The 7.4-Mev State of Li^7

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Blair and Holland have reported preliminary measurements of the cross section for the $Li^6(n,\alpha)$ reaction as a function of the neutron energy. Roberts *et al.* have measured the angular distribution of the tritons for a neutron energy of 270 kev. Assuming that the nonresonance part of the cross section is due entirely to s-neutrons and that there is no accidental degeneracy of the resonance state, this state is shown to have spin $\frac{3}{2}$.* Experiments which might test the assumptions are discussed.

1. INTRODUCTION

THE compound state of Li^7 formed in the $Li^6(n,\alpha)$ reaction at resonance is of interest because the shell model leads to a spin of 5/2 for this state, while the model of Inglis[1] permits 3/2. Connected with the interpretation of this reaction is the question whether the radius of the Li^6 nucleus is of normal size ($\sim 1.4A^{\frac{1}{3}} \times 10^{-13}$ cm) or whether it is abnormally large, as might be expected if it consists, roughly speaking, of an alpha-particle and a deuteron.

The absorption cross section measurements of Blair and Holland[2] show a resonance maximum of 3.1 barns, 2.5 barns of which is actually due to the resonance, at 250 kev with width at half-maximum about 100 kev. We understand from Dr. Holland that these results are regarded as preliminary and that the absolute calibration may be in error by as much as 20 percent. Roberts and co-workers[3] have measured the angular distribution $I(\theta)$ of the tritons at 270 kev and found (in the c.m. system)

$$I(\theta) = 103 + 83 \cos\theta + 192 \cos^2\theta. \tag{1}$$

Roberts' relative coefficients are estimated to be accurate to about 20 percent.

We propose to show that the combined results of Roberts and of Blair and Holland are, within their present uncertainties, compatible with the simplest possible assumptions about the $Li^6(n,\alpha)$ reaction. We assume that the nonresonance part of the cross section at 270 kev is due to s-neutrons alone and that there is no accidental degeneracy of the compound state. Then we will show that the data are compatible with, and only with, spin 3/2 for the compound state. Only a small improvement of the data is needed to clarify the situation appreciably. In particular, if it turns out that the total cross section is really as high as the present most probable value and that our first assumption is confirmed by new measurements of the angular distribution, then the compound state must be degenerate.[4]

2. PRELIMINARY REMARKS

Our first assumption implies that the nuclear radius r of Li^6 is smaller than $\lambda\!\!\!^{-} = (1/2\pi) \times$(neutron wavelength)$= 0.95 \times 10^{-12}$ cm at 250 kev. The existence of the $\cos\theta$-term requires that at least one odd and one even component of the incident neutron wave are absorbed, so that the excited Li^7 nucleus does not have definite parity. The resonant part must now be due to the p-wave. Higher odd angular momenta are excluded by the large height and width of the resonance. The neutron width Γ_n is in fact limited[5] by

$$\Gamma_n \leqq (3\hbar^2/mr\lambda\!\!\!^{-})T_l = 1500 T_l \text{ kev}, \tag{2}$$

where m is the neutron mass and the factors T_l depend only on $r/\lambda\!\!\!^{-}$, rising for f-neutrons from $T_3 = 1/225000$ for $r/\lambda\!\!\!^{-} = 0.3$ to $T_3 = 1/275$ when $r/\lambda\!\!\!^{-} = 1$. For higher angular momenta T_l decreases rapidly with increasing l. Even if $r/\lambda\!\!\!^{-} = 1$ (which would actually contradict our assumption of s-waves only for the nonresonance absorption), we would thus have $\Gamma_n \leqq 4.1$ kev, while the actual width is 100 kev, so that not even several closely spaced f-neutron resonances could account for the observed line width. If, contrary to the usual situation in this energy range, the absorption width $\Gamma_a \gg \Gamma_n$ so that Γ_a would be the total width, Γ_n/Γ_a would have to be less than 0.04 for f-neutrons and the cross section would be limited to much less than one barn. These considerations strengthen the observation of Roberts that no $\cos^4\theta$-term appears in the angular distribution.[6]

Since the ground state of Li^6 has angular momentum 1, this limits us to the possibilities of one or more p-neutron resonances or to compound nuclei of total angular momentum 1/2, 3/2, and 5/2. The state $J = 1/2$ will lead to a triton distribution without a $\cos^2\theta$-term and can therefore be excluded unless it is nearly degenerate with a state of $J = 3/2$ or 5/2.

* *Footnote added in proof:*—Dr. Roberts has recently informed us that his preliminary result may require revision. Any effect of this revision on our conclusions will be seen by substituting the revised coefficients into our equations (12) and (16).

[1] D. R. Inglis, Phys. Rev. **85**, 492 (1952). We wish to thank Dr. Inglis for several enlightening discussions on this point.

[2] J. M. Blair and R. E. Holland (to be published). See R. K. Adair, Revs. Modern Phys. **22**, 249 (1950) for preliminary data.

[3] Roberts, Darlington, and Haugsnes, Phys. Rev. **82**, 299 (1951). We are indebted to Dr. Roberts for supplying us with his corrected data prior to publication.

[4] One such case is calculated in the Appendix.

[5] E. P. Wigner, Am. J. Phys. **17**, 99 (1949).

[6] By virtue of a theorem derived by E. Eisner and R. G. Sachs, Phys. Rev. **72**, 680 (1947) and L. Wolfenstein and R. G. Sachs, Phys. Rev. **73**, 528 (1948).

The total absorption cross section is limited by the inequality,[7]

$$\sigma_a \leqq \tfrac{1}{6}(2J+1)\pi\lambda\!\!\!^{-2}; \quad (3)$$

i.e., $\sigma_a \leqq 2.12$ barns for $J=3/2$ and $\sigma_a \leqq 3.07$ barns for $J=5/2$. To compare these figures to the measured values we must subtract the nonresonance part of the experimental cross section and multiply the result by 13.3 to correct for the abundance of Li^6. This corrected experimental value is about 2.5 barns, which is consistent with either spin value within the uncertainty of the calibration. To obtain further information we must calculate the angular distribution.[8]

3. CALCULATIONS

We denote by X_{jm} ($j=1/2,\ 3/2$) the spin wave functions of the combined neutron and Li^6 spins, and by $Y_{00}{}^N + AY_{10}{}^N$ the s and p parts of the orbital wave function of the incident neutron wave. Those "outside" wave functions of Li^6 and n which are contained in the incident plane wave are denoted by

$$u_{jm} \equiv X_{jm}(Y_{00}{}^N + AY_{10}{}^N). \quad (4)$$

Of all outside wave functions having orbital angular momentum 0 or 1, we form linear combinations $\phi_{Jm}{}^{(e)}$, $\varphi_{Jm}{}^{(0)}$, $\phi_{Jm}{}^{(0)}$ of definite angular momentum J. The superscripts indicate the parity and $\varphi_{Jm}{}^{(0)}$ and $\phi_{Jm}{}^{(0)}$ contain the spin functions $X_{\frac{1}{2}m}$ and $X_{\frac{3}{2}m}$, respectively. For the wave functions of even parity $l=0$, so that $J=j$ and there is no ambiguity:

$$u_{\frac{1}{2}m} = \phi_{\frac{1}{2}m}{}^{(e)} + A\sum_{J=\frac{1}{2}}^{\frac{3}{2}} C_J{}^{(\frac{1}{2})}\varphi_{Jm}{}^{(0)}, \quad (5)$$

$$u_{\frac{3}{2}m} = \phi_{\frac{3}{2}m}{}^{(e)} + A\sum_{J=\frac{1}{2}}^{\frac{5}{2}} C_{Jm}{}^{(\frac{3}{2})}\phi_{Jm}{}^{(0)}, \quad (6)$$

where $C_J{}^{(\frac{1}{2})}$ and $C_{Jm}{}^{(\frac{3}{2})}$ are known coefficients.[9] In this form the incident wave functions can be tied in with "inside" wave functions of the excited nucleus Li^7. Through these, they will be coupled to "outside" T, α-wave functions $\psi_{Jm}{}^{(0)}$ and $\psi_{Jm}{}^{(e)}$ with unknown coefficients $p^{(j)}$ and $q_J{}^{(j)}$ which depend on J, j and parity, but not upon m:

$$u_{\frac{1}{2}m} \rightarrow p^{(\frac{1}{2})}\psi_{\frac{1}{2}m}{}^{(e)} + \sum_{J=\frac{1}{2}}^{\frac{3}{2}} q_J{}^{(\frac{1}{2})}C_J{}^{(\frac{1}{2})}\psi_{Jm}{}^{(0)}, \quad (7)$$

$$u_{\frac{3}{2}m} \rightarrow p^{(\frac{3}{2})}\psi_{\frac{3}{2}m}{}^{(e)} + \sum_{J=\frac{1}{2}}^{\frac{5}{2}} q_J{}^{(\frac{3}{2})}C_{Jm}{}^{(\frac{3}{2})}\psi_{Jm}{}^{(0)}. \quad (8)$$

Apart from the improbable event of an accidental degeneracy or near degeneracy, the coefficients $q_J{}^{(j)}$ vanish except for one value of J. We shall, therefore, proceed with the alternatives $J=3/2$ or $5/2$, since $J=1/2$ could not yield a $\cos^2\theta$-term in the angular distribution.

The functions $\psi_{Jm}{}^{(0)}$ and $\psi_{Jm}{}^{(e)}$ must be expressed as linear combinations $\chi_\pm{}^T Y_{lm'}$ where $\chi_\pm{}^T$ is the spin wave function of the triton and $Y_{lm'}$ is the orbital wave function for the T, α-wave. There is no excited bound state for either triton or α-particle, and the ground states are even 1/2 and even 0, respectively.

Case I: $J=5/2$.

Using the abrreviations $p^{(\frac{3}{2})}=a$, $p^{(\frac{1}{2})}=b$, and $q_{\frac{5}{2}}{}^{(\frac{3}{2})}=\sigma$, we obtain

$$u_{\frac{1}{2}m} \rightarrow b\delta_{m\frac{1}{2}}Y_{00}\chi_+{}^T, \quad (9)$$

$$u_{\frac{3}{2}m} \rightarrow \left[a\left(\frac{5-2m}{10}\right)^{\frac{1}{2}} Y_{2,m-\frac{1}{2}} + \sigma\left(\frac{25-4m^2}{40}\right)^{\frac{1}{2}}\left(\frac{7-2m}{14}\right)^{\frac{1}{2}} Y_{3,m-\frac{1}{2}}\right]\chi_+{}^T, \quad (10)$$

where we have written the $C_{Jm}{}^{(j)}$ explicitly and omitted the terms with $\chi_-{}^T$. The intensity I is equal to the sum of the absolute squares of the coefficients of $\chi_+{}^T$, summed over j and m:

$$I(\theta) = |a|^2 + \tfrac{1}{2}|b|^2 + (3/10)|\sigma|^2 + (3/\sqrt{10})(a^*\sigma + a\sigma^*)\cos\theta + \tfrac{3}{5}|\sigma|^2\cos^2\theta. \quad (11)$$

By comparing Eq. (11) with Roberts' experimental results we have $|\sigma|^2=320$, $|a|^2+\frac{1}{2}|b|^2=7$. From this we obtain the ratio of the resonance part of the total absorption cross section $[(3/10)|\sigma|^2+\frac{1}{3}\times\frac{3}{5}|\sigma|^2]$ to the nonresonance part $[|a|^2+\frac{1}{2}|b|^2]$ as

$$\tfrac{1}{2}|\sigma|^2/(|a|^2+\tfrac{1}{2}|b|^2) = 23. \quad (12)$$

The data of Blair and Holland, however, give this ratio as about 3. The coefficient of $\cos\theta$ in Eq. (1) must on the same basis be smaller in magnitude than

$$(3/10^{\frac{1}{2}})2(320)^{\frac{1}{2}}7^{\frac{1}{2}} = 90,$$

which is not in contradiction with present data.

Case II: $J=3/2$.

Using $p^{(\frac{3}{2})}=a$, $p^{(\frac{1}{2})}=b$, $q_{\frac{3}{2}}{}^{(\frac{3}{2})}=\rho$, and $q_{\frac{3}{2}}{}^{(\frac{1}{2})}=\tau$ and again writing only the terms in $\chi_+{}^T$, we have

$$u_{\frac{1}{2}m} \rightarrow \left[b\delta_{m\frac{1}{2}}Y_{00} + \tau\left(\frac{3+2m}{6}\right)^{\frac{1}{2}} Y_{1,m-\frac{1}{2}}\right]\chi_+{}^T, \quad (13)$$

$$u_{\frac{3}{2}m} \rightarrow \left[a\left(\frac{5-2m}{10}\right)^{\frac{1}{2}} Y_{2,m-\frac{1}{2}} + \rho\frac{2m}{(15)^{\frac{1}{2}}}\left(\frac{3+2m}{6}\right)^{\frac{1}{2}} Y_{1,m-\frac{1}{2}}\right]\chi_+{}^T. \quad (14)$$

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[7] Freshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).

[8] Although the general problem of angular distributions has been calculated by J. M. Blatt and L. C. Biedenharn [Phys. Rev. **82**, 123 (1951) and preliminary mimeographed publication], it is more convenient in our case to write down the wave functions explicitly.

[9] B. L. v. d. Waerden, *Die gruppentheoretische Methode in der Quantenmechanik* (Verlag Julius Springer, Berlin, Germany, 1932), p. 70. Our normalization is different, however.

This yields for the intensity $I(\theta)$

$$I(\theta)=|a|^2+\tfrac{1}{2}|b|^2+(\tfrac{1}{4}|\tau|^2+(7/15)|\rho|^2)+\left(\frac{a^*\rho+a\rho^*}{(15)^{\frac{1}{2}}}+\frac{b^*\tau+b\tau^*}{(2)^{\frac{1}{2}}}\right)\cos\theta+(\tfrac{3}{4}|\tau|^2-\tfrac{2}{5}|\rho|^2)\cos^2\theta. \quad (15)$$

If we compare with Eq. (1), we get $|\tau|^2=256+(8/15)|\rho|^2$ from the $\cos^2\theta$ term and

$$|a|^2+\tfrac{1}{2}|b|^2=103-[\tfrac{1}{4}|\tau|^2+(7/15)|\rho|^2]$$

from the constant term. The ratio of resonance to nonresonance part of the total absorption is

$$(\tfrac{1}{2}|\tau|^2+\tfrac{1}{3}|\rho|^2)/(|a|^2+\tfrac{1}{2}|b|^2)=(128+\tfrac{3}{5}|\rho|^2)/(39-\tfrac{3}{5}|\rho|^2). \quad (16)$$

This is in agreement with Blair and Holland's data if ρ is assumed to be zero. Because of the unknown phase relations we have for the $\cos\theta$-term in Eq. (15) only an inequality, which is satisfied by, e.g., $\rho=0$ and any $|a|^2<29$.

4. DISCUSSION

Assuming first that the assumptions underlying our calculations are correct, we note that our main conclusions are not very sensitive to possible experimental inaccuracies. In order to make spin 5/2 possible, for instance, we must assume either that the measurement of the ratio of resonance to nonresonance cross section is too small by about a factor 6 or that in Roberts' measurement the ratio of the $\cos^2\theta$-term to the constant term is too large by about 45 percent. Roberts' ratio is estimated to be correct within twenty percent. While there is some doubt about the calibration in the experiment of Blair and Holland, the experimental error cannot account for a factor six in the *ratio* of resonance to nonresonance cross section. Furthermore, the nonresonance cross section is in fair agreement with the extrapolated $1/v$ law and to have a ratio of 23 would require a resonance peak higher than the maximum possible for p-neutrons.

The truth of our assumptions should be clarified by certain further experiments. An improvement of the total cross section measurement to 10 percent could rule out a nondegenerate state of spin 3/2 if it turns out that the resonance cross section exceeds 2.12 barns. If Roberts' angular distribution measurements at 100 kev, now in progress, show too large an angle dependence, our first assumption is wrong. It may then still be possible to determine the spin of the compound nucleus by an extension of our calculations to include d-neutrons. As yet, there is no indication of a term in $\cos^4\theta$ up to 400 kev.

The unlikely possibility of an accidental near degeneracy can of course never be excluded by considerations of this type. We have in fact treated in the Appendix the case of a 1/2, 5/2 degeneracy and found that even this combination is compatible with all the data.

APPENDIX

For the combined 1/2 and 5/2 states we use Eqs. (7) and (8) with the abbreviations $p^{(\frac{3}{2})}=a$, $p^{(\frac{1}{2})}=b$, $q_{\frac{1}{2}}{}^{(\frac{1}{2})}=\xi$, $q_{\frac{1}{2}}{}^{(\frac{3}{2})}=\eta$, and $q_{\frac{5}{2}}{}^{(\frac{3}{2})}=\sigma$, and obtain

$$u_{\frac{1}{2}m}\rightarrow[b\delta_{m\frac{1}{2}}Y_{00}+\xi 2m(3-2m)^{\frac{1}{2}}Y_{1,m-\frac{1}{2}}]\chi_+{}^T, \quad (17)$$

$$u_{\frac{3}{2}m}\rightarrow\left[a\left(\frac{5-2m}{10}\right)^{\frac{1}{2}}Y_{2,m-\frac{1}{2}}+\eta(9-4m^2)^{\frac{1}{2}}(3-2m)^{\frac{1}{2}}Y_{1,m-\frac{1}{2}}+\sigma\left(\frac{25-4m^2}{40}\right)^{\frac{1}{2}}\left(\frac{7-2m}{14}\right)^{\frac{1}{2}}Y_{3,m-\frac{1}{2}}\right]\chi_+{}^T, \quad (18)$$

and consequently,

$$I(\theta)=|a|^2+\tfrac{1}{2}|b|^2+(3/10)|\sigma|^2+3|\xi|^2+24|\eta|^2-6(3/20)^{\frac{1}{2}}(\sigma^*\eta+\sigma\eta^*)+[(3/10^{\frac{1}{2}})(a^*\sigma+a\sigma^*)+(3/2)^{\frac{1}{2}}(b^*\xi+b\xi^*)+2\sqrt{6}(a^*\eta+a\eta^*)]\cos\theta+[\tfrac{3}{5}|\sigma|^2+18(3/20)^{\frac{1}{2}}(\sigma^*\eta+\sigma\eta^*)]\cos^2\theta. \quad (19)$$

The ratio of resonance to nonresonance cross section is then

$$\text{Res./nonres.}=(\tfrac{1}{2}|\sigma|^2+3|\xi|^2+24|\eta|^2)/(|a|^2+\tfrac{1}{2}|b|^2)=Z/(167-Z), \quad (20)$$

where

$$Z=\tfrac{1}{2}|\sigma|^2+3|\xi|^2+24|\eta|^2. \quad (21)$$

To obtain a resonance to nonresonance ratio of 3 we must have $Z=125$. This must be compatible with an inequality obtained from

$$18(3/20)^{\frac{1}{2}}(\sigma^*\eta+\sigma\eta^*)=192-\tfrac{3}{5}|\sigma|^2, \quad (22)$$

as

$$36(3/20)^{\frac{1}{2}}|\sigma||\eta|\geqq|192-\tfrac{3}{5}|\sigma|^2| \quad (23)$$

or

$$|192-\tfrac{3}{5}|\sigma|^2|\leqq[(125-\tfrac{1}{2}|\sigma|^2)/24]^{\frac{1}{2}}. \quad (24)$$

This inequality is fulfilled for, e.g., $|\sigma|=13$. To explain all the present data with a 1/2, 5/2 degeneracy it is, therefore, sufficient to assume that the 1/2 state contributes one third of the measured resonance cross section, which is in agreement with Eq. (3).