Moments of the Angular Distribution for High Energy Nuclear Collisions

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(Received May 16, 1952)

Detailed results for the mean square angle of scattering of nucleons in high energy nucleon-nucleus collisions are presented using the accurate expressions previously determined by Messel and Green. Curves are given both for light and heavy elements. Confirmation of the results presented would provide valuable information on the differential cross section for nucleon-nucleon collisions.

1. INTRODUCTION

DURING a calculation of the angular and radial distribution functions for the nucleon component of the cosmic radiation in the atmosphere, Messel and Green[1–3] were led to consider the problem of determining the angular moments of scattered nucleons in high energy nuclear collisions.

The preliminary results of this calculation were reported in reference 2 and in the first approximation in reference 3. In 2 and 3 a differential cross section for nucleon-nucleon collisions of the form $R+S'\cos^2\bar{\theta}$ in the center-of-mass frame of reference was used, where R and S' were functions of the energy only, and $\bar{\theta}$ the scattering angle in the center-of-mass frame. This form of cross section was taken from the predictions of field theories. Messel and Green[1] have since shown that a differential cross section of this form is incompatible with experimental observations on the radial spread of the nucleon component of the cosmic radiation, and that theory could only be reconciled with experiment when the number of scattered particles decreases exponentially from the direction of motion of the incident nucleon in the laboratory frame. The purpose of this paper is to present results for the mean square angle of scatter of nucleons resulting from high energy nuclear encounters in light and heavy elements, using the accurate expression for the second moment and the new differential cross section taken from 1. These results should prove to be of considerable value in interpreting high energy collisions observed in photographic plate work.

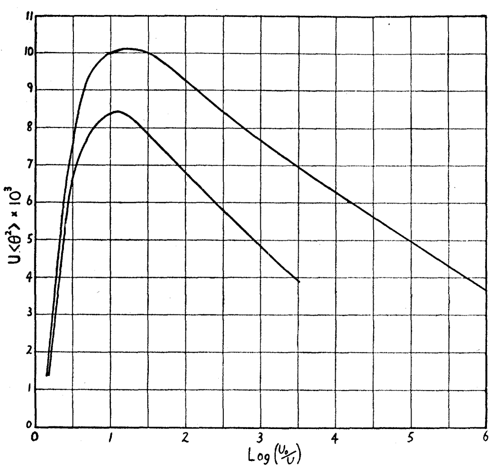

FIG. 1. A plot of $U\langle\theta^2(U_0, U)\rangle\times10^3$ against the logarithm of the ratio of the primary energy U_0 and the energy U above which particles are emitted from a nucleus. The energies are measured in proton mass units. $\langle\theta^2(U_0, U)\rangle$ gives the mean square angle of emission in radians squared of nucleons resulting from a nucleon-nucleus collision. The upper curve is valid for the heavy elements silver and bromine, the lower curve for the light nuclei carbon, oxygen, and nitrogen.

2. FUNDAMENTAL EQUATIONS AND SOLUTIONS

The probability $\nu(U, C, z)dUdC$ of finding a nucleon at a depth z (measured in the direction of motion of the incident particle) in homogeneous nuclear matter, with energy U, dU and direction of motion making an angle of cosine C, dC with that of the incident particle satisfies the integro-differential equation

$$C\frac{\partial\nu}{\partial z}(U, C)+\nu(U, C)=2\int_U^\infty dU'\int_{c0}^1 dc\int_0^{2\pi}\frac{d\omega}{2\pi}\nu(U', C')W(U', U, c), \quad (1)$$

$$C'=Cc+Ss\cos\omega,\ S=(1-C^2)^{\frac{1}{2}},\ s=(1-c^2)^{\frac{1}{2}},\ c_0=1-1/U,$$

where

$$W(U_0, U, c)dUdc=UG(U_0, U)y\{U(1-c)\}dUdc \quad (2)$$

is the differential cross section for nucleon-nucleon collisions and

$$y\{U(1-c)\}=144(1-e^{-144})^{-1}\exp\{-144U(1-c)\}, \quad (3)$$

$$G(U_0, U)=20U_0^{-5}U(U_0-U)^3. \quad (4)$$

All energies are measured in proton mass units ($\sim$1 Bev), and U_0 is the primary energy. The nth moment of $W(U_0, U, c)$ is defined by

$$W_{(n)}=2^n\int_{c0}^1 W(U_0, U, c)(1-c)^n dc$$
$$=(2/U)^n\int_0^1 G(U_0, U)y(x)x^n dx$$
$$=(2/U)^n G(U_0, U)y_{(n)}, \quad (5)$$

[1] H. Messel and H. S. Green, Phys. Rev. **87**, 738 (1952).
[2] H. Messel and H. S. Green, Phys. Rev. **83**, 1279 (1951).
[3] H. Messel and H. S. Green, Proc. Phys. Soc. (London) **A65** 245 (1952).

and the lth moment of $\nu(U, C, z)$ by

$$\nu_{(l)}(U, z)=2^l\int_{-1}^{1}(1-C)^l\nu(U, C, z)dC. \quad (6)$$

Using the initial condition,

$$\nu(U, C, z=0)=\delta(U_0-U)\delta(1-C), \quad (7)$$

Messel and Green[1] have shown that the accurate expression for $\nu_{(l)}(U, z)$ is given by

$$U_0\nu_{(l)}(U, z)=(2\pi i)^{-2}\int_{v_0-i\infty}^{v_0+i\infty}dv\int_{p_0-i\infty}^{p_0+i\infty}dp \times(U_0/U)^{v+1}e^{pz}4^l(l!)^2A_v{}^l, \quad (8)$$

where

$$A_v{}^l=U_0{}^{-l}\{p+\alpha(v-l)\}^{-l} \times\sum{}'\prod_{i=1}^{l}\alpha^{a_i}_{v-(a_1+\cdots+a_{i-1})}, \quad (9)$$

$$\alpha_v{}^l=y_{(l)}W(v-l)2^{-l}(l!)^{-2}\{p+\alpha(v)\}^{-1},\ l=1,\ 2,\ \text{etc.}$$

$$\alpha(v)=1-240\{(v+2)(v+3)(v+4)(v+5)\}^{-1}, \quad (10)$$

$$W(v)=1-\alpha(v). \quad (11)$$

The summation $\sum'$ in (9) is applied to all different products for which $a_1+a_2+\cdots+a_l=l$. The a's may take the positive integral values, including zero and $\alpha^0=1$.

The distribution of particles emitted by a nucleus may be obtained from $\nu(U, C, z)$ by applying the averaging operator $\mathbf{N}(D_A)=\int_0^{D_A}2zdz/D_A{}^2$ to it. D_A is the average number of collisions suffered by a nucleon on making a diametrical passage through a nucleus of atomic weight A. Thus

$$n(U, C)=\mathbf{N}(D_A)\nu(U, C, z), \quad (12)$$

and the lth moments of this nuclear distribution function are given in

$$n_{(l)}(U)=\mathbf{N}(D_A)\nu_{(l)}(U, z). \quad (13)$$

If we consider only those emitted particles whose energies are in excess of U, we have

$$N(U, C)=\int_U^{U_0}n(U', C)dU', \quad (14)$$

and for the lth moments of $N(U, C)$,

$$N_{(l)}(U)=\mathbf{N}(D_A)(2\pi i)^{-2}\int_{v_0-i\infty}^{v_0+i\infty}dv/v\int_{p_0-i\infty}^{p_0+1\infty}dp \times(U_0/U)^ve^{pz}4^l(l!)^2A_v{}^l. \quad (15)$$

3. RESULTS

The mean square angle of scatter of nucleons with energies $>U$, resulting from a nucleon-nucleus collision, is given by

$$\langle\theta^2\rangle=N_{(1)}(U)/N_{(0)}(U), \quad (16)$$

where, from (15),

$$N_{(0)}(U)=(2\pi i)^{-1}\int_{v_0-i\infty}^{v_0+i\infty}(U_0/U)^v\{1-h(v)\}dv/v, \quad (17)$$

$$N_{(1)}(U)=(2\pi iU_0)^{-1}\int_{v_0-i\infty}^{v_0+i\infty}(U_0/U)^v2y_{(1)}W(v-1) \times\{\alpha(v)-\alpha(v-1)\}^{-1}\{h(v)-h(v-1)\}dv/v, \quad (18)$$

and

$$h(v)=1-2[1-\{1+D_A\alpha(v)\}\exp\{-D_A\alpha(v)\}] \times\{D_A\alpha(v)\}^{-2}. \quad (19)$$

In Fig. 1 we have given the results of a calculation for $\langle\theta^2\rangle$ defined by (16). The values of $U\langle\theta^2\rangle$ for the light elements carbon, nitrogen, and oxygen ($D_A=3.7$) and the heavy elements silver and bromine ($D_A=6.8$) are plotted against $\log(U_0/U)$. The qualitative behavior of the curves is similar to that already discussed in reference 2. Quantitatively, however, there is little similarity in the values obtained, the values of $\langle\theta^2\rangle$ are now smaller by an order of magnitude than those previously obtained.

It should be pointed out that the values of $\langle\theta^2\rangle$ are independent of the theory of meson production employed for nucleon-nucleon collisions. They are equally valid on the plural and multiple hypotheses of meson production. If, however, one assumes that the mesons are produced according to the plural theory, then the values of $\langle\theta^2\rangle$ for mesons would be very close to those we have given for the nucleons.

Messel and Green[1] have pointed out that the differential cross section given by (2), (3), and (4) is quite different from what had hitherto been expected. It imposes upon the scattering angle a condition more stringent than that obtained from relativistic transformations. Strong support for their conclusions is given by the recent results of Branch,[4] who found during an experiment on the angular distribution of penetrating particles, with respect to the shower axis, a mean angle of only $2\frac{1}{2}$ degrees. Further quantitative experimental verification of our results would therefore provide very valuable confirmation of the differential cross section and at the same time allow these results to be used with confidence in interpreting high energy nuclear phenomena.

[4] G. M. Branch, Phys. Rev. 84, 147 (1951).