geometrical cross section of the respective nucleus. Thus, it appears that the ratio between measured and calculated collision lengths should vary from 1 with large nuclear masses to about 1.3 with the lighter nuclear masses. This assumes that this ratio does not vary strongly with the energy of the primary radiation and, as shown by Walker *et al.*,[4] that the collision lengths for non-ionizing and ionizing radiation are the same.

In order to bring the present results into agreement with those quoted above, one is led to assume that the hydrogen nuclei in water have a very small cross section for the production of penetrating showers by non-ionizing primaries. In fact, if one assumes that oxygen nuclei alone are responsible for the penetrating showers observed in this experiment, the calculated collision length becomes 77 g/cm^2, giving just the ratio of 1.3 between measured and calculated collision lengths. George and Jason[2] measured the collision length as $\sim$80 g/cm^2 for the ionizing primaries in paraffin. This result also indicates an extremely low cross section for hydrogen when compared with the result obtained by Walker[4] in carbon. Harding,[14] working with the production of π-mesons in ice, also concluded that π-mesons are produced only in the oxygen nuclei of ice.

It should be pointed out that the results obtained here are contrary to those of Meyer *et al.*,[3] who measured the collision length of the total (ionizing as well as non-ionizing) radiation producing penetrating showers in water as (54 ± 19) g/cm^2. Pomeroy's[1] results in paraffin are also below those of George and Jason.[2]

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[14] J. B. Harding, Phil. Mag. **42**, 621 (1951).

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Possibilities of Heavy Ion Bombardment in Nuclear Studies*

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Bombardment of nuclei by multiply charged ions of medium atomic weight such as C^{12} or O^{16} is considered as a possible means of studying nuclear structure. Estimates are made regarding the approximate magnitude of: (a) distortion effects in target nuclei produced by the incident particles, (b) consequences of the distortion such as effects on thresholds of reactions having their origin in Coulomb barriers, (c) stimulation to fission, (d) effects characteristic of the leakage of neutrons and protons out of the two colliding nuclei by wave-mechanical penetration of the regions of negative kinetic energy; an exploration of these effects should amount to a study of the halo of neutrons and protons surrounding the more compact nuclear interior and might be helpful in determining the number of nuclear particles at the nuclear surface having a given energy. The treatment is qualitative and the mathematical discussion involves many approximations. General design characteristics of a 60-inch cyclotron that should be capable of imparting the necessary energy to multiply charged ions are considered.

I. INTRODUCTION

In the early development of nuclear physics it was important to bombard nuclei with charged particles under conditions which would insure the penetration of the Coulomb barrier. The limited energies available and technical difficulties with ion sources made it desirable, therefore, to choose relatively light particles as the bombarding projectiles, minimizing the loss of useful energy in recoil action and simplifying interpretation of the elementary processes involved.[1] Instrumentation for work along lines of "classical" nuclear physics has developed, therefore, along lines especially suitable for the acceleration of protons, deuterons, and alpha-particles. The interpretation of experiments performed by these means often involves a large amount of mathematical work on account of the necessity of taking into account the wave-mechanical nature of the initial collision process.

We have investigated the possibility of obtaining information regarding nuclear structure by bombarding nuclei with much heavier projectiles. In this case the wave-mechanical features of collision processes are considerably less important, and one may hope that the interpretation of most experiments could be made by considering the approach stage of the process by means of classical mechanics. Of course, quantum mechanics still will have to be used, but its application can be expected to be primarily concerned with questions of the structure of nuclei themselves rather than with the phenomena of the diffraction of waves representing the relative motion of the two colliding parts. The study of the possibilities of obtaining new information

* Assisted by the joint program of the ONR and AEC.
[1] G. Breit, Phys. Rev. **34**, 817 (1929).

by means of heavy ion bombardment was made about two years ago. Since then some of the phases discussed have been touched on in the literature[2] and it appears advisable to publish the considerations in an abridged form, in the hope that the possibilities of gaining knowledge regarding nuclear constitution by means of heavy ion bombardment will be more critically examined.

The possibility of forming transuranic elements by heavy ion bombardment has been demonstrated at Berkeley in experiments with ions of C^{12}. It is only natural to expect that in this new field of work different methods of bombardment will show advantages for different purposes. From the point of view of barrier penetration one would expect sextuply charged C^{12} ions to be about as effective as doubly charged He^4 ions. The new elements berkelium (97) and californium (98) have been formed by He^4 bombardment according to published work.[3] A heavier projectile can be expected to have the advantage of not having to come quite so close to the bombarded nucleus because its radius is greater so that its center may be farther while surface contact is made. This advantage is partly offset by the fact that a greater fraction of the energy goes into kinetic energy of the motion of the center of mass. Thus in the bombardment of U^{238} the factors contributed to the reciprocals of required energies by the increase in radius from He^4 to C^{12} to O^{16} are, respectively, 1.089 and 1.118 on the basis of the $A^{\frac{1}{3}}$ approximation to nuclear radii. The reduced mass factors for the same quantity are 0.968, 0.952. The net gain is 1.054 for C^{12} and 1.064 for O^{16}. A serious disadvantage of the use of heavy ions is the difficulty in obtaining highly charged heavy ions in large quantities. But sextuply charged ions of C are known to exist in cyclotrons and have been successfully used. It can be expected that improved methods will be found.

From the viewpoint of energy available for a reaction at the instant of contact between the surface of the projectile and the target, the advantage is with the single alpha-particle because of the binding of alpha-particles to each other within C^{12} or O^{16}. This energy is 7 Mev for C^{12} and 14 Mev for O^{16}. Per alpha-particle, however, the energy is only about 3 Mev. After the projectile has partially penetrated the target, this energy is not very significant since the energy available for α-emission is often greater than 3 Mev.

At this point the small binding energy per nucleon in a heavy nucleus becomes a disadvantage, since it must be energetically unfavorable in many cases for the alpha-particles to leave the relatively tight binding arrangements which they have either in C^{12} or O^{16}. The bombardment does not have to be made exclusively with nuclei consisting of an integral number of alpha-particles however. By employing such projectiles as B^{11} or C^{13} one can have a lower binding energy per nucleon in the projectile than in the target and the binding energy per nucleon may possibly be lower in the projectile than in the end product. There is besides some of the kinetic energy supplied by the accelerating device to draw on. It may be expected, therefore, that the formation of new elements can be effectively studied by this means. A special advantage is the much larger jump in the atomic number Z which becomes possible in a single reaction. The possibility of moving up into so far unexplored regions of the periodic table appears worth mention. Research along these lines might lead to discoveries of new fissionable elements, and it is likely to result in an improved understanding of the structure of nuclei of transuranic elements. It should give more information concerning the role played by stable shells and it should provide additional tests of the liquid drop model of nuclear structure. There are other related fields such as the chemistry of transuranic elements, β-decay, internal conversion of γ-rays with especially large relativistic effects, additional information on nuclear spins, magnetic moments, and possibly electric quadrupole and octupole moments.

There are furthermore some features of the initial reaction which takes place which appear to deserve attention on their own merits. These are discussed below under the headings of: Distortion Effects in Heavy Nuclei, Distortion Effects in Light Nuclei, Neutron and Proton Penetration Effects, Meson Penetration.

II. DISTORTION EFFECTS IN HEAVY NUCLEI

For purposes of orientation the distortion of a heavy nucleus caused by a charge of Ze will be estimated by means of the liquid drop model. The effect under immediate consideration is that of the distortion which has been held responsible by Bohr and Wheeler[4] for the initial stage of the fission process. The distortion will be first assumed to be of the form

$$\delta r(\theta)=[\alpha_0+\alpha_2 P_2(\cos\theta)],\ (\alpha_0\cong-\alpha_2^2/5), \tag{1}$$

where r is the distance of a point of the nucleus from the center, θ is the angle made by r with the line of collision, and $P_2(x)=(3x^2-1)/2$ is the Legendre coefficient of order 2. The reason for considering this type of distortion first is its connection with fission.

For a point located along line of collision at distance a from center of bombarded nucleus the mutual potential energy between Ze and the distorted charge $Z'e$ is the average over θ and r of

$$\delta\{ZZ'e^2/[r^2+a^2-2ar\cos\theta]^{\frac{1}{2}}\}, \tag{2}$$

where δ represents the change caused by the distortion. On the incompressible fluid model the part of the mutual potential energy attributable to α_2 is thus

[2] Norman F. Ramsey, Phys. Rev. **83**, 659 (1951).

[3] Thompson, Ghiorso, and Seaborg, Phys. Rev. **80**, 781 (1950); Thompson, Street, Ghiorso, and Seaborg, Phys. Rev. **80**, 790 (1950).

[4] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

found to be

$$(3/5)(R^2/a^3)\alpha_2 ZZ'e^2, \tag{3}$$

where R is the radius of the bombarded nucleus. According to Bohr and Wheeler,[4] the electrostatic self energy of the nucleus contains an α_2-dependent part,

$$-(3/25)\alpha_2^2(Z'e)^2/R, \tag{4}$$

which is overcompensated by an effect of the surface tension so as to result in

$$(3/25)q\alpha_2^2(Z'e)^2/R, \tag{5}$$

where q is a positive number for stable nuclei. Static distortion corresponds to a minimum in the potential energy. The value of α_2 for static distortion is thus found to be

$$\alpha_2 = -(5/2)(R/a)^3(Z/Z'q). \tag{6}$$

The nuclear distortion caused by electrostatic effects is seen to increase with Z, the charge on the projectile. It is sensitive to the distance of approach a, increasing rapidly as a decreases and is proportional to $1/q$. The latter quantity is a measure of approach to instability to fission. For the collision of O^{16} with U^{238} the values are $Z=8$, $a/R=1+(16/238)^{\frac{1}{3}}=1.406$. The value of q may be formally calculated according to the considerations in the paper of Bohr and Wheeler, making use of the value of the surface tension constant given in their paper. The value thus obtained depends somewhat on the choice of the constant $r_0 \cong 1.4\times10^{-13}$ cm and varies between 1.29 and 1.37 in reasonable agreement with Gamow and Critchfield's book.[5] The value 0.17 stated in the paper by Bohr and Wheeler[4] is low but is likely to correspond more closely to the physical situation as will be discussed later. Employing $q=0.33$ one finds for the static value of α_2,

$$(\alpha_2)_{\text{Static}} = -0.24 \quad (q=0.33). \tag{7}$$

The value of α_2 gives the maximum fractional change of the distance from the center, and the rather large number obtained represents a change which may have effects on the reaction mechanism which can be distinguished from effects expected on the compound nucleus view. In the expreme case of contact when the bombarded nucleus is ready to separate by fission, the relative probabilities of different reaction products would have some of the characteristics of the fission fragments. The estimated distortion is not large enough to expect this phenomenon in a clear cut way but the approach to this condition should be accessible to investigation.

The change is largely geometrical. With the numbers used above, it corresponds to a change in the electrostatic self-energy of only

$$(3/25)(92)^2(0.510/3.3)\alpha_2^2 = 157\alpha_2^2 \text{ Mev} = 8.8 \text{ Mev}. \tag{8}$$

It should be mentioned that the value of q used in this estimate corresponds to an over-all view concerning nuclei and does not take account of the occurrence of spontaneous fission. The latter phenomenon is a sign of nuclear instability, and a theory accounting for it may be expected to give larger distortions than those obtained above. The elementary model employing only α_2 requires, for example, a vanishing q to make a nucleus barely stable. While actually the instability is connected with higher modes, the net effect is similar to that of a smaller q. It may be expected, therefore, that the static values of $|\alpha_2|$ are in many cases appreciably greater than those calculated.

The effective Coulomb barrier height is diminished by the addition of the amount

$$-(3/25)((Z'e)^2/R)q\alpha_2^2, \tag{9}$$

where the value of α_2 for the potential minimum is meant. For $Z'=92$, $Z=8$ this is approximately $-158q\alpha_2^2 \cong -2.9$ Mev. Calculation on the approximate basis of employing α_2 only gives

$$-(3/4q)(R/a)^5(Z^2e^2/a). \tag{10}$$

This formula shows that the effect is sensitive to Z and is inversely proportional to q. It is expected, therefore, that electrostatic polarization effects will decrease the barrier height more rapidly as the charge of the bombarding nucleus is increased and that the effect will be largest for the less stable bombarded nuclei. An experimental test of the conditions for barrier decrease when combined with a more complete theory should throw light on the features of nuclear structure connected with fission.

Another estimate of the order of magnitude of the expected effects can be made by considering the energy evolved when an alpha-particle is moved across a diameter of the bombarded nucleus in the electric field of the bombarding particle. This energy is

$$4Ze^2R/(a^2-R^2), \tag{11}$$

where R is the radius of the target nucleus and a is the distance between the centers of the bombarding and target nuclei. The reason for considering an alpha-particle is that it is a relatively stable part of the nucleus and that its displacement as a unit is more probable than that of other composite nuclear parts. The only property of the alpha-particle used here is its charge and the question of preservation of identity of the alpha-particle does not enter directly. For $R=3e^2/mc^2=8.4\times10^{-13}$ cm the energy evolved is

$$(4mc^2/3)Z/((a/R)^2-1), \tag{11'}$$

where Ze is the charge on the bombarding particle. This quantity is seen to be sensitive to (a/R) and the maximum value is reached for the smallest a. Taking $a/R=1+(16/238)^{\frac{1}{3}}=1.406$ so as to correspond to contact between O^{16} and U^{238}, this energy is $1.375Zmc^2$. For O^{16} one has $Z=8$ and the energy is $11mc^2$. This amount

[5] G. Gamow and C. L. Critchfield, *Theory of Atomic Nucleus and Nuclear Energy Sources* (Oxford University Press, London, 1949).

of energy corresponds to a change in electrostatic self-energy for a value of α_2 obtainable from

$$(3/25)\alpha_2^2(Z'e)^2/R=11mc^2, \qquad (12)$$

which gives $\alpha_2=0.19$, an amount roughly equal to that obtained from the consideration of the mutual energy of a $P_2(\cos\theta)$ type of distortion with the field of the charge Z. Since there are many α-particles, the mutual energy with the charge Ze for a P_2 type of distortion is seen to be a relatively unfavorable criterion of the amount of energy available to the bombarded nucleus. If there should be conversion of energy by dynamic internal effects from one Legendre function mode of distortion to another then the energy change caused by the displacement of a small fraction of the nuclear charge across a nuclear diameter can give rise to energy changes which when expressed in terms of the self-energy of the mode P_2 correspond to very appreciable deformations.

The estimates for the α-particles are seriously modified by corrections for dynamic effects and other factors, as will be seen presently.

So far all effects have been considered statically. They will be reduced by inertia effects. An estimate of the reduction for the P_2 mode can be made as follows. The incident nucleus of mass M at the turning point is performing approximately uniformly accelerated motion with the acceleration

$$ZZ'e^2/Ma^2. \qquad (13)$$

The time required to cover a distance Δa is then

$$t=[2Ma^2\Delta a/ZZ'e^2]^{\frac{1}{2}}. \qquad (14)$$

The potential energy of the P_2 mode for U^{238} according to Bohr-Wheeler[4] is

$$V=(3/25)((Z'e)^2/R)\alpha_2^2q, \qquad (15)$$

where $q\cong0.17$. The kinetic energy is approximately

$$(3M'/20)R^2(d\alpha_2/dt)^2. \qquad (16)$$

The angular frequency is therefore

$$\omega=[(4q/5M')((Z'e)^2/R^3)]^{\frac{1}{2}} \qquad (17)$$

and

$$\omega t=[(8/5)q(MZ'/M'Z)(e^2/Rmc^2) \times(\Delta a/e^2/mc^2)]^{\frac{1}{2}}(a/R). \qquad (18)$$

For $q=0.17$, $M=16$, $M'=238$, $a/R=1+(16/238)^{\frac{1}{3}}$, $R=3.3e^2/mc^2$ this formula reduces to

$$\omega t\cong0.35(\Delta a/e^2/mc^2)^{\frac{1}{2}}. \qquad (19)$$

The value $q=0.17$ rather than 0.33 is used here so as to take partial account of the smaller stability of nuclei indicated by spontaneous fission. It would have been more consistent to use the same q everywhere, but the whole effect amounts to a factor 2 only while the smaller effective mass and higher frequency of other modes will give smaller dynamic effects. The time involved is approximately doubled on account of the return path. It will also be effectively somewhat longer because the acceleration decreases as the distance increases. Since $a\cong1.4\times3.3e^2/mc^2=4.6e^2/mc^2$, it appears fair to use $\Delta a=2e^2/mc^2$ leading to $2\omega t=0.35(2\sqrt{2})=1.00$. The distortion attained at the end of the period $2t$ can be expected to be of the order $1-\cos(2\omega t)$ of the static value, i.e., about 0.47 of the static value. The potential barrier may be expected to be reduced by only 0.47×2.9 Mev$=1.4$ Mev according to this estimate. It should be pointed out that for small values of ωt the dynamic correction makes the whole effect independent of q, because to a first approximation the dynamic correction factor is $(\omega t)^2/2$ which is proportional to q while the static effect contains the factor $1/q$. It is unlikely, however, that the distortion of the whole nucleus can follow the liquid drop model so closely as to make this result more than formally right. Deviations of the order of the whole shift can be expected as a result of differences in the structure of bombarded nuclei. Such differences would be detectable as irregularities in the variation of barrier height with mass number which would not correlate with irregularities of corresponding curves for alpha-emission and bombardment by lighter projectiles. The assumptions made above are admittedly crude and the only object of the estimates is that of drawing attention to the possibility that such effects may be detectable.

An alpha-particle inside a big nucleus moves through the distance

$$\tfrac{1}{2}(2Ze^2t^2)/M_\alpha(a+xR)^2 =2(a/(a+xR))^2(M/M_\alpha)(\Delta a/Z') \qquad (20)$$

during the approach time t. Here $-1<x<1$ and changes in the number x correspond to varying the position of the alpha-particle in the nucleus. For uranium bombarded by oxygen and $\Delta a=2e^2/mc^2$ this formula gives for the distance traveled by the alpha-particle the values $(0.23, 0.37, 8.3)(e^2/mc^2)$ for $x=(1, 0, -1)$ including the time taken for recession of the oxygen nucleus. The last of these numbers corresponds to an alpha-particle located in the most favorable position and is an overestimate because the distance from the bombarding oxygen changes rapidly while the alpha-particle is moving. Correcting for this fact one obtains a much smaller value for the third number. The numbers $(0.23, 0.37)e^2/mc^2$ correspond to $\alpha_2\sim0.1$, in general agreement with the estimate made on the liquid drop model. The effects on parts of a nucleus are seen to be of the same order as the general α_2 distortion and can combine with it. The result can be expected to depend on details of nuclear structure. It appears justifiable to conclude that there is likelihood of appreciable distortion of a heavy nucleus such as U^{238} when it is bombarded by O^{16}. The effects of distortion can be expected to be:

(a) A change in relative intensities of reaction products and ejected particle groups corresponding to levels of the residual nucleus in comparison with bombardment by projectiles having a small Z.

(b) Some modes of excitation which can be produced by the influence of the electric field in the approach stage should be capable of excitation independently of the charge of the projectile provided the energy available in the transfer process is sufficient. This energy decreases with the charge Z and it is not intended to claim that for low Z such effects will occur.

(c) It appears possible that fission into fragments of roughly equal size might become stimulated by such bombardment. Once contact between projectile and target has been established nuclear forces will cause additional polarization and distortion effects.

(d) A change in the Coulomb barrier. Such a change should show itself in the effective thresholds of reactions. For large masses leakage through the barrier is a relatively small effect and to a good approximation an exoergic reaction may be considered to have a threshold at an energy just sufficient to overcome the Coulomb barrier.

A study of the barrier thresholds may be expected to be of value also in connection with theories predicting certain nuclear radii, shell structure, quadrupole moments, etc.

III. DISTORTION EFFECTS IN LIGHT NUCLEI

The order of magnitude of expected effects can be seen by considering the collision of two oxygen nuclei with each other. The difference in potential energy of an alpha-particle at opposite ends of a nuclear diameter is in the notation of the preceding section

$$\delta V = 32(e^2/R)/((a/R)^2-1). \tag{21}$$

Taking $R=1.5\times(16)^{\frac{1}{3}}\times10^{-13}\,\text{cm}=3.78\times10^{-13}\,\text{cm}=1.34\times(3)e^2/mc^2$,

$$\delta V = 23.8mc^2/((a/R)^2-1). \tag{21'}$$

For contact $a=2R$ and

$$\delta V = 7.9mc^2 = 4.0(5)\ \text{Mev}. \tag{21''}$$

The binding energy of an alpha-particle in O^{16}, for dissociation into $C^{12}+He^4$, is 7.2 Mev and is greater than δV. The binding energy of the last neutron in O^{17} is about 4 Mev. Binding energies of this order are not uncommon for the last proton or neutron. A transfer of the whole energy from the displacement of an internal alpha-particle to a proton or neutron would result in ionization in some cases. Occasional strong polarization effects may be expected, therefore, on a purely static basis.

Since the charge-mass ratio is the same for the alpha particle as for the oxygen nucleus one expects the velocities acquired by the internal alpha-particle and by the bombarding O^{16} nucleus to be comparable.[6] The distance traveled by the alpha-particle will be decreased by the binding and the effect cannot be expected to be as large as above estimates seem to indicate. Nevertheless, the dynamic effects are not expected to decrease the effect considered in a profound manner.

In light nuclei as well as in heavy ones the direct transfer of one or two protons from one part of a nucleus to another is also of possible importance, especially in cases of loose binding.

IV. NEUTRON AND PROTON PENETRATION EFFECTS

The penetration effects to be discussed in this section are closely related to the Oppenheimer-Phillips[7] process. Even when the nuclei are not in direct contact a neutron or a proton can be expected to be able to change partners.

In this connection it is especially interesting to consider nuclei with a low binding energy of the last neutron or proton. The wave function of a neutron in the region outside the nucleus; i.e., outside the region within which it is acted on by other nuclear particles contains the factor

$$e^{-\alpha r},$$

where

$$\alpha = (2|E|/mc^2)^{\frac{1}{2}}/[\hbar(M_n m)^{\frac{1}{2}}c]. \tag{22}$$

Here E is the binding energy of the neutron, M_n is its mass, and m is the electronic mass. The length $\hbar/(M_n m)^{\frac{1}{2}}c=9.0\times10^{-13}$ cm. In the distance $1/\alpha$ the chance of finding a neutron per unit distance from center decreases by $(1/2.718)^2=1/7.4$. The chance of the neutron being farther than this distance from the nuclear surface is 13 percent, a non-negligible amount. For Be^9, $|E|=3.3mc^2$ and $1/\alpha=3.5\times10^{-13}\,\text{cm}=1.25e^2/mc^2$. For O^{17}, $|E|=8.1mc^2$ and $1/\alpha=2.2\times10^{-13}\,\text{cm}=0.8e^2/mc^2$. For reactions which depend only on contact of the bombarded nucleus with a neutron from the incident projectile, the effective Coulomb barrier may be considered for this reason as being lower by roughly 25 percent in the bombardment of heavy nuclei. It will be noted that:

(a) On account of neutron penetration effects the Coulomb barrier threshold may be expected to become diffuse. The influence of the Coulomb barrier is much more pronounced than the relatively mild exponential decrease associated with neutron penetration.

(b) The energy dependence of the reaction, when analyzed in terms of the expected and calculable Coulomb barrier penetration effect, should give an indication of the presence of a milder low energy cutoff when the reaction may be caused by contact with a neutron of the bombarded nucleus. The energy de-

[6] If the charge of the bombarded nucleus is $Z'e$, the acceleration of the bombarding nucleus is $ZZ'e^2/M'a^2$, where Ze and M are the charge and mass of the projectile. The acceleration of the alpha-particle is $2Ze^2/M_\alpha a^2$, where M_α is the mass of the alpha-particle. The ratio of the two accelerations is $(Z'/M)/(2/M_\alpha)$. In cases under discussion $Z'\cong Z$ and the accelerations are comparable, therefore.

[7] J. R. Oppenheimer and M. Phillips, Phys. Rev. 48, 500 (1935).

pendence of the reaction may thus become an indicator of the reaction's origin.

(c) By performing experiments capable of detecting small collision cross sections one can extend the region within which the reaction is detectable considerably below the threshold which would be estimated from Coulomb barrier considerations. If by increasing the sensitivity of the experiment the Coulomb barrier requirement is decreased by the factor 1/1.25, then increasing the sensitivity by a factor $7.4^3=400$ the energy requirements are decreased by the factor $(1/1.25)^3=1/2$. Sensitive measuring methods may be expected to increase the variety of reactions beyond the limits attainable on the basis of barrier estimates and could do part of what otherwise would have to be accomplished by the more expensive method of building bigger machinery.

(d) In reactions which have been studied so far there is seldom a possibility of investigating the diffuseness of the nuclear boundary. The reason is that α-particles, protons, and neutrons are themselves very compact structures. In the deuteron the Oppenheimer-Phillips process does indeed give evidence of the neutron halo. The theory cannot be made very clear cut, however, because the ratio of the masses of the deuteron and neutron is only 2 so that a definite separation into classical and quantum mechanical components is impossible.

Protons should also show penetration phenomena under suitable conditions. These are harder to satisfy than for neutrons. The leakage of a proton out of a light nucleus with $Z\sim 8$ is not very different from that of a neutron, the Coulomb barrier being approximately $3mc^2$. The entrance of the proton into a nucleus such as U is seriously affected, however, by a potential barrier of $\sim 30mc^2$. Special resonance conditions are needed to make the penetration into the heavier nucleus effective. For lighter target elements, however, the penetration effects become appreciable. They offer possibilities of enriching the knowledge of nuclear reactions and structure.

Penetration of particles through regions of negative kinetic energy can be expected to be affected by nuclear shell structure. Thus, for example, there is evidence in Schmidt's suggestion concerning nuclear magnetic moments[8] that an odd neutron or proton is mainly outside the completed shells. The penetration of such a particle through the region of negative kinetic energy outside the nucleus should take place more easily as a result of such a favorable geometrical arrangement. The contributions to the neutron atmosphere arising from different shells may be expected to act in different ways for different reactions.

V. MESON PENETRATION

The present section is written with much reservation, in the spirit of speculative consideration of a possibility which is hard to exclude and without emphasis on its probability. Since π-mesons are lighter than neutrons or protons they should penetrate regions of negative kinetic energy more readily than either of the nucleons, presupposing equal energy differences. In a light nucleus the energy needed to make the emission of a π-meson possible is of the order of the meson mass energy and the penetration distance is of the order of the range of nuclear forces. In this case there is no clear way of differentiating between meson and nucleon penetration. For heavy nuclei, however, one is in a region of instability towards disintegration by fission. The fission process liberates an energy of $\sim$200 Mev which is in excess of the meson rest mass. There exist, therefore, states for which the energy is nearly the initial one and for which the kinetic energy of a π-meson is positive or only slightly negative. It is possible that the chance of formation of such states is too small for them to be of practical importance. An estimate of this chance depends on the relative importance of binding of mesons to individual nucleons as compared with binding to the nucleus originating in the cooperative action of many nucleons. If the latter is important then one may expect the states with a free meson to participate in the reactions and the escape of mesons out of the heavy nucleus to be a factor also. The whole question is intimately connected with the relative importance of two-body in comparison with many-body forces and the allied distinctions between weak-coupling and strong-coupling meson theories.

Avoiding mathematical complexities the difference between the points of view which could be distinguished by such experiments can be somewhat crudely described in the following manner. Two extreme and oversimplified pictures of the role played by mesons in a heavy nucleus are: (a) The mesons are permanently attached to one or another nucleon. Their participation in the mechanism of nuclear binding does not change appreciably the density of virtual mesons around an individual nucleon. The exchange of mesons between a pair of nucleons is not appreciably affected by the presence of other nucleons. (b) The virtual mesons form essentially an envelope or cement around a large group of nucleons. The density of virtual mesons in a particular location has no simple relation to that expected for any one nucleon. A given meson can have an appreciable energy exchange with a whole group of nucleons at once.

From a theoretical viewpoint these questions are difficult and full of speculation. An experimental result containing an indication concerning the actual state of things would be welcome.

VI. APPROXIMATE DESIGN CHARACTERISTICS OF EQUIPMENT SUITABLE FOR THE SUGGESTED INVESTIGATIONS

There are good possibilities of obtaining the necessary energies by accelerating ions in a cyclotron and

[8] T. Schmidt, Z. Physik **106**, 358 (1937).

TABLE I. Values of maximum energy in Mev obtainable with $\mathcal{H}_{k\text{ gauss}}=15$, $D_{\text{in.}}=60$.

Z^*	$M=1$	2	4	12	16	20
1	49.46	24.78	12.34	4.11	3.18	2.57
2			49.4	16.4	12.3	9.9
3				37.0	27.8	22.1
4				66	49	39.5
5				103	77	62
6				148	111	89

there are some possibilities of attaining the same end by means of Van de Graaff electrostatic machines. In the following discussion, estimates of the energies attainable by the cyclotron method are made and requirements for producing intimate contact between nuclear surfaces are indicated.

Notation

D = diameter of pole piece,
$D_{\text{in.}}$ = diameter of pole piece in inches,
ρ = radius of particle orbit,
Z^* = charge carried by ion in units of the positron charge,
$\mathcal{H}_{\text{gauss}}$ = magnetic field in gauss,
$\mathcal{H}_{k\text{ gauss}}$ = magnetic field in kilogauss,
f = frequency of revolution of particle in orbit-frequency of alternating accelerating field,
$f_{\text{Mc/sec}}=f$ measured in megacycles/sec,
M = mass of ion on nuclear scale,
E_{Mev} = energy of ion on emergence measured in Mev,
Z = atomic number of bombarding nucleus,
Z' = atomic number of bombarded nucleus,
A = mass number of bombarded nucleus.

It is assumed below that

$$D=2.26\rho. \tag{23}$$

The following approximate relations hold:

$$f_{\text{Mc/sec}}D_{\text{in.}}=196.(6)(E_{\text{Mev}}/M)^{\frac{1}{2}}, \tag{24}$$

$$\mathcal{H}_{k\text{ gauss}}D_{\text{in.}}=128.1(ME_{\text{Mev}})^{\frac{1}{2}}/Z^*, \tag{25}$$

$$f_{\text{Mc/sec}}=1.535\mathcal{H}_{k\text{ gauss}}Z^*/M. \tag{26}$$

For $D_{\text{in.}}=60$, $\mathcal{H}_{k\text{ gauss}}=15$ Eq. (3) gives the maximum energy as

$$E_{\text{Mev}}=49.3(6)Z^{*2}/M, \tag{25'}$$

so that the energies obtainable are as in Table I.

TABLE II. Values of frequency in megacycles per second corresponding to operation at maximum energy.

Z^*	$M=1$	2	4	12	16	20
1	23.0(2)	11.5	5.8	1.92	1.44	1.15
2			11.5	3.8	2.88	2.30
3				5.8	4.3	3.46
4				7.7	5.8	4.6
5				9.6	7.2	5.8
6				11.5	8.6	6.9

The frequencies which correspond to this operation are given in Table II. It is seen from this table that the frequency required in the acceleration of heavy ions to the energies listed in Table I are lower than those required for protons. Since beyond $M=1$ there are no nuclei with $Z/M>2$, the frequency does not exceed that required in the acceleration of deuterons. The values $Z^*=5$, 6 would be used only infrequently and it is seen, therefore, that the frequencies required are very moderate. The technical difficulty of obtaining the dee voltage required for good focusing is reduced and reliable operation may be expected. In Tables I and II only bombarding nuclei of masses 1, 2, and multiples of 4 are considered.

The values of the maximum attainable energy may be compared with the Coulomb barrier height B and the energy E_B which must be given to the bombarding particle in order that the kinetic energy available in the center-of-mass system be equal to B. In the absence of distortion and polarization effects the two nuclei may be expected to come in contact for $E\geqq E_B$. Values of B and E_B for bombardment by C^{12} are listed in Table III. In the calculation of numbers for Table III the radius of a nucleus with mass number A was assumed to be $1.5\times10^{-13}A^{\frac{1}{3}}$ cm. On this basis

TABLE III. Values of barrier height B and of threshold energy E_B for bombardment by C^{12}.

Bombarded nucleus	C^{12}	Ne^{20}	Ca^{40}	Zn^{64}	Sn^{120}	Hg^{200}	U^{238}
B (Mev)	7.5	11.5	20.1	27.4	40	56	62
E_B (Mev)	15.0	18.3	26.2	32.5	44	60	65

$$B=0.96[ZZ'/(M^{\frac{1}{3}}+A^{\frac{1}{3}})]\text{ Mev}, \tag{27}$$

$$E_B=[(M+A)/A]B. \tag{28}$$

Comparison of Table I with Table III shows that contact of the nuclear surface can be expected to be established for $Z^*=4$ even for $Z'=92$. By increasing Z^* one increases E quadratically while according to Eq. (5) B increases with Z only linearly. For $Z^*=6$ one can expect contact with U^{238} for bombardment with F^{19}.

It has been attempted above to cover in outline some of the more obvious subjects which suitable equipment would make it possible to investigate. As is usual in new fields the most important may have remained unthought of. It is hoped, however, that the essentially exploratory character of this field of work has been brought out.

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