16) vanishes at $\cos\theta\approx 0.94$, or

$$B[\cos\beta+2.9\cos(\beta+\delta)]+0.17\approx 0. \quad (17)$$

In order that the scattering amplitudes represent only elastic scattering and absorption but not creation of mesons, one must have $-\pi\leqslant\beta$, $(\beta+\delta)<0$. For $\cos\delta=-\frac{1}{2}$ and $B\geqslant 0.18$ there is only one such solution of (17):

$$\left.\begin{array}{r}\beta=-13° \text{ to } 0° \\ (\beta+\delta)\approx-133° \text{ to } -120°\end{array}\right\} \text{ as } B=0.18 \text{ to } 0.38. \quad (18)$$

The upper limit on B given by (18) is larger than that implied by the measured p-π^+ cross section.

The rough values (18) allow the following inferences: if the p-wave scattering is represented by a one-level resonance formula, $b^1=|M|^2/(E-E_c+\frac{1}{2}i\Gamma_c)$, the phase angle $(\beta+\delta)$ indicates that $E_\pi=60$ Mev is below the p-wave "resonance energy" E_c by an amount on the same order as the corresponding "half-width." This is not in disagreement with observation.[3] For the s-scattering the amplitude is small and has a relatively small imaginary part. This would suggest a "far-away" resonance level with $E_c\ll E_\pi$ and accordingly is probably better described by a simple repulsive potential.

As a final check on the internal consistency of this treatment, we may estimate the π^+ cross section at the Coulomb minimum, relative to the nuclear cross section at 0°. From (16) and (9) one has

$$\frac{[d\sigma^c(20°)]}{[d\sigma(0°)]}=\left|\frac{\bar{\rho}(20°)}{\bar{\rho}(0°)}\right|\frac{|\sin\beta+0.34\alpha\sin(\beta+\delta)|^2}{1+2\alpha\cos\delta+\alpha^2}\approx 0.1, \quad (19)$$

using the values $\beta\approx-13°$, $(\beta+\delta)\approx-133°$. Thus $(d\sigma^c)/(d\omega)\sim 15$ mb/sterad at the minimum, which is the measured value.[5] This must be regarded as only order-of-magnitude agreement, since angular resolution difficulties will tend always to make the measured minimum too high.

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