current distribution associated with the particle, then these coefficients themselves can be finite only if this charge and current density ultimately fall off faster with distance from the particle than any inverse power of this distance. When this is not the case our representation obviously fails, for some coefficients in the series will then be infinite.

Our primary purpose in setting up the characterization of the electromagnetic properties of Dirac particles presented above has been to provide a framework for the interpretation of the experimental results on the electron-neutron interaction as given in the following paper. However, we regard our results as quite tentative and have emphasized the shortcomings of our characterization to indicate how urgently a more satisfactory characterization is needed.

APPENDIX

We consider the problem of constructing all possible Lorentz scalars formed from the Dirac matrices γ_μ and the four-vector A_μ of the electromagnetic potentials, and its derivatives, which are linear in A_μ. We employ a Lorentz gauge so that A_μ satisfies the equation

$$\partial A_\mu/\partial x_\mu = 0. \qquad \text{(A-1)}$$

We proceed by examining in succession invariants of the form

$$\gamma_\tau \cdots \gamma_\sigma \gamma_\nu \gamma_\mu \partial^n A_\mu/\partial x_\tau \cdots \partial x_\sigma \partial x_\nu.$$

The first of these containing one γ is $\gamma_\mu A_\mu$. The next containing two γ's is

$$\gamma_\nu \gamma_\mu \partial A_\mu/\partial x_\nu,$$

which we may transform by the use of the commutation relations of the γ's,

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu},$$

in the following way:

$$\begin{aligned}\gamma_\nu \gamma_\mu \partial A_\mu \partial x_\nu &= \tfrac{1}{2}(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu + 2\delta_{\mu\nu})\partial A_\mu/\partial x_\nu \\ &= \tfrac{1}{2}\gamma_\nu\gamma_\mu(\partial A_\mu/\partial x_\nu - \partial A_\nu/\partial x_\mu).\end{aligned}$$

With three γ's we may form and reduce the invariant

$$\begin{aligned}\gamma_\sigma\gamma_\nu\gamma_\mu\partial^2 A_\mu/\partial x_\sigma \partial x_\nu &= \tfrac{1}{2}[\gamma_\sigma\gamma_\nu - \gamma_\nu\gamma_\sigma + 2\delta_{\nu\sigma}]\gamma_\mu \partial^2 A_\mu/\partial x_\sigma \partial x_\nu \\ &= \gamma_\mu \partial^2 A_\mu/\partial x_\nu \partial x_\nu = \gamma_\mu \Box A_\mu.\end{aligned}$$

By continuation of this process one readily finds that the most general invariants which may be formed belong to one of the two classes:

$$\gamma_\mu \Box^n A_\mu, \quad \gamma_\nu\gamma_\mu\Box^n(\partial A_\mu/\partial x_\nu - \partial A_\nu/\partial x_\mu),$$

where n is any non-negative integer.

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The Electron-Neutron Interaction*

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The known electromagnetic properties of nucleons, assuming that the electron-neutron interaction is fundamentally of electromagnetic origin, are fitted into the phenomenological framework developed in the preceding paper and the results compared with predictions of weak coupling meson theories. The detailed comparison shows that the intrinsic electron-neutron interaction is somewhat smaller than predicted and it is suggested that even in the more favorable cases, the rough agreement as to order of magnitude may be largely due to a fortuitious cancellation of different contributions, which may easily be upset when higher order effects are included in the theory. Even apart from the detailed calculations, it is indicated that the observed intrinsic electron-neutron interaction is considerably smaller than order-of-magnitude expectations from general meson-theoretical principles. The results emphasize the importance of more accurate experimental determinations of the electron-neutron interaction, since a smaller value of the intrinsic interaction will either pose a very stringent test for any meson theory or require a critical re-evaluation of our present ideas regarding nucleonic structure. Some phenomena related to the electron-neutron interaction and the possibility that the intrinsic interaction may be nonelectromagnetic in origin are briefly discussed.

RECENT measurements of the magnitude of the electron-neutron interaction by Hughes[1] and by Hamermesh, Ringo, and Wattenberg,[2] when combined with previous measurements by Fermi and Marshall,[3] and by Havens, Rabi, and Rainwater,[4] now yield an experimental value for this quantity with an accuracy of the order of ten percent. While there appear prospects for a considerably more accurate determination of this interaction in the near future, it appears appropriate, nevertheless, to examine the available results in the

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[1] D. J. Hughes, New York meeting of the American Physical Society [Phys. Rev. **86**, 606 (1952)].

[2] Hamermesh, Ringo, and Wattenberg, Phys. Rev. **85**, 483 (1952).

[3] E. Fermi and L. Marshall, Phys. Rev. **72**, 1139 (1947).

[4] Havens, Rainwater, and Rabi, Phys. Rev. **82**, 345 (1951).

light of current notions concerning the origin of this interaction at the present time. Such an examination as given in this paper indicates the possibility of a rather severe conflict between these experimental results and current ideas which ascribe the electron-neutron interaction to the charge cloud of virtual mesons surrounding the neutron. The importance of further and more accurate measurements of the electron-neutron interaction is thus emphasized.

We begin with a summary of experimental results concerning the electromagnetic properties of nucleons and their characterization in the phenomenological scheme developed in the preceding paper. The results are then compared with expectations from meson theory and some of the difficulties discussed. Finally the possibility of a nonelectromagnetic origin of the electron-neutron interaction is considered and some other phenomena which bear on this question are briefly discussed.

At the present time we have experimental knowledge concerning four purely electromagnetic properties of nucleons, namely, (1) the charge on the proton $=|e|$ where e is the electronic charge, (2) the magnetic moment of the proton $=2.7896$ nuclear magnetons, (3) the magnetic moment of the neutron $=-1.9135$ nuclear magnetons, and (4) the electron-neutron interaction. The last quantity is a short range interaction between a neutron and an electron which is given (perhaps prematurely) an electromagnetic interpretation by considering it to be a direct interaction of the neutron with the charge density responsible for an electromagnetic field. The experimental determinations of this interaction have been executed only in the case where this charge density is that associated with electrons in atoms, hence the ascription of the name electron-neutron interaction. Its magnitude is usually specified by the convention of giving the magnitude of the potential which must extend over a spherical volume of radius equal to the classical electron radius e^2/mc^2 which will give the same scattering matrix element at low energies as does the actual interaction.[5] The experimental determinations of this potential V_0 are given in Table I together with their weighted mean. At present we have no information concerning the corresponding interaction for the proton.

TABLE I. Experimental determinations of the electron-neutron interaction V_0.

Investigators	V_0, ev
Fermi and Marshall	-300 ± 5000
Havens, Rabi, and Rainwater	-5300 ± 1000
Hughes	-4200 ± 700
Hammermesh, Ringo, and Wattenberg	-4100 ± 1000
Weighted mean	-4400 ± 400
Contribution of magnetic moment	-4080
Residual (intrinsic interaction)	-320 ± 400

We may note first by a comparison of these results with the phenomenological framework developed in the preceding paper[6] for characterizing electromagnetic properties of Dirac particles, that this framework is adequate for the description of experimental results. Thus for the proton we may immediately identify ϵ with the charge of the proton and μ_0 with the anomalous magnetic moment of the proton:

$$\epsilon_0{}^p=|e|, \quad \mu_0{}^p=1.7896|e|\hbar/2Mc.$$

We have no information concerning the values of the succeeding coefficients $\epsilon_1{}^p$, $\mu_1{}^p$, etc.

Correspondingly, we have for the neutron the identifications:

$$\epsilon_0{}^n=0; \quad \mu_0{}^n=-1.9135|e|\hbar/2Mc.$$

The electron-neutron interaction, however, allows us to obtain a rough value for the next coefficient $\epsilon_1{}^n$. To obtain this we note that the volume integral of the term in $\mathrm{div}\mathbf{E}=-\Box\varphi$ in Eq. (8) of the preceding paper, where $\mathbf{E}$ is the electric field due to an electron, may be immediately identified with V_0 multiplied by the volume over which it is assumed to exist:

$$\frac{4\pi}{3}\left(\frac{e^2}{mc^2}\right)^3 V_0=|e|\left\{\epsilon_1{}^n+\frac{\hbar}{2Mc}\mu_0{}^n+\frac{1}{2}\left(\frac{\hbar}{2Mc}\right)^2\epsilon_0{}^n\right\}.$$

The last term in the brackets on the right vanishes for the neutron, and the second term in the brackets, as shown in a previous publication,[7] accounts for -4080 ev of the observed potential V_0. The remainder -320 ± 400 ev then yields for $\epsilon_1{}^n$ the value

$$\epsilon_1{}^n=-(0.08\pm0.10)|e|(\hbar/Mc)^2.$$

Before discussing these results further we compare them with the predictions obtained from meson theory in the limit of weak coupling.[8] A direct comparison is made somewhat difficult by the fact that the meson theory results depend on the value of the coupling constant assumed. To circumvent this difficulty we have tabulated in Table II the values of the ratios $\epsilon_1{}^n/(\mu_0{}^n\hbar/2Mc)$ and $\mu_0{}^p/\mu_0{}^n$ which, in weak coupling theory, are independent of the choice of coupling constant. However, it must be remembered that this

[5] The use of the classical electron radius is very arbitrary and somewhat inappropriate, since this radius presumably has nothing to do with the actual range of the interaction. In fact, as will be clearer later, the Compton wavelength of the neutron would be a considerably more appropriate radius to employ.

[6] L. L. Foldy, Phys. Rev. 87, 688 (1952) (preceding paper).

[7] L. L. Foldy, Phys. Rev. 83, 688 (1951). The difference between the present value quoted for the contribution of the neutron's magnetic moment and that quoted in this reference (-3900 ev) is due simply to the use of very rough values for fundamental constants in the previous calculation. The present value should be correct to approximately 1 ev. See also G. Breit, Proc. Nat. Acad. Sci. U. S. 37, 837 (1951).

[8] J. M. Luttinger. Phys. Rev. 74. 893 (1948): M. Slotnick and W. Heitler, [illegible] Phys. Acta 21, 1645 (19[illegible]949); S. M. Dancoff and S. D. Drell, Phys. Rev. 76, 205 (1949); S. Borowitz and W. Kohn, Phys. Rev. 76, 818 (1949); B. D. Fried (to be published).

will no longer be the case if higher order terms in the meson theory were retained. Calculations have been made for the neutral, charged, and symmetrical scalar and pseudoscalar theories. The calculations for the neutral theory have been omitted from Table II since they lead to zero neutron magnetic moment and zero electron-neutron interaction.

An examination of the table shows immediately that it is not possible to fit simultaneously the nucleon magnetic moments and the electron-neutron interaction to the observed results by any choice of the coupling constant in any of these theories. The best fit is provided by the charged pseudoscalar theory but this can hardly be taken as evidence for its validity. Of particular interest, however, is the sensitivity of the electron-neutron interaction to the type of meson theory. There is a difference of a factor of thirty between the results in the scalar and the pseudoscalar theories. The pseudoscalar result is somewhat smaller than one would expect on rough dimensional grounds and this suggests that its smallness is at least partly the result of an accidental cancellation. This conjecture appears to be substantiated by some calculations of the author of the value of the ratio $\epsilon_1{}^n/(\mu_0{}^n\hbar/2Mc)$ as a function of the ratio of meson to nucleon mass. These calculations indicate this quantity is positive for small mass ratio but becomes negative for larger mass ratios, and the result passes through zero not far from the experimental ratio of pi-meson to nucleon mass. This is an important point, since if we do have an accidental near cancellation in the pseudoscalar theory, it is likely to be upset when other effects (higher order corrections, for example) are included and the discrepancy with the experimental value thereby increased. The smallness of the observed intrinsic electron-neutron interaction would then be a very stringent test for any meson theory. In this connection, the discrepancy by a factor of one hundred (even apart from the discrepancy in sign) between the experimental value of $\epsilon_1{}^n/(\mu_0{}^n\hbar/2Mc)$ and that predicted by the scalar meson theory would seem to be a strong argument against this theory, since it indicates that either the meson charge cloud in this theory is far too extended or the region over which the currents that give rise to the anomalous moment are distributed is far too compact.[9]

The discrepancies between the experimental results and meson theory calculations is not too surprising in view of the other well-known deficiencies of meson theory. However, there is nevertheless a rather striking anomaly in the experimental results which can be stated in a manner which is independent of any direct reference to meson theory. We may see this by remembering that at the present time we know of only two electromagnetic interactions of the neutron (remembering that the neutron has no electric charge) which are measured by the two fundamental parameters: $\mu_0{}^n$ the magnetic moment of the neutron, and $\epsilon_1{}^n$ the intrinsic Darwin coefficient which measures the intrinsic electron-neutron interaction. The ratio of these two parameters $\epsilon_1{}^n/\mu_0{}^n$ is a quantity of the dimensions of a length and represents the only "electromagnetic radius" of the neutron of which we have present knowledge. The experimental ratio is $\epsilon_1{}^n/\mu_0{}^n=(0.04\pm0.05)\hbar/Mc$ and is, therefore, twenty-five times smaller than the "mechanical radius" of the neutron given by the neutron's Compton wavelength. This is certainly much smaller than would be expected from any *a priori* notions. This odd disparity can be demonstrated in another way: if we assume that q represents the effective distributed electric charge of the neutron and that the charge and current of the neutron are both spread over a spherical volume whose radius is of the order a, then we would expect

$$\epsilon_1{}^n\sim qa^2, \quad \mu_0{}^n\sim qa.$$

Solving these for q and a we find

$$a\sim\epsilon_1{}^n/\mu_0{}^n\sim0.04\hbar/Mc, \quad q\sim(\mu_0{}^n)^2/\epsilon_1{}^n\sim-25|e|,$$

which is a result quite out of line with current notions. If we assume that the effective distributed charge of the neutron is actually of the order of $-|e|$, then we must assume that the charge density of the neutron is spread over a region whose radius is of the order of 1/25 of the radius of the region over which the current density of the neutron is distributed.

The situation is not quite so dark as painted above, however, when we note that in the pseudoscalar meson theory we find also a small theoretical value for the ratio $\epsilon_1{}^n/\mu_0{}^n$, namely, $0.16\hbar/Mc$. An explanation for the smallness of this result may be formulated as follows: In the pseudoscalar theory, the emission of a negative meson by a neutron (which simultaneously is converted into a proton) is accompanied by a relatively large recoil of the proton. Hence, not only is the negative meson charge spread out over a finite volume, but so also is the positive proton charge. If the spatial spread of the two is nearly the same, the intrinsic electron-neutron interaction will be considerably reduced over its value when the recoil of the proton is neglected. Hence the smallness of the interaction in the pseudoscalar theory probably results from a cancellation

TABLE II. Comparison of experimental results on electromagnetic properties of nucleons with predictions of weak coupling meson theories.[a]

Theory	$\epsilon_1{}^n/(\mu_0{}^n\hbar/2Mc)$	$\mu_0{}^p/\mu_0{}^n$
Symmetrical pseudoscalar	0.318	−0.128
Charged psuedoscalar	0.318	−0.422
Symmetrical scalar	−9.05	0.627
Charged scalar	−9.05	0.088
Experimental	0.08±0.10	−0.935

[a] The theoretical values in this table have been compiled from the publications listed in reference 8.

[9] This is due, of course, to the fact that in the scalar theory, the charge cloud is produced principally by the mesons, but the mesons do not contribute to the magnetic moment.

between the negative meson charge distribution and the positive proton charge distribution. This conjecture is substantiated by the fact that the intrinsic electron-neutron interaction changes sign when the meson mass becomes comparable with the nucleon mass, indicating that in this case because of the larger proton recoil, the proton charge is actually spread over a larger volume than the meson charge. Also in line with this explanation is the fact that in the scalar theory where the proton recoil is relatively smaller, the computed electron-neutron interaction is considerably larger than in the pseudoscalar theory. However, it should also be mentioned that the failure of the pseudoscalar meson theory to give approximately equal but opposite anomalous magnetic moments to the proton and neutron, as observed experimentally, seems also to be due to the large nucleon recoil effects;[10] this would suggest a difficulty in formulating a meson theory which simultaneously yields a small intrinsic electron-neutron interaction and approximately equal and opposite anomalous magnetic moments for the proton and neutron.

Our discussion above has been based on the assumption that the intrinsic electron-neutron interaction is essentially an electromagnetic interaction between the neutron and the charge density producing an external electromagnetic field. If this is a valid assumption, then the short range interaction of the neutron with an electron is not specific to the electron but would be present in the interaction of the neutron with any charged particle to the same magnitude but with a sign corresponding to an attraction between the neutron and negatively charged particles and repulsion between the neutron and positively charged particles. It is, in principle, possible to detect this interaction in the case of the neutron-proton interaction, but the presence of the large specifically nuclear interaction whose exact nature is not known makes this unfeasable at the present time.[11] The possibility of detecting a neutron-positron interaction or the interaction of a neutron and mu-mesons also appears very remote at the present time in view of the experimental difficulties. The interaction of the neutron and charged pi-mesons is also obscured, in this case by a strong meso-nucleonic interaction of nonelectromagnetic origin about which we also know very little. Hence, it is not likely that we may verify the assumption that the observed intrinsic electron-neutron interaction is fundamentally electromagnetic in nature in the near future. The detection of an intrinsic short range electron-proton interaction arising from a nonzero intrinsic Darwin coefficient for the proton is also possible, in principle, from its contribution to the Lamb shift in hydrogen, but some refinement of both the theory and the experimental determination of the Lamb shift is necessary before a quantitative result would be available. In this connection, we may also note that the electron-neutron interaction would be expected to contribute to a very slight difference in the Lamb shift in hydrogen and deuterium.

Before closing, it is of interest to consider the possibility of nonelectromagnetic contributions to the intrinsic electron-neutron interaction. Since this interaction can be formulated as a direct interaction between two incoming and two outgoing Dirac particles, one can express it in a manner analogous to beta-decay interactions—that is, as a Fermi-type coupling. It might even be expected that, in view of other evidence for the existence of a universal Fermi interaction between all fermions, such a Fermi interaction between neutrons and electrons should exist. If the entire intrinsic electron-neutron interaction were represented by a Fermi interaction, the corresponding coupling constant G_{e-n}, which has the dimensions erg-cm^3, can be directly identified with the observed volume integral of the electron-neutron interaction and, expressed in absolute units, would have the value

$$G_{e-n}=(5\pm6)\times10^{-47}\ \text{erg-cm}^3.$$

Comparing this with the Fermi constant for beta-decay $G_\beta=2.5\times10^{-49}$ erg-cm^3 we see that the observed intrinsic electron-neutron interaction is, at most, two orders of magnitude greater than beta-decay interactions, and, in fact, the large error in the experimental results does not yet preclude the possibility of them being identical. If the last possibility were actually the case, then the intrinsic electron-neutron interaction would have a magnitude of only 1.6 ev.

We summarize with the observation that the intrinsic electron-neutron interaction is somewhat smaller than would be expected on the basis of semiquantitative conclusions obtained from meson theory. Its experimental determination is not yet sufficiently precise to indicate how much of a hurdle this fact represents for meson theory, and the need for more precise measurements is clearly indicated. Should the interaction turn out to be appreciably smaller than -300 ev, a real challenge will be presented to currently popular forms of meson theory, since it appears that only a rather unlikely accidental cancellation of contributions to the interaction would lead to a predicted interaction as small as this. In such an eventuality, a complete and critical re-evaluation of our current ideas concerning the structure of nucleons would be necessitated.

[10] K. M. Case, Phys. Rev. **76**, 1 (1949).
[11] See, however, J. Schwinger, Phys. Rev. **78**, 135 (1950).