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Gyromagnetic Ratios of Iron, Cobalt, and Many Binary Alloys of Iron, Cobalt, and Nickel

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Gyromagnetic ratios for iron, cobalt, and many binary alloys of iron, cobalt, and nickel were determined by measurements on the Einstein-de Haas effect much as made in the Norman Bridge Laboratory previously. Improvements were made in the method of eliminating large quadrature torques, in the method of reducing disturbances due to the vibrations of the building, in the method of frequency control, in the design of the square wave commutator, and in the design of the rotors. For each series of binary alloys the gyromagnetic ratio was found to vary linearly, or nearly linearly, with the concentration. For the small number of these materials studied in the Norman Bridge Laboratory previously there is good agreement between the new results and the older, both those obtained by the Barnett effect and those obtained by the Einstein-de Haas effect. For the three series of alloys the ranges of $\rho e/m$ are approximately as follows: iron-cobalt, 1.03 to 1.08 or 1.09; iron-nickel, 1.03 to 1.05; and nickel-cobalt, 1.05 to 1.08 or 1.09.

I. INTRODUCTION

This paper is a sequel of a series of articles on the Barnett[1,2] and Einstein-deHaas[3–5] gyromagnetic effects published by one of us in earlier years. The principal object of the new work was to make, by means of the Einstein-de Haas effect, as nearly complete an investigation as practicable, in the time at our disposal, of the gyromagnetic ratios of the binary alloys of the three principal ferromagnetic elements, of which only a few were studied in the earlier work (see Table I) and of which almost none have been studied by others. A few new measurements were made on iron and cobalt, chiefly as a check on the other work.

II. EARLIER WORK IN THE NORMAN BRIDGE LABORATORY

It will be conducive to brevity in the discussion of this new work to refer in the text for details to some of the earlier papers, and by the numbers affixed to their titles given below:

I. Researches on the rotation of permalloy and soft iron by magnetization and the nature of the elementary magnet, Proc. Am. Acad. Arts Sci. **66**, 273 (1931).
II. The rotation of cobalt and nickel by magnetization and the gyromagnetic ratios of their magnetic elements, Proc. Am. Acad. Arts Sci. **69**, 119 (1924).
III. Gyromagnetic and electron inertia effects, Revs. Modern Phys. **7**, 129 (1935).
IV. Gyromagnetic ratios for ferromagnetic substances; new determinations and a new discussion of earlier determinations, Proc. Am. Acad. Arts Sci **73**, 401 (1940).
V. New researches on magnetization by rotation and the gyromagnetic ratios of ferromagnetic substances, Proc. Am. Acad. Arts Sci. **75**, 109 (1944). See also Phys. Rev. **66**, 224 (1944).
VI. Magnetization and rotation, Am. J. Phys. **16**, 140 (1948).

The gyromagnetic ratios as obtained previously in the Bridge Laboratory from the Einstein-de Haas effect in iron, nickel, cobalt, and their alloys, and published in IV, are collected in Table I.

These values of ρ are in substantial agreement with those obtained from measurements on the Barnett effect. See IV, V, and VI.

III. RESULTS OBTAINED IN OTHER LABORATORIES

The nearly identical values for iron, given above, are also in close agreement with all measurements made elsewhere on this material (see IV) (or the ferrites)[6] in recent years, and comparable with them in the precision claimed, with one exception: A. J. P. Meyer[7] has recently obtained for ρ about $1.00\times m/e$ from a small number of measurements on iron, nickel, and a few Co—Ni—Fe alloys. For one alloy (Fe_3Ni) his result is $0.966\times m/e$. His observations on nickel are the only recent ones made outside the Bridge Laboratory.†

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* Now at George Fox College, Newberg, Oregon.

[1] S. J. Barnett, Phys. Rev. **6**, 239 (1915).

[2] S. J. Barnett and L. J. H. Barnett, Proc. Am. Acad. Arts Sci. **60**, 125 (1925); S. J. Barnett, **75**, 109 (1944).

[3] A. Einstein and W. J. de Haas, Verhandl. deut. physik Ges. **17**, 152 (1915); A. Einstein, Verhandl. deut. physik Ges. **17**, 203 (1915).

[4] W. J. de Haas, Proc. Amsterdam Acad. Sci. **18**, 1280 (1916); A. Einstein, Verhandl. deut. physik Ges. **18**, 17 (1916).

[5] See especially S. J. Barnett, Proc. Am. Acad. Arts Sci. **73**, 401 (1940).

[6] D. P. Raychaudhuri, Indian J. Phys. **9**, 383 (1935).

[7] A. J. P. Meyer, Ann. phys. **6**, 171 (1951).

† *Note added in proof by senior author.*—Two erroneous statements made in Meyer's papers should be corrected here, *viz:* (1) That I had recently acknowledged a systematic error of 1 percent in my gyromagnetic ratios on account of results obtained by Major Webber and myself on the torsion modulus of German silver wire [see S. J. Barnett and D. S. Webber, Phys. Rev. **71**, 896 (1947)]. These results have nothing at all to do with my own gyromagnetic experiments, but they bring the results obtained long ago by Messrs. Sucksmith and Bates 1 percent closer to mine. (2) That I had found a variation in the gyromagnetic ratio of pure iron in samples of different origin. Within the limits of the experimental error all specimens of pure, or nearly pure, iron give the same value. In some of the earlier work there was a discrepancy between ρ for a specimen of electrolytic iron obtained from the Barnett effect and ρ for nearly identical material obtained from the same effect; but this was long ago traced to a defect in a coil used with electrolytic iron in the experiments on the Barnett effect.

TABLE I.[a] Values of $\rho e/m$ from paper IV, Table 43, Einstein-de Haas effect.

Armco iron	1.032	Hopkinson's alloy (75% Fe, 24½% Ni)	1.024
Honda iron	1.032[b]	Permalloy (Ca 80% Ni, 20% Fe)	1.047
Yensen electrolytic iron	1.034	Cobalt (99.9% Co)	1.090
Cold rolled steel	1.039	Cu-cobalt (92.4% Co)	1.082
Nickel	1.052	Cobalt-iron (34% Co)	1.026
Hipernik (50% Fe, 50% Ni)	1.052	Cobalt-nickel (54% Co)	1.077

[a] *Note added in proof by the senior author.*—The referee on this paper suggested to the Editors that it might be well to include in this table, in addition to the *gyromagnetic ratios*, the so-called g' values recently introduced into the literature by certain authors interested in microresonance. To this I made to the Editors on March 5 the following reply, which I hope will help to clarify the situation:

> I have always thought it far better to use the term *gyromagnetic ratio*, defined as the ratio of the angular momentum of the element to its magnetic moment, a simple physical quantity, than the terms "g-factor," "Landé splitting factor," etc.; and also I do not like to encourage those physicists who write as if the microresonance experiments were not gyromagnetic experiments—or as if there were anything strange in the gyromagnetic ratio (for the same substance) being different under different circumstances. All the experiments have the same dynamical basis, and there is no reason why the relative contributions of electron and orbit should not differ in different cases (as they appear to do).

[b] In the table of IV, 1.023 is incorrectly printed for this number.

Special mention should be made of a very elaborate and precise investigation on pure iron by G. G. Scott,[8] who gives as his final value $\rho e/m = 1.0278 \pm 0.0014$, in close agreement with the values quoted in Table I for pure and nearly pure iron. His values for 1948 are in even closer agreement (1.031 and 1.028) with the table.

IV. METHODS USED IN THE NEW WORK

Methods used in the new work and details of theory are essentially identical with those described in I, p. 302; II, pp. 403–404; III, p. 30; and IV, p. 3, with certain modifications (mostly of a minor character) described below. A very brief discussion follows:

The rotor C, Fig. 1, rigidly attached to a double-faced mirror E and a small permanent magnet F with axis normal to the mirrors, is magnetized by a fixed solenoid B fed by a storage battery V through a square wave commutator W running at the natural frequency of the rotor. The alternations of the magnetization produce a gyromagnetic torque g upon the rotor.

Also the changing magnetization of C produces a current in the induction solenoid A and the torque coil HH, which, acting on F, alters the amplitude and phase of the vibration according to the value of the adjustable conductance $X = 1/R$ of the circuit.

The quadrature coil GG, fed by a battery P through a square wave commutator Q, produces on F a torque, whose phase and magnitude are adjustable until it is in, or nearly in, quadrature with the gyromagnetic torque g. The two commutators are mounted on the same shaft. Mutual inductance compensators $A'B'$, $H'H'$, $G'G'$ reduce to zero the mutual inductances between the different circuits.

Observations are made in a region neutral, or nearly neutral, magnetically. The conductance $X_0 + \delta X_0$ of the secondary circuit at which the resonance amplitude is a minimum is determined by measuring accurately the amplitudes A_1 and A_2 (=approximately A_1), respectively, for $X = 0$ and $X = X_2$ [=approximately $2(X_0 + \delta X_0)$]; by measuring approximately A_0 the minimum amplitude for X=approximately $X_0 + \delta X_0$; and by substituting in the formula

$$X_0 + \delta X_0 = \frac{X_2(A_1^2 - A_0^2)^{\frac{1}{2}}}{(A_1^2 - A_0^2)^{\frac{1}{2}} + (A_2^2 - A_0^2)^{\frac{1}{2}}},$$

which become

$$X_0 + \delta X_0 = X_2 A_1/(A_1 + A_2)$$

when $A_0 = 0$.

X_0 is the conductance at which the amplitude would vanish if there were no extraneous torques; and δX_0, the error due to such torques, is eliminated by making observations for both directions I and II of the asymmetry reversing switch U in the magnetizing circuit, and for two azimuths, NPE and NPW, differing by 180°, of the rotor and permanent magnet attached to it. The mean value of δX_0 for these four arrangements is zero, and the gyromagnetic ratio is determined from the equation

$$-\rho = X_0 \Gamma_0 \gamma_0 m_0,$$

where Γ_0 and γ_0 are the constants of the torque coil and induction solenoid, respectively, and m_0 is the moment of the permanent magnet. Under good conditions the use of the quadrature coil made A_0 negligible.

In calculating the results the simpler formula was always used first. This sufficed for the final calculation in nearly all cases because of the smallness of A_0/A_1 and $|(A_0 - A_2)|/A_1$. In the few cases in which appreciable error resulted from the use of the simple formula, the longer formula was used for the final calculation.

V. PRINCIPAL SOURCES OF ERROR

Most of the sources of error and the methods of eliminating or avoiding them have been discussed in references I, II, and IV. Only a brief discussion will be given here. The principal disturbing torques may be divided into three groups: (1) torques due to the residual earth's field, (2) torques due to the action of the fixed magnetizing coil on the vibrating system, and (3) torques due to magnetostriction.

(1) The horizontal component of the residual earth's field acting on the horizontal component of the magnetic moment of the rotor (h) will produce a torque about a vertical axis. If the horizontal component of the mag-

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[8] G. G. Scott, Phys. Rev. 82, 542 (1951).

netic moment h is in phase with the vertical component μ which produces the gyromagnetic torque, this disturbing torque will be in quadrature with the gyromagnetic torque and cause no error. If h is not exactly in phase with μ, this disturbing torque will have in-phase components which would increase or decrease the observed value for the gyromagnetic ratio. If the suspended system is turned through 180° about its axis, the torque will be unaltered in magnitude but will be reversed in sign and will be eliminated, provided the residual field remains constant.

(2) If the field of the magnetizing solenoid is not strictly vertical, it will produce a torque about a horizontal axis by acting on the horizontal component of the permanent magnetism of the rotor. This torque also is reversed in sign by changing the azimuth 180°. There is a similar torque on the fixed magnets of the magnet-mirror holder. In this work the magnets of the magnet-mirror holder were much stronger than in the earlier work, about 5.5 emu instead of some 0.7 emu. The stronger magnets gave certain advantages discussed later, but they increased the effects of the torques due to the magnetizing solenoid. Since square waves were supplied to the magnetizing solenoid, the magnetic moment of the rotor was nearly in phase with the fundamaental of the square wave. Hence the fundamental of the square wave was nearly in quadrature with the gyromagnetic torque, which is proportional to the rate of change of the magnetic moment. So any torque on the magnets due to the magnetizing solenoid would be largely a quadrature torque. The principal quadrature torques were due to magnetostriction (see below) and the residual earth's field. No certain effects due to the torque on the permanent magnets were observed. In any event, the torque is reversed in sign but not in magnitude when the suspended system is turned 180° in azimuth, and the effect is thus eliminated. Great care was taken to insure straightness of the suspended system. The action of the magnetizing current on the alternating magnetization of the rotor produces a double-frequency torque which is without effect except in the case of asymmetry. In this case it is eliminated along with magnetostriction. [See (3) below.]

(3) Longitudinal magnetostriction was large in most of the alloys investigated. The vertical motions produced by magnetostriction may be partly converted into axial vibrations because of asymmetries present in rotor and wave form. Since the change in length due to magnetostriction is independent of the direction of magnetization, the frequency of the effect will be twice that of the magnetizing current and thus not in resonance. However, if the two half-cycles of the magnetizing current are dissimilar, the difference between the two effects will have the frequency of the magnetizing current and may have any phase relation to the gyromagnetic torque. The effect may be eliminated by a reversing switch in the magnetizing circuit which interchanges the half-cycles of current producing a particular direction of magnetization.

The quadrature torque usually reversed its phase while retaining nearly the same amplitude when the asymmetry reversing switch was thrown, indicating that most of the quadrature torque was due to magnetostriction.

VI. ELIMINATION OF QUADRATURE TORQUES

The presence of torques in quadrature with the gyromagnetic torque flattens the vibration amplitude *vs* conductance curve and makes the precise determination of the value of $X_0 \pm \delta X_0$ difficult. To eliminate such torques, a quadrature coil was first used by de Haas. One of us used a quadrature coil in his earlier work on nickel and cobalt (see II) and in some of the later work. Like de Haas he obtained the current for the quadrature coil from the current supply for the magnetizing solenoid. If the magnetic moment of the rotor is exactly in phase with this current, the gyromagnetic torque is in quadrature with the magnetizing current. The emf across a nearly noninductive resistance in the magnetizing circuit was used to supply the quadrature coils. Since the resistance in the quadrature circuit was high and the inductance small, the current in this circuit was assumed to be in phase with the magnetizing current. Various time lags, however, may still be present on account of hysteresis, eddy currents within the material, and other effects. If such a time lag is present, the torque of the quadrature coils on the magnets will not be strictly in quadrature with the gyromagnetic torque and will produce in-phase effects. It was sometimes practicable to eliminate these effects by determining the apparent values of ρ for several values of the applied quadrature torque, and by interpolation or extrapolation to zero quadrature torque. This was not always possible, however, and it appeared desirable to supply the quadrature coils from a source with arbitrary phase angle with respect to the magnetizing current.

In preparation for the present investigation a double set of commutators was prepared (see I for details of

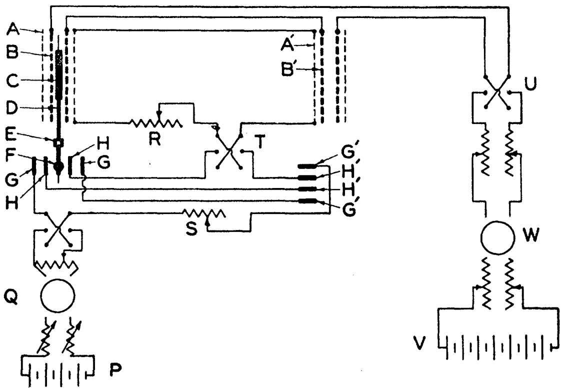

FIG. 1. Primary, secondary, and quadrature circuits.

design, also XII below), mounted on the same shaft to give two square waves of exactly the same frequency (Fig. 2). They were so constructed that one set could be rotated to an arbitrary position with respect to the other, and the position was indicated on an accurately made scale and vernier. The two sets of commutator segments were supplied from independent banks of lead storage cells. Both commutators were driven on the same shaft by a $\frac{1}{2}$-horse power dc motor whose frequency was accurately controlled by a method referred to below. This apparatus gave two square waves having identical frequencies but arbitrary phases and independent amplitudes. These will be designated as the *main square wave* (which was supplied to the magnetizing helix) and the *quadrature wave.*

Let the fundamental of the main square wave (current) be given by

$$I=I_0\cos(\omega t+\alpha),$$

and the fundamental of the magnetic moment of the rotor by

$$\mu=\mu_0\cos\omega t.$$

The gyromagnetic torque then is given by

$$G=-\rho d\mu/dt=\mu\rho\omega\sin\omega t.$$

The phase angle α was unknown and varied from rotor to rotor. To make effective use of the new quadrature supply it was necessary to determine the phase angle α or the phase of the first harmonic of quadrature wave with respect to the gyromagnetic torque. This was done by making use of the current in the secondary circuit.

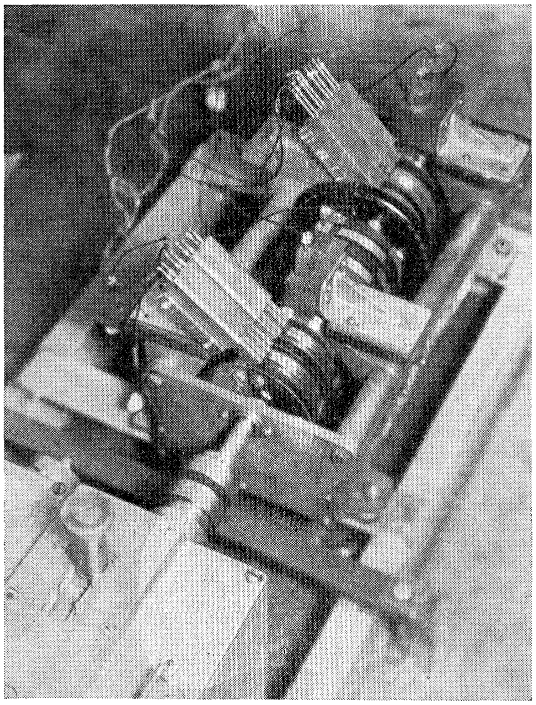

FIG. 2. Main and quadrature square wave commutators.

The first harmonic of the current in the secondary circuit is given by

$$i=-(\gamma/R)d\mu/dt=-(\gamma\omega\mu_0/R)\sin\omega t,$$

and is in phase with the gyromagnetic torque because of the negligible reactance of the secondary circuit.[9] The current in the secondary circuit was quite different in form from the quadrature wave (a square wave) (see Figs. 8–11); and the problem was to determine the phase angle between the fundamentals of the waves in these circuits.

To measure this angle a phase adjuster was constructed as follows: Two sets of coaxial coils were constructed, wound on a single Bakelite tube (Fig. 3). An Alnico magnet with its axis horizontal and a mirror were suspended midway between these by a piano wire suspension of adjustable length. The piano wire suspension passed through a No. 0 Brown and Sharpe pin vise and out the top. The hollow handle was threaded with a 5–40 tap to a depth of some $1\frac{1}{2}$ inches. The piano wire passed through a hole in a special brass screw inserted in the pin vise. A collar around the wire above the brass screw was used to fix its length. With this arrangement it was possible to tune the system to the natural frequency of the rotor to within 1 part in 2500. The inductance of the inner set of coils was 15 millihenrys, the outer set 12 millihenrys. Usually each set was connected in series with 4000 to 6000 ohms resistance. When in use, the flat topped waves from the quadrature commutators were supplied to the outer set of coils, the current from the secondary circuit was supplied to the inner set of coils.

The main square wave was first tuned to the natural frequency of the rotor; then the magnet-mirror system of the phase-meter was tuned to the frequency of the main square wave by adjusting the length of the suspension. By adjusting the resistance and phase of the quadrature circuit it was possible to tune the system to a minimum amplitude of vibration. The quadrature wave was then 180° out of phase with the fundamental of the current in the secondary circuit. Rotation of the quadrature commutator 90° from this position put the first harmonic of the quadrature wave in quadrature with the first harmonic of the current in the secondary circuit and thus with the gyromagnetic torque. With care, the null position could be determined to within $\pm\frac{1}{4}$ degree, but was usually determined only to within $\pm\frac{1}{2}$ degree.

The settings for quadrature were constant (to within 1°) for one rotor for a period of several days, and appeared to change only as the brushes gradually wore down. The settings varied from rotor to rotor, however, by angles up to 7°, according to the effects of hysteresis,

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[9] The frequency used was about 10 cycles per second, the inductance of the secondary circuit less than 0.05 henry, resistance 10,000 ohms or more. The reactance $\omega L/R$ then is 3×10^{-4} radian.

eddy currents, and other factors on the forms of the waves they produce in the secondary circuit. (See Figs. 7–11.)

The phase adjuster made it possible to set the fundamental of the quadrature wave in almost exact quadrature with the gyromagnetic torque, and thus the quadrature torques could be eliminated without introducing in-phase components.‡

VII. TORQUE AND QUADRATURE COILS

The torque and quadrature coils were used in part of the earlier work and were designated torque and quadrature system *E*. (See page 410, reference IV for a diagram.) The constant of the torque coil was measured again and found to be unchanged from its original value of 1009.6 emu. The mutual inductance between the torque and quadrature coils was again compensated by connecting a duplicate set of coils in such a way that the mutual inductance of the second set was opposed to that of the main set. Tests showed the residual mutual inductance to be entirely negligible.

VIII. INDUCTION SOLENOID AND FIXED MAGNETIZING COILS

The induction solenoid and fixed magnetizing coils also had been used in the earlier work (see IV, Sec. 12, last paragraph). Their mutual inductance was compensated as before by that of a duplicate set of coils. The balance was adjusted by moving the magnetizing coil of the compensator axially with respect to the induction solenoid. When the coils were adjusted properly, the residual mutual inductance was less than 0.02 percent of that of either set alone. The magnetic intensity applied to the rotor in all measurements was about 50 gausses. The insulation between primary and secondary circuits was exceedingly high.

IX. MAGNET-MIRROR HOLDER

The magnet-mirror holder used was the one designated as No. 7 in reference IV. The steel magnets previously used were replaced by magnets made from "Cunife" kindly presented by the General Electric Company. The moment of these magnets was about 5.3 emu instead of the 0.6 to 0.8 emu of the spring steel magnets previously used. This increased moment made the magnetic moment itself much easier to measure. No control magnet was necessary to increase the sensitivity of the magnetometer. The current in the torque coils was reduced in the same ratio as the increase in magnetic moment. Thus, any direct effect of the magnetic field of the torque coil on the rotor was reduced. The increased resistance in the secondary circuit decreased the already negligible time lag between the current in the secondary circuit and the gyromagnetic torque.

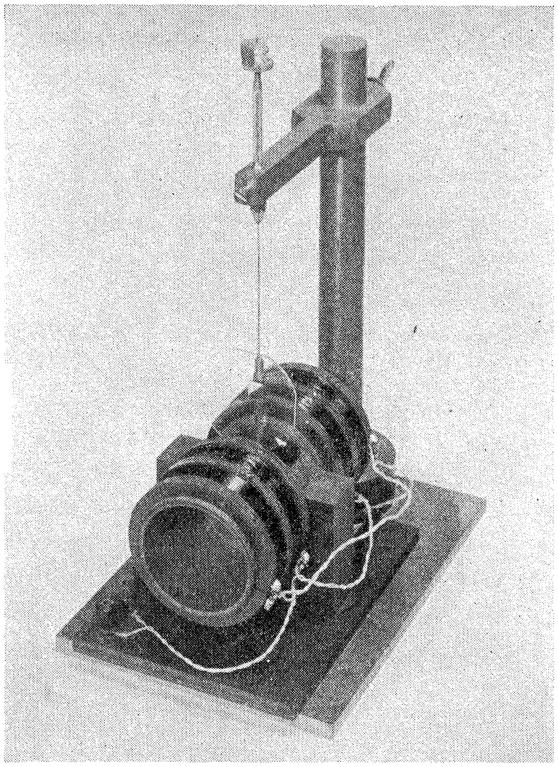

FIG. 3. The phase adjuster.

X. ROTOR AND SUSPENSION CONSTRUCTION

All of the rotors were of the standard short type, unwound, and were constructed on the antimagnetostriction principle described in references II and IV, but with improvements, as indicated in Fig. 4. The upper and lower suspensions were made of No. 30 German silver wire 6.9 cm long and 5 cm long, respectively. Since the rotors were of nearly identical construction, the same upper and lower suspensions could be used for all of them. Their resonance frequencies were all near 11 cycles per second.

XI. THE VIBRATION-FREE MOUNTING

The vibrations in the building due to traffic outside and people and equipment in other parts of the labora-

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‡ If *Q* designates the amplitude of the quadrature torque which has to be annulled, and *G* that of the gyromagnetic torque, then the fractional error in ρ due to the angular error $\Delta\theta$ in setting the quadrature circle correctly is

$$\delta\rho/\rho=\sin\Delta\theta\cdot(Q/G).$$

If *C* designates the constant of the quadrature coil, *I* the amplitude of the current which traverses it when the quadrature torque is annulled, and A'_0 the amplitude of the quadrature vibration produced by *Q*, we may write

$$\delta\rho/\rho=\sin\Delta\theta\cdot(CIm_0/\rho\omega\mu)=\sin\Delta\theta\cdot(A'_0/A_1).$$

The absolute sign of $\Delta\theta$, whose magnitude will be assumed to be one-half degree, or 1/115 radian, is of course not known, but the relative signs for all the constituents NPE_I, NPE_{II}, NPW_I, NPW_{II} of each set are known from the directions of the emf and the directions of the quadrature current switch. Thus $\overline{\delta\rho/\rho}$ can be obtained for each set. This quantitity proved to be entirely negligible for every *rotor;* and indeed for every *set* but three, where it reached about one part in a thousand, it is entirely negligible.

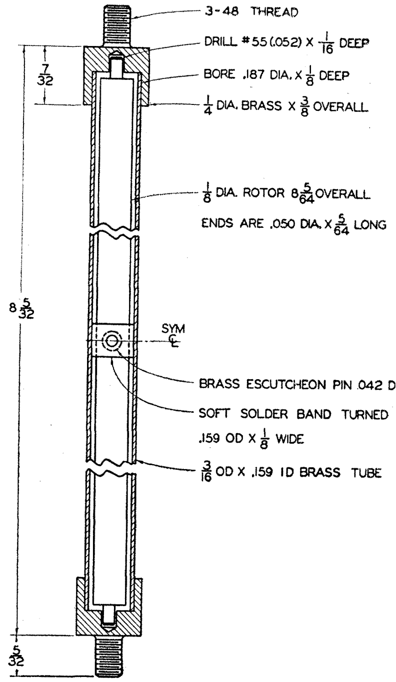

FIG. 4. Construction of the improved type of rotor. All lengths are in inches.

tory disturbed the moving system and made it necessary to protect it from the vibrations. As the vibration free support used in the earlier investigations was in use, it was necessary to construct a new support.

A modification of the form developed by Becker[10] was adopted (see Figs. 5 and 6). The necessity of using nonmagnetic material in the apparatus caused difficulties. The greatest freedom from vibrations is obtained when the suspended system has a low natural frequency. Loading the suspension to increase the mass caused the rods to buckle before the period was sufficiently long to give good protection from vibration. In order to get rid of vibrations, the marble slab (15 in.×30 in.×$2\frac{1}{2}$ in.) on which the sensitive apparatus rested was suspended from the ceiling by 8 brass springs 44 in. long (before extension) about 1 in. o.d., made from 0.128 in. brass spring wire. In use, the springs were extended to 62 inches, making the period for vertical oscillations about 1.4 seconds. To provide for damping, the springs were wrapped with flat gum-rubber strips 2 inches wide. The high internal friction of the rubber and friction between the rubber and the springs damped the vertical oscillations very effectively. To remove the asymmetry in Becker's system two similar pendulums, arranged symmetrically, were used instead of one.

Horizontal vibrations were a much more serious source of disturbance than vertical oscillations, and it was necessary to suppress their effects as much as possible. As is well known, internal damping is much superior to external damping in obtaining freedom from vibrations.

The vibrations of the system in a horizontal direction, or oscillations about a vertical axis, were damped by pendulums of the type developed by Becker mentioned previously. Figure 6 shows the top of one of the pendulums. The two arms at right angles to each other carry pistons which dissipate the energy of oscillation of the pendulum by friction with heavy oil. The pendulum was supported by a wire instead of a pivot. At small amplitudes of vibration the friction at the pivot increased greatly so the pendulums did not swing but moved with the apparatus and provided no damping. The wire support, however, does not have increased friction for small amplitudes and the pendulums remained effective.[11] It was necessary to tune the pendulums by adjusting the lengths until the pendulum period was nearly equal to that of the suspended system. The pendulum bobs weighed about 3.8 kg each; their length in proper adjustment was about 170 cm. The half time for horizontal vibrations was 14 seconds, for vertical motions about 30 seconds.

The freedom from mechanical disturbances was sufficient to permit work not requiring precise annullment of the earth's magnetic field to be done at any time of day or night except late afternoon, when traffic was too heavy.

XII. SQUARE WAVES, COMMUTATORS, BRUSHES, AND FREQUENCY CONTROL

The square waves of electromotive force which supplied the current in the magnetizing coil and the quadrature coils at the natural frequency of vibration of the rotor were obtained much as in the earlier investigations, but with several improvements. The Wenner system of speed control was modified to great advantage by placing the resistor which was automatically short-circuited in the armature circuit instead of in the field circuit. This change greatly strengthened the control.

The 3-hp motor used in the previous investigations required too much current for the present battery bank in the Norman Bridge Laboratory and was replaced by a $\frac{1}{2}$-hp motor. One gear box was used instead of two.

Although the motor ran at the same average frequency as the tuning fork, it tended to hunt and thus produced changes in the phase of the square wave, which affected the amplitude of the vibrating rotor in the main experiment. To increase the inertia, a flywheel, 1 foot in diameter and 65 pounds in weight, was constructed and connected rigidly to the motor shaft. It was mounted in ball bearings and carefully aligned so that it added little to the load on the motor after the system reached its final speed. The voltage of the current supplied to the motor was stabilized by a set of 20 6-volt lead storage batteries across the line.

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[10] H. E. R. Becker, Z. Tech. Physik **21**, 195 (1940).

[11] This improvement in the apparatus was suggested to us by Mr. Sheldon J. Brown.

The commutator construction has been discussed in the earlier papers and in VI above (see Fig. 2). New brushes and brush-holders were constructed. Two independent brushes pressed on each commutator at an angle of 45° with the surface, so that the frictional drag would tend to pull the brushes into closer contact. The pair of brushes insured more nearly uniform contact since it was unlikely that both would miss contact at the same time. Much less force was required to give satisfactory contact, the power input to the motor was reduced by one-third, and the reduced load made the speed control much easier. After all the changes mentioned were made the square waves and frequency control were quite satisfactory.

XIII. NEUTRALIZATION OF THE EARTH'S MAGNETIC FIELD

The three components of the earth's magnetic intensity were neutralized by the three Helmholtz coils shown in Fig. 5, much as in the earlier investigations. The currents for the three sets of coils were furnished by three independent banks of lead storage batteries.

The proper compensating currents were determined from the nullification of the electromotive force induced in a small rotating coil mounted inside a Lucite cylinder and driven by an air stream. The voltage induced in the coil was read by a vacuum tube voltmeter. The sensitivity was sufficient to compensate the field intensity to 1 part in 5000, but the earth's field fluctuations were much larger than this. The compensating currents for the north-south and vertical components were determined to 1 part in 1000, that of the small east-west component to 1 part in 250.

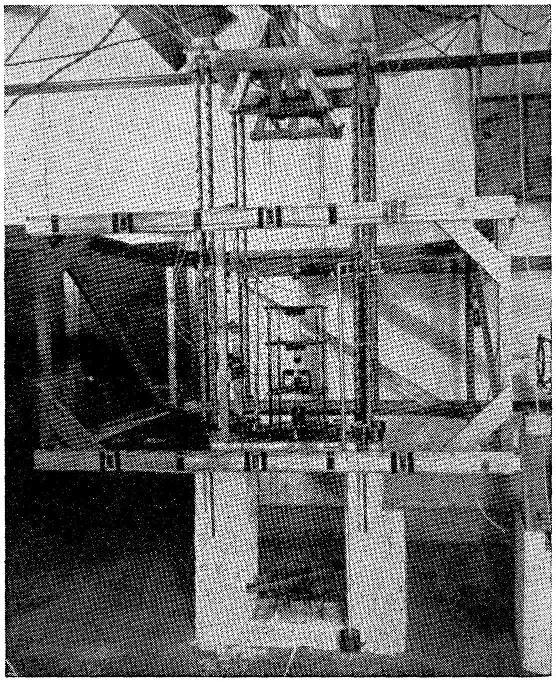

FIG. 5. The coils for neutralizing the earth's magnetic field, the antivibration equipment, the coils in which the rotor was suspended, the torque and quadrature coils, etc.

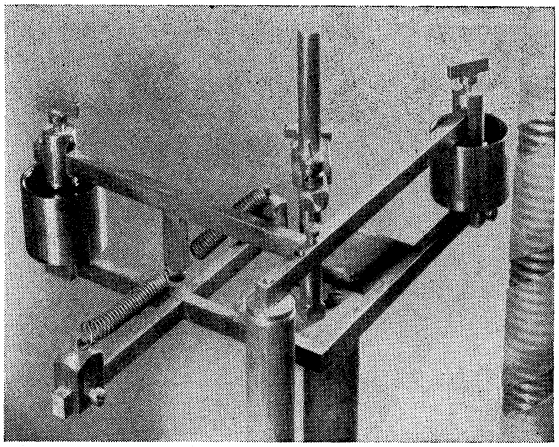

FIG. 6. Details of devices for annulling horizontal and vertical vibrations.

In order to make this calibration, it was necessary to remove and again replace the magnetizing and induction solenoids, a time-consuming operation. It was also necessary to have the rotor in place several hours before the measurements began (see below). Thus it was impossible to make a direct measurement of the compensating currents immediately before beginning a run. The difficulty was avoided, as in the earlier work, by making supplementary measurements with an earth inductor and a Hibbert magnetic standard.

There were only slight changes in the vertical component over a period of months. Also, slight changes in the vertical intensity produce no effect on the observed value of the gyromagnetic ratio. The compensating currents were left the same for both azimuths of the rotor. Except in the cases of two rotors which were unsymmetrical and had very large horizontal moments, the residual earth's field introduced no serious difficulties.

XIV. DETERMINATION OF CONSTANTS

The constants of the torque coil Γ_0 and the induction solenoid γ_0, the resistance $R_0=1/X_0$ of the secondary circuit, and the moment of the magnets m_0 which enter into the fundamental equation for ρ, *viz.*,

$$-\rho=X_0\gamma_0\Gamma_0m_0,$$

must be accurately known. The constants of the torque coil and induction solenoid were measured with precision in the earlier work and were checked for this work. The moment m_0 of the magnet was measured bi-monthly by the magnetometer method described in I. The moment changed by only 0.1 percent per month, and the value for any particular night was found from the curve.

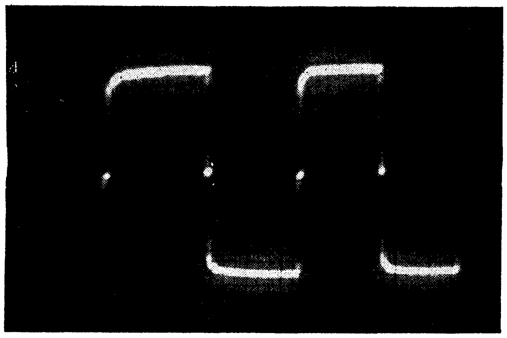

FIG. 7. Current in primary coil *vs* time for Bell Telephone iron rotor.

The resistance R_0 was 8500 ohms or more, 124 ohms in the coils and leads, the rest in the resistance boxes. The standard resistance boxes were checked carefully and found to be accurate to 0.1 percent or better for the high resistance used. Below 40 ohms, the resistances were good to only 0.5 percent or 1 percent, but this error was a negligible contribution to the total resistance. Thus the total resistance of the induction circuit was known to an accuracy of at least 0.1 percent.

XV. OSCILLOGRAMS

For one of the rotors Fig. 7 shows the current in the primary circuit. The commutator gaps are visible as dots. Figures 8–11 show the electromotive forces in the secondary circuit due to the rotor *F* (41 percent Ni, 60 percent Fe), the rotor *A* (15 percent Ni, 85 percent Co) and an iron rotor, respectively. The primary current was the same for each of these four rotors. The differences in shape show how the first harmonic of this wave form (which is essentially the same as the gyromagnetic torque on the rotor) may be shifted in phase by several degrees from exact quadrature with the fundamental of the magnetizing current. The differences in form are caused by the differences in the properties of the materials. Most of the lag in iron appears to be due to eddy currents since the decay is exponential. Ideally, one should want to have a very sharp peak and no lag to reduce the influence of the in-phase components of the disturbing torques. Figure 11 is for the same material as Fig. 8. The peak is shown broadened to illustrate its double nature. With mechanical commutators, the current first falls to zero when the brush passes over the gap and then rises to a maximum in the opposite direction. The first part of the double peak is due to the change in magnetization when the current falls to zero, the second part is caused by the change in magnetization when the current rises to a maximum in the opposite direction. The change is so rapid that the rising part of the curve does not appear, but the decay is clearly visible.

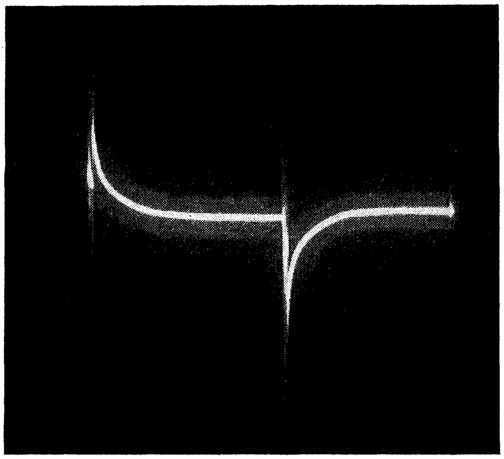

FIG. 9. Secondary emf *vs* time for 15 percent Ni 85 percent Co rotor.

XVI. OBSERVATIONAL PROCEDURE

In order to insure the necessary stable conditions before beginning observations, the motor driving the

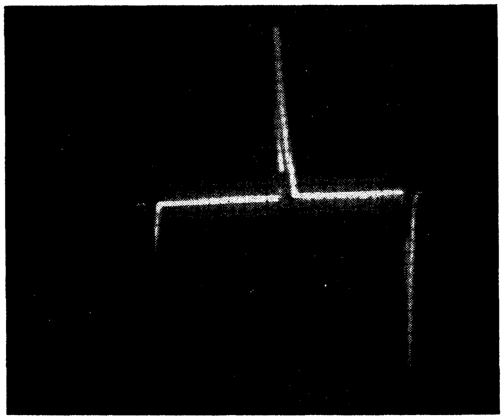

FIG. 8. Secondary emf *vs* time for 41 percent Ni 60 percent Fe rotor.

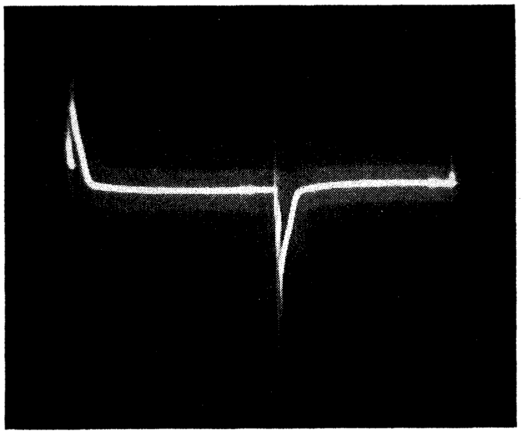

FIG. 10. Secondary emf *vs* time for electrolytic iron rotor.

TABLE II. Composition of Bell Telephone cobalt-iron alloys.

Percent Cobalt	Percent Manganese	Percent Iron	Percent Silicon	Percent Carbon
10.17	0.31	89.41	0.01	
20.04	0.47	79.22	Nil	
30.19	0.45	69.37	Nil	
40.09	0.44	59.44	Nil	
50.11	0.47	49.18	0.01	
59.82	0.58	39.45	0.01	
69.62	0.39	29.80	0.01	
79.36	0.48	20.18	0.01	
89.78	0.63	9.56	0.03	
98.90	0.63	0.18	0.09	0.20

commutators, the tuning fork, and the rotor current were all turned on for at least four hours before observations began. After the motor was brought under the frequency control, the frequency was adjusted until the rotor amplitude was a maximum. Then the resistances of the secondary circuit and the quadrature circuit which gave minimum amplitude (see discussion of method above) were determined for both positions of the asymmetry reversing switch. The measurements then proceeded as described in detail in IV.

Most of the measurements were made after midnight; some were made in the evening before midnight, and a few were attempted in the daytime. On a few occasions two complete sets of observations were made in one night. Work was made impossible on a few nights by magnetic storms, but the conditions between 1:00 A.M. and 4:30 A.M. were usually very good.

XVII. OBSERVATIONS AND RESULTS

A. Cobalt-Iron Alloys

Through the kindness of the Bell Telephone Laboratories, a series of iron-cobalt alloys was obtained. The analysis provided with the alloys is given in Table II. The samples were all annealed for one-half hour at 900°C after swaging.

1. *10 percent cobalt-iron rotor.*—This rotor had a very large horizontal magnetic moment which made measurements impossible. Repairs to the rotor were not completed in time for this work.

2. *20 percent cobalt-iron rotor.*—(See Table III.) The band of light on the scale was crossed by faint lines, indicating that the effects of magnetostriction were not completely eliminated by the complex rotor construction (see I, p. 332). The pattern was steady, but it was necessary to use the quadrature coil.

TABLE III. 20 percent cobalt-iron rotor. Double amplitude 3.1–3.8 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho e/m$
1940							
June 5	nt	1	1.016	1.025	0.989	1.032	1.016
6	nt	1	0.974	1.093	0.992	1.091	1.030
7	day	$\frac{1}{2}$	0.983	1.057	0.944	1.032	1.004
7	eve	$\frac{1}{2}$	0.964	1.093	0.959	1.087	1.026
10	nt	1	0.904	1.103	0.903	1.104	1.004
14	nt	1	1.021	1.089	0.954	1.107	1.042

The weighted mean is $\rho e/m=1.021\pm0.013$.

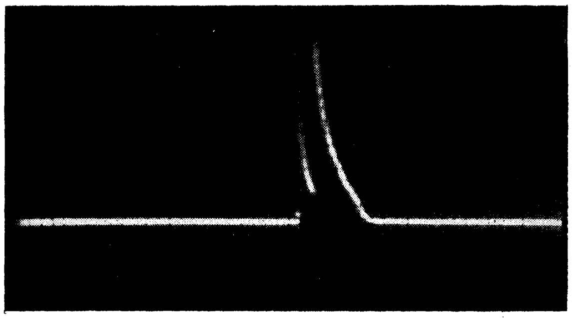

FIG. 11. Secondary emf *vs* time for 41 percent Ni—60 percent Fe rotor of Fig. 8, but with a different time scale.

3. *30 percent cobalt-iron rotor.*—(See Table IV.) As first constructed, this rotor had a large horizontal magnetic moment and the effects of magnetostriction were very evident. The rotor was remade, in an attempt to make it more nearly symmetrical. It then behaved quite well, although the I, II asymmetry was larger than usual. The quadrature coil was used. The rotor was inverted for the observation of July 27.

4. *40 percent cobalt-iron rotor.*—This rotor broke during construction and could not be repaired or replaced.

5. *50 percent cobalt-iron rotor.*—(See Table V.) All observations on this rotor were made during the daytime or early evening. It had a rather small horizontal magnetic moment, however, and the observations were fairly good. The quadrature coil was used.

6. *60 percent cobalt-iron rotor.*—(See Table VI.) The earth's magnetic field was unsteady through this series of observations, but the small horizontal moment of the rotor reduced the effects of the fluctuations. Quadrature torques were small. The quadrature coil was used.

7. *70 percent cobalt-iron rotor.*—(See Table VII.) The earth's magnetic field was steady during this series. The quadrature coil was used.

8. *80 percent cobalt-iron rotor.*—(See Table VIII.) This rotor had a large magnetic moment and gave large amplitudes. The quadrature coil was used.

9. *90 percent cobalt-iron rotor.*—(See Table IX.) This rotor was very good. The quadrature coil was used.

TABLE IV. 30 percent cobalt-iron rotor. Double amplitude 4.4–5.6 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho e/m$
1950							
July 22	eve	$\frac{1}{2}$	1.141	0.921	1.113	0.902	1.019
24	nt	1	1.158	0.968	1.127	0.923	1.044
27	nt	1	0.645	1.443	0.669	1.408	1.041

The weighted mean is 1.038±0.007,

Table V. 50 percent cobalt-iron rotor. Double amplitude 2.7–3.3 cm.

			NPE		*NPW*		
Date	*T*	Obs	I	II	I	II	$\rho e/m$
1950							
Mar. 14	day	½	1.126	1.010	1.054	1.029	1.054
16	day	½	1.160	1.053	1.085	1.054	1.088
16	eve	½	1.163	1.032	1.079	0.068	1.061

The mean of these results is 1.068 ±0.013.

10. *Cobalt rotor.*—(See Table X.) This rotor was exceptionally good. Quadrature torques were small and the amplitude very stable. The quadrature coil was used.

B. Cobalt-Nickel Alloys

Measurements of the gyromagnetic ratio were made on three cobalt-nickel alloys. The alloys were obtained from the General Electric Company, to which we are much indebted for them. They were deoxidized with 0.2 percent aluminum, 0.2 percent silicon, and 0.5 percent manganese. The percentages are by addition; the alloys were not analyzed. We are much indebted to Dr. Duwez of the Metallurgical Laboratory for annealing all these rotors. They were annealed in hydrogen for 4 hours at 1350°C.

Table VI. 60 percent cobalt-iron rotor. Double amplitude 3.3–4.2 cm.

			NPE		*NPW*		
Date	*T*	Obs	I	II	I	II	$\rho e/m$
1950							
Aug. 6	nt	1	1.086	1.046	1.031	0.977	1.035
7	eve	1	1.075	1.083	1.033	1.009	1.050
7	nt	1	1.105	1.065	1.019	0.993	1.046
8	nt	1	1.076	1.095	0.994	1.013	1.045
8	nt	1	1.103	1.074	1.022	1.007	1.052

From these observations the mean value of $\rho e/m$ is 1.046 ±0.004.

1. *20 percent cobalt-nickel rotor.*—(See Table XI.) The band of light on the scale was crossed by lines showing that the effects of magnetostriction were present. The pattern was steady. After 3 runs which were rejected because of large asymmetries and poor magnetic conditions, four good sets of measurements were obtained. The quadrature coil was used.

Table VII. 70 percent cobalt-iron rotor. Double amplitude 3.0–3.8 cm.

			NPE		*NPW*		
Date	*T*	Obs	I	II	I	II	$\rho e/m$
1950							
Aug. 1	eve	1	1.087	1.049	0.998	1.046	1.045
4	eve	1	1.016	1.075	1.085	1.061	1.059
5	nt	1	1.052	1.077	1.025	1.062	1.054
5	nt	1	1.012	1.074	1.076	1.105	1.066

The mean of these results is $\rho e/m = 1.056 \pm 0.007$.

Table VIII. 80 percent cobalt-iron rotor. Double amplitude 4.7–7.0 cm.

			NPE		*NPW*		
Date	*T*	Obs	I	II	I	II	$\rho e/m$
1950							
Mar. 20	nt	½	1.249	0.945	1.185	0.962	1.085
22	eve	1	1.159	1.012	1.060	1.079	1.077
23	day	1	1.195	1.028	1.091	1.036	1.088

The weighted mean for these observations is $\rho e/m = 1.083 \pm 0.005$.

2. *40 percent cobalt-nickel rotor.*—(See Table XII.) Quadrature torques were small. The quadrature coil was used.

3. *70 percent cobalt-nickel rotor.*—There were enormous effects from magnetostriction in this rotor. Measurements were not attempted.

4. *85 percent cobalt-nickel rotor.*—(See Table XIII.) This rotor gave rather small amplitudes, but very steady readings. Quadrature effects were small. The quadrature coil was used.

Table IX. 90 percent cobalt-iron rotor. Double amplitude 3.6–4.4 cm.

			NPE		*NPW*		
Date	*T*	Obs	I	II	I	II	$\rho e/m$
1950							
April 6	day	½	1.117	1.124	1.044	1.067	1.088
7	nt	1	1.126	1.110	1.062	1.064	1.091
8	nt	1	1.116	1.100	1.053	1.062	1.083
11	nt	1	1.100	1.096	1.059	1.067	1.081

The weighted mean of these results is $\rho e/m = 1.086 \pm 0.004$.

C. Nickel-Iron Alloys

Measurements of gyromagnetic ratios were made on 4 nickel-iron alloys obtained from the General Electric Company, to which we desire to express our thanks. The alloys were deoxidized with 0.2 percent silicon and 0.2 percent aluminum. The percentages listed are by addition. We desire to express our thanks also to Dr. Duwez, who annealed these rotors in the same manner in which the Co–Ni rotors were annealed.

1. *15 percent nickel-iron rotor.*—(See Table XIV.) Quadrature torques were so small with this rotor that the quadrature coil was not used.

Table X. Cobalt rotor. Double amplitude 2.0–2.2 cm.

			NPE		*NPW*		
Date	*T*	Obs	I	II	I	II	$\rho e/m$
1950							
June 27	nt	1	1.067	1.110	1.084	1.019	1.070
28	nt	1	1.059	1.102	1.099	1.040	1.075
30	eve	½	1.088	1.089	1.084	1.053	1.078
July 1	eve	1	1.101	1.075	1.049	1.097	1.081

The weighted mean of these results is $\rho e/m = 1.076 \pm 0.004$. (On the curves of Figs. 12 and 13 this point is placed one-tenth percent too low.)

TABLE XI. 20 percent cobalt-nickel rotor. Double amplitude 1.7–2.4 cm.

Date	T	Obs	NPE		NPW		$\rho e/m$
			I	II	I	II	
1950							
Aug. 16	eve	1	0.970	1.132	1.046	1.091	1.060
16	nt	1	0.953	1.133	1.046	1.096	1.057
17	nt	1	0.927	1.172	1.068	1.097	1.066
17	nt	1	0.967	1.135	1.025	1.079	1.052

The mean of these results is $\rho e/m = 1.059 \pm 0.005$.

2. *40 percent nickel-iron rotor.*—(See Table XV.) At small amplitudes, the band of light on the scale was crossed by faint lines, indicating small effects from magnetostriction were present. Quadrature torques were very small and the quadrature coil was not used.

3. *65 percent nickel-iron rotor.*—(See Table XVI.) One measurement was rejected because of very unstable conditions. Two good measurements were obtained. The rotor was very nearly symmetrical. The quadrature coil was used.

4. *90 percent nickel-iron rotor.*—(See Table XVII.) The earth's magnetic field was not very steady during this series of measurements. The rotor was very true mechanically. The quadrature coil was used.

TABLE XII. 40 percent cobalt-nickel rotor. Double amplitude 2.6–3.6 cm.

Date	T	Obs	NPE		NPW		$\rho e/m$
			I	II	I	II	
1950							
Aug. 22	nt	½	0.975	1.189	0.964	1.159	1.072
	nt	½	0.959	1.247	0.909	1.172	1.072
	nt	½	1.085	1.118	1.034	1.097	1.083
	nt	1	1.121	1.090	1.072	1.073	1.089

The weighted mean of the results is $\rho e/m = 1.080 \pm 0.008$.

D. Iron Rotors

Gyromagnetic ratios were measured for two pure iron rotors. The material for one was obtained from Westinghouse, that for the other was obtained from the Bell Telephone Laboratories.

1. *Westinghouse rotor.*—(See Table XVIII.) This rotor was reconstructed from the same material used in earlier researches on the Einstein-de Haas effect, and was cut out from a rod used in Columbus and Washington on the Barnett effect. In the process of reconstruction the rotor broke in the middle but was welded togther again. The quadrature coil was used.

TABLE XIII. 85 percent cobalt-nickel rotor. Double amplitude 1.1–1.3 cm.

Date	T	Obs	NPE		NPW		$\rho e/m$
			I	II	I	II	
1950							
Aug. 26	nt	1	1.036	1.162	1.096	1.031	1.081
26	nt	1	1.031	1.184	1.098	1.030	1.080
27	nt	1	1.051	1.153	1.102	1.025	1.083
27	nt	1	1.032	1.184	1.106	1.010	1.083

The mean of these observations gives $\rho e/m = 1.082 \pm 0.001$.

TABLE XIV. 15 percent nickel-iron rotor. Double amplitude 3.1–3.4 cm.

Date	T	Obs	NPE		NPW		$\rho e/m$
			I	II	I	II	
1950							
Aug. 30	nt	1	1.075	1.014	1.041	0.983	1.028
Sept. 1	nt	1	1.086	1.018	0.999	1.032	1.034
	nt	1	1.057	1.057	1.011	1.027	1.038

The mean of these results is $\rho e/m = 1.033 \pm 0.004$.

2. *Bell Telephone rotor.*—(See Table XIX.) Quadrature effects were small, but the quadrature coil was used.

The mean value for pure iron is 1.037±0.003.

XVIII. CONCLUSIONS

Figures 12, 13, and 14 show the gyromagnetic ratio plotted against the concentration for each of the three series of alloys. The solid circles represent the values obtained in the present investigation; the open circles, values obtained in the Norman Bridge Laboratory in the course of earlier measurements on a few of the same materials, or closely related materials, by the Einstein-de Haas effect; and the squares, values obtained in the same laboratory in the course of measurements by the Barnett effect. The open circle values are considered the most precise. All are in excellent agreement in view of the difficulties involved.

TABLE XV. 40 percent nickel-iron rotor. Double amplitude 4.6–5.2 cm.

Date	T	Obs	NPE		NPW		$\rho e/m$
			I	II	I	II	
1950							
Sept. 8	nt	1	1.016	1.070	1.056	0.989	1.033
	nt	1	1.077	1.011	1.061	0.983	1.035
	nt	1	1.072	1.018	1.082	0.972	1.038

The mean of these results is $\rho e/m = 1.035 \pm 0.002$.

The average errors for all the new rotors are given immediately after the tables. The average for all new

TABLE XVI. 65 percent nickel-iron rotor. Double amplitude 2.4–3.2 cm.

Date	T	Obs	NPE		NPW		$\rho e/m$
			I	II	I	II	
1950							
Sept. 14	nt	1	0.917	1.205	1.037	1.041	1.050
15	nt	1	1.000	1.112	0.999	1.074	1.046

The mean from these observations is $\rho e/m = 1.048 \pm 0.002$.

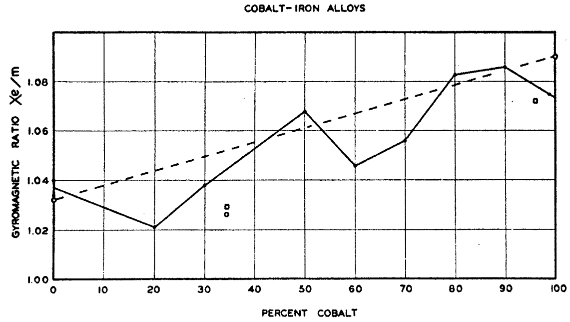

Fig. 12. Results for cobalt-iron alloys.

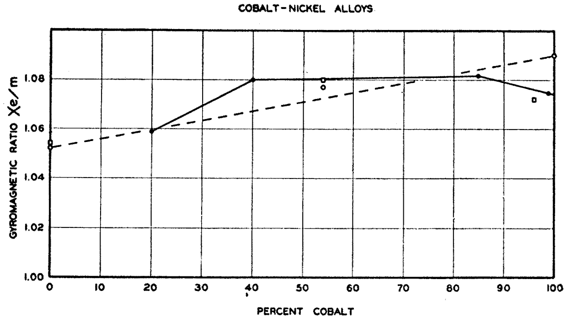

Fig. 13. Results for cobalt-nickel alloys.

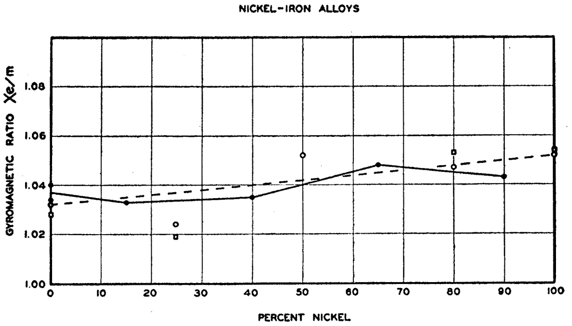

Fig. 14. Results for nickel-iron alloys.

rotors is 0.006±0.004. The errors range from 0.001 for Bell Telephone iron and 85 percent cobalt-nickel, to 0.018 for 90 percent nickel-iron.

Table XVII. 90 percent nickel-iron rotor. Double amplitude 2.3–3.1 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho e/m$
1950							
Sept. 2	nt	1	1.015	1.062	0.897	1.139	1.028
4	nt	1	0.949	1.124	0.873	1.148	1.024
4	nt	1	0.987	1.156	0.888	1.154	1.047
5	nt	1	0.989	1.172	0.882	1.195	1.060
7	nt	1	0.970	1.135	0.939	1.182	1.056

The mean from these observations is $\rho e/m=1.043\pm0.018$.

Table XVIII. Westinghouse pure iron rotor. Double amplitude 3.0–3.9 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho e/m$
1950							
July 5	eve	½	1.027	1.025	1.024	1.043	1.030
6	eve	1	1.055	1.058	1.015	1.044	1.043
7	nt	1	1.068	1.073	1.005	1.030	1.044
11	nt	1	1.078	1.060	1.014	1.021	1.043

The mean for these observations is $\rho e/m=1.040\pm0.005$.

Table XIX. Bell Telephone pure iron rotor. Double amplitude 3.5–4.8 cm.

Date	T	Obs	NPE I	NPE II	NPW I	NPW II	$\rho e/m$
1950							
July 12	nt	½	1.030	1.073	1.002	1.034	1.035
15	nt	1	1.017	1.070	1.000	1.041	1.032
20	nt	1	0.974	1.042	1.030	1.097	1.035

The mean of these results is $\rho e/m=1.034\pm0.001$.

The most important characteristic of each of the curves is the general increase which it shows in the value of ρ with increasing concentration of the element which has the larger gyromagnetic ratio. The linear or nearly linear relation[12] between the gyromagnetic ratio and the concentration is unmistakable.

XIX. SPECIAL ACKNOWLEDGMENTS

For important help in this work we are indebted to many others, a number of whom are mentioned in the text. We have to thank the Carnegie Institution of Washington for the use of much of the fixed equipment and the U. S. Navy for support in all parts of the work. The experiments have been made in the Norman Bridge Laboratory of the California Institute of Technology.

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[12] In a note in the Comptes Rendus [S. J. Barnett, Compt. rend. **231**, 761 (1950)], in the French translation, the gyromagnetic ratio in the case of each series was said to increase linearly with the concentration. The original had linearly or nearly linearly, as above.

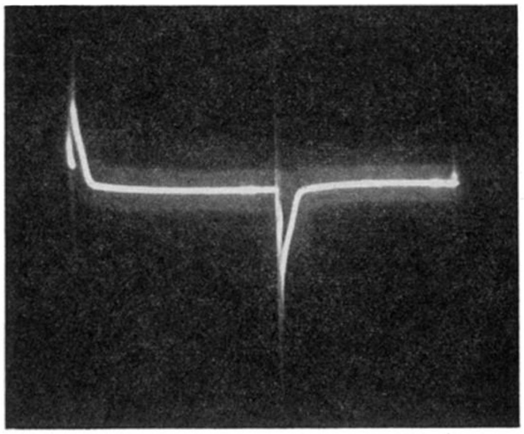

FIG. 10. Secondary emf *vs* time for electrolytic iron rotor.

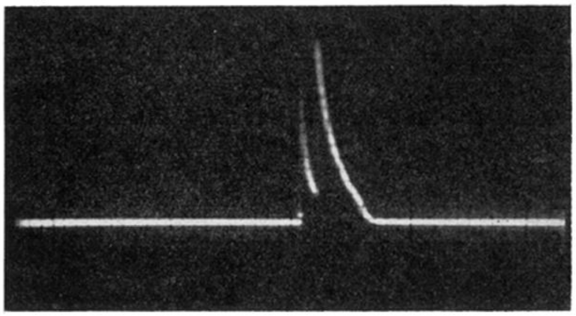

FIG. 11. Secondary emf *vs* time for 41 percent Ni—60 percent Fe rotor of Fig. 8, but with a different time scale.

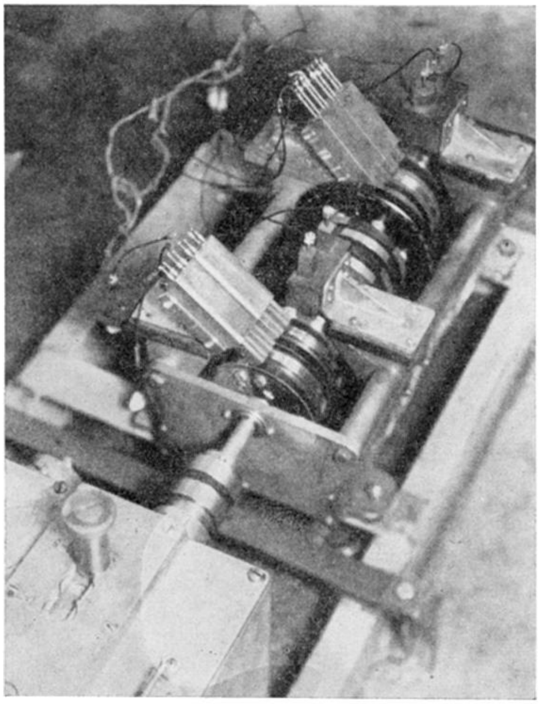

FIG. 2. Main and quadrature square wave commutators.

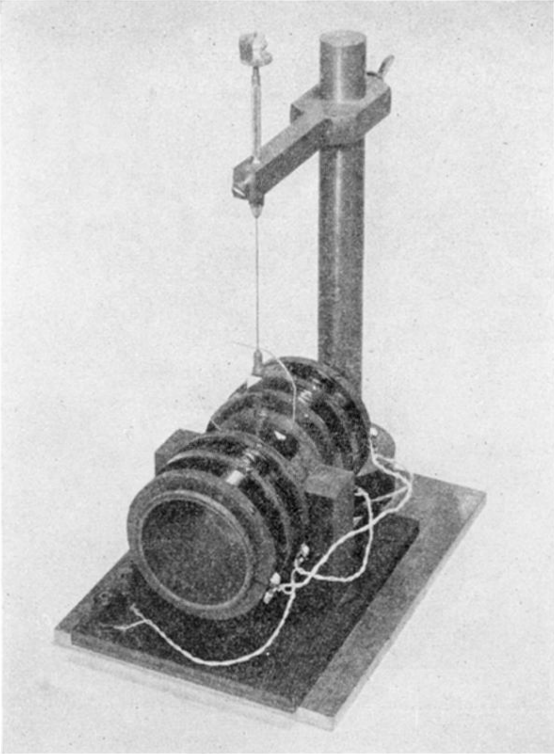

FIG. 3. The phase adjuster.

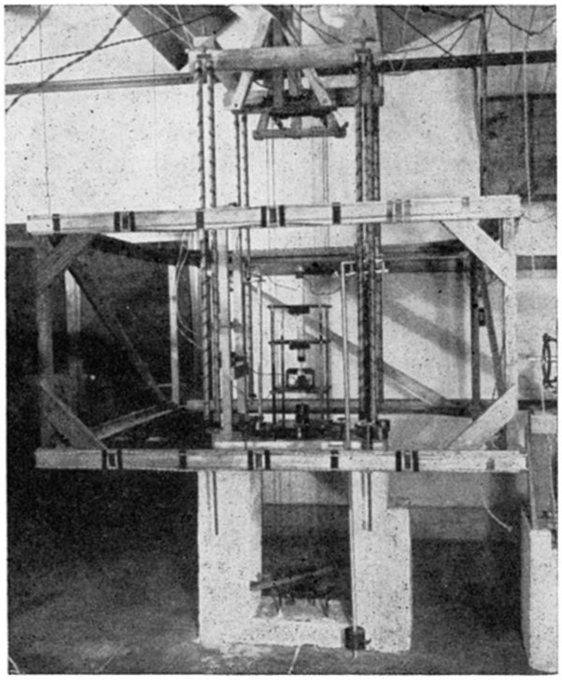

FIG. 5. The coils for neutralizing the earth's magnetic field, the antivibration equipment, the coils in which the rotor was suspended, the torque and quadrature coils, etc.

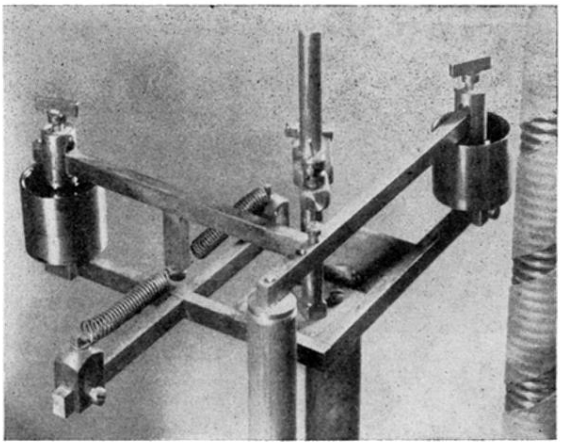

FIG. 6. Details of devices for annulling horizontal and vertical vibrations.

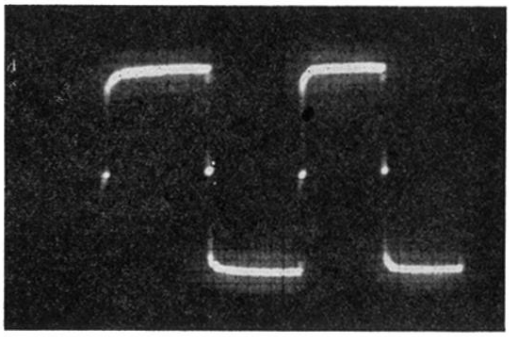

Fig. 7. Current in primary coil *vs* time for Bell Telephone iron rotor.

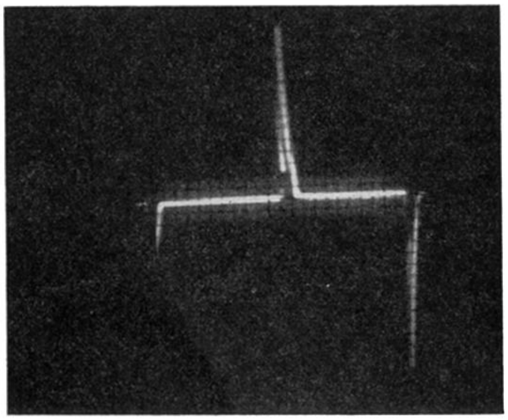

FIG. 8. Secondary emf *vs* time for 41 percent Ni 60 percent Fe rotor.

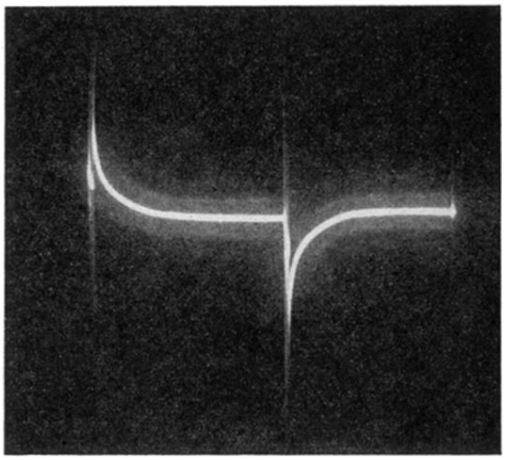

FIG. 9. Secondary emf *vs* time for 15 percent Ni 85 percent Co rotor.