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The Electromagnetic Properties of Dirac Particles*

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A framework for describing those electromagnetic properties of Dirac (spin-$\frac{1}{2}$) particles which determine their behavior when moving with low momentum through weak, slowly varying, external electromagnetic fields is developed by finding the most general interaction terms which may be added to the Dirac equation for the particle subject to appropriate conditions. The interaction terms found form an infinite series involving arbitrarily high derivatives of the electromagnetic potentials evaluated at the position of the particle. The series of coefficients of these terms then characterize the properties to be described and can be interpreted as a series of moments of the charge and current distribution associated with the Dirac particle. The first coefficient represents the charge of the particle, the second its anomalous magnetic moment. The third is a measure of the spatial extent of the particle's charge distribution, and the corresponding term describes a direct interaction of the particle and the charge distribution responsible for the external electromagnetic field. Higher terms in the series describe direct interactions of the particle with various derivatives of the external charge and current distribution. The correct physical interpretation of the terms is examined by transforming to the Foldy-Wouthuysen (nonrelativistic) representation of the Dirac equation. The consistency of the framework developed with field theoretical results is discussed. Limitations of the characterization derived here and the possibilities of broadening the assumptions on which it is based are examined. The results are applied in the succeeding paper to the interpretation of the electromagnetic properties of nucleons with particular reference to the electron-neutron interaction.

INTRODUCTION

THE present paper represents an attempt to characterize those electromagnetic properties of Dirac (spin-$\frac{1}{2}$) particles which determine their behavior when moving with low momentum through weak, slowly varying, external (classically describable) electromagnetic fields. For this purpose we assume that the particle satisfies a Dirac equation with terms representing the interaction with the electromagnetic field. The nature of the interaction terms which are included is determined by the following restrictions:

(a) that the equation be Lorentz covariant and gauge invariant;
(b) that the interaction be linear in the electromagnetic potentials (the assumption of weak fields);
(c) that the terms do not vanish in the limit of vanishing momentum of the Dirac particle;
(d) that the charge and current distribution associated with the Dirac particle be sufficiently localized that its interaction with slowly varying electromagnetic fields may be expressed, through an expansion in "moments," in terms of the electromagnetic potentials and arbitrarily high derivatives of these potentials evaluated at the position of the particle.

It is found that an infinite series of interaction terms can be found satisfying these restrictions. The coefficients of these terms then characterize the electromagnetic properties of the particle which we set out to describe and represent "moments" of various order of the charge and current distribution associated with the Dirac particle. The first two coefficients can be identified with the charge and anomalous (intrinsic) magnetic moment of the particle. The next term represents a direct interaction between the Dirac particle and the charge distribution which produces the external electromagnetic field. In the case of the neutron, the coefficient of this term characterizes the "intrinsic" electron-neutron interaction, that is, the part of the electron-neutron interaction which is not associated with the anomalous magnetic moment of the neutron.[1] Higher terms in the series describe the interaction of the particle with various derivatives of the charge and current distribution responsible for the electromagnetic field.

While the characterization of the electromagnetic properties of a Dirac particle by the series of coefficients mentioned above is based on the assumption that the particle satisfies the Dirac equation, it is believed that this characterization actually has greater validity and applies under more general assumptions. The S-matrix formalism provides a more general domain in which this problem may be examined, and the author hopes to investigate this more general problem in the future.

Although from the point of view of Lorentz covariance it is convenient to deal with the problem posed above in the usual Dirac representation of the Dirac equation, the physical interpretation of the interaction terms is facilitated by passing to the Foldy-Wouthuysen (nonrelativistic) representation of the Dirac equation.[2] The coefficients defining the interaction take different forms in the two representations and in describing an electromagnetic property of a Dirac particle it is necessary to specify the representation which is being employed in order to avoid ambiguity. The confusion concerning the electron-neutron interaction stems primarily from a confusion of representations, and it is hoped that the discussion given below will clarify this point. The results derived herein are applied in the subsequent paper to an analysis of the significance of

* The research was supported by the AEC. Some of the results contained herein were presented at the Columbus, Ohio, meeting of the American Physical Society, March 20–22, 1952.

[1] L. L. Foldy, Phys. Rev. **83**, 688 (1951).
[2] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

the experimental results on the electron-neutron interaction.

ELECTROMAGNETIC INTERACTIONS OF DIRAC PARTICLES

The Dirac equation for a free particle can be written in the form

$$H\psi=\{\beta mc^2+c\boldsymbol{\alpha}\cdot\mathbf{p}\}\psi=i\hbar\partial\psi/\partial t, \tag{1}$$

where $\mathbf{p}=-i\hbar\nabla$, $\mathbf{x}$ is the position vector of the particle, β and $\boldsymbol{\alpha}$ are the familiar Dirac matrices, and m, c, and $\hbar$ are, as usual, the rest mass of the particle, the velocity of light, and Planck's constant (divided by 2π), respectively. It is convenient for our purposes to write Eq. (1) in manifestly covariant form:

$$\gamma_\mu\partial\psi/\partial x_\mu+(mc/\hbar)\psi=0, \tag{2}$$

by the use of the relations

$$x_\mu=(\mathbf{x},\, ict),\quad \gamma_\mu=(-i\beta\boldsymbol{\alpha},\, \beta),$$

with the usual summation convention applying in expressions involving repeated Greek indices.

If the particle has a charge e and interacts with an electromagnetic field specified by the four-vector potential $A_\mu(x)$, then Eq. (2) must be modified to read

$$\gamma_\mu\partial\psi/\partial x_\mu+(mc/\hbar)\psi-i(e/\hbar c)\gamma_\mu A_\mu\psi=0. \tag{3}$$

Although the addition of this term appears to imply that the Dirac particle has only a point charge, it is well known that in its interactions the particle behaves as if it also had a magnetic moment of one Bohr magnetron. Pauli[3] showed that Eq. (3) can be still further modified so as to represent a particle having an arbitrary magnetic moment by adding to the left side the term $-(i\mu/\hbar c)\gamma_\mu\gamma_\nu(\partial A_\mu/\partial x_\nu-\partial A_\nu/\partial x_\mu)$, whereupon the particle behaves as if it had an "anomalous" moment μ in addition to its "normal" moment.

One may inquire as to the degree to which one can add further interaction terms to Eq. (3) without destroying its relativistic covariance and gauge invariance. One easily finds that a great variety of terms may be added. If one restricts oneself to such terms as satisfy the conditions specified in the introduction, namely, terms which are linear in the electromagnetic potentials, which depend on these potentials and their derivatives evaluated only at the position of the particle, and which do not vanish in the limit of vanishing momentum of the particle (quasi-static interactions) and hence do not involve derivatives of the wave function, then one can easily construct all possible terms. This calculation is given in the appendix; the result for the most general equation is[4]

$$\gamma_\mu\frac{\partial\psi}{\partial x_\mu}+\frac{mc}{\hbar}\psi-\frac{i}{\hbar c}\sum_{n=0}^{\infty}\left[\epsilon_n\Box^n\gamma_\mu A_\mu+\mu_n\Box^n\gamma_\mu\gamma_\nu\left(\frac{\partial A_\mu}{\partial x_\nu}-\frac{\partial A_\nu}{\partial x_\mu}\right)\right]\psi=0, \tag{4}$$

where $\Box$ stands for the d'Alembertian operator:

$$\Box=\partial^2/\partial x_\sigma\partial x_\sigma=\nabla^2-\partial^2/c^2\partial t^2.$$

The coefficients ϵ_n and μ_n are constants characterizing the interactions, of which ϵ_0 may immediately be identified as the electric charge of the Dirac particle and μ_0 as its anomalous magnetic moment. The higher terms in the series represent a direct interaction between the Dirac particle and the charge and current distribution which produces the external electromagnetic field as may be seen by employing the equations

$$\Box A_\mu=-4\pi j_\mu,\quad F_{\nu\mu}=\partial A_\mu/\partial x_\nu-\partial A_\nu/\partial x_\mu, \tag{5}$$

with

$$j_\mu=(\mathbf{j},\, i\rho),\quad F_{\nu\mu}=(\mathbf{H},\, i\mathbf{E}),$$

to rewrite Eq. (4) in the form

$$\gamma_\mu\frac{\partial\psi}{\partial x_\mu}+\frac{mc}{\hbar}\psi-\frac{i}{\hbar c}\left\{\epsilon_0\gamma_\mu A_\mu+\mu_0\gamma_\mu\gamma_\nu F_{\nu\mu}-4\pi\sum_{n=1}^{\infty}\left[\epsilon_n\Box^{n-1}\gamma_\mu j_\mu+\mu_n\Box^{n-1}\gamma_\mu\gamma_\nu\left(\frac{\partial j_\mu}{\partial x_\nu}-\frac{\partial j_\nu}{\partial x_\mu}\right)\right]\right\}\psi=0. \tag{6}$$

The remainder of this paper is concerned with a more direct physical interpretation of these interaction terms.

However, before proceeding with this aspect of the problem, we wish to comment briefly on the domain of validity of an equation of the type (4). It is very unlikely that there exist any real particles which satisfy an equation of this type rigorously. However, it is not necessary that the equation be satisfied rigorously for the characterization of the electromagnetic structure of the particle by the coefficients ϵ_n and μ_n to be valid. In fact we may assume that the interaction terms in (4) must be treated only by first-order perturbation theory (Born approximation) since we have limited ourselves to the case of terms linear in the electromagnetic field. In this case, Eq. (4) may be used to calculate the S-matrix in the Born approximation for the elastic scattering of the Dirac particle in an external electromagnetic field, and the elements of the S-matrix will then depend on the coefficients ϵ_n and μ_n in a definite way. Equation (4) may then be considered as reduced to playing the role of an equation which gives the same S-matrix for weak electromagnetic fields as will be yielded by some more elaborate and fundamental theory. That Eq. (4) can actually play such a role is demonstrated by the usual treatment of the problem of

[3] W. Pauli, Revs. Modern Phys. **13**, 203 (1941).

[4] That the meson theory expression for the modification of the convection current of a nucleon due to interaction with mesons can be put into the form of the term in ϵ_1 in Eq. (4) was pointed out to the author by Professor F. Villars in a private communication.

the modification of the interaction of a nucleon with an electromagnetic field due to the interaction of the nucleon with a meson field. In this case an equation of the form (4) can be written down with determined values of the coefficients which will yield the same result in the Born approximation for the elastic scattering part of the S-matrix as is yielded by the more fundamental methods of field theory. The characterization by the coefficients ϵ_n and μ_n of certain of the electromagnetic properties of a Dirac particle may thus have a domain of validity much wider than would be indicated by the basis on which it was derived. This is the reason for our earlier remark that it would be of interest to reinvestigate this problem directly from the point of view of the S-matrix and its invariance properties rather than from a wave equation.

PHYSICAL INTERPRETATION OF THE DIRAC EQUATION

The physical interpretation of the interaction terms contained in Eq. (4) requires considerable care; that this is the case can already be seen in the special case represented by Eq. (3), for an examination of this equation itself would lead one to the conclusion that the interaction of the Dirac particle with the electromagnetic field is simply that of a point electric charge. Actually, as mentioned above, it is well known that the Dirac particle described by this equation exhibits in its electromagnetic interactions not only the properties of a point charge but also more complex properties, ordinarily characteristic of a particle possessing an extended charge and current distribution, as, for example, a magnetic moment.

As has been demonstrated in a previous publication,[2] the electromagnetic properties of a Dirac particle may be exhibited in a very direct way by transforming the Dirac equation into a new representation in which states of positive and negative energy for the particle are separately represented by two-component wave functions. The unitary transformation which generates the new representation leads to the introduction of a new position operator (the Newton-Wigner position operator[5]) for the particle which can more readily be identified with the conventional position operator for a particle. The modification in form of the electromagnetic interaction of the particle can then be traced directly to the fact that the interaction in the new representation is expressed in terms of the electromagnetic potentials and their derivatives evaluated at this new position which is displaced by a finite distance from the Dirac position which occurs in the usual representation of the Dirac equation. Actually, in the presence of interaction, the generating function for the transformation to the new representation can only be obtained as a series expansion in powers of the Compton wavelength of the particle and consequently the Hamiltonian occurring in the new form of the Dirac equation is also obtained as a power series in the same parameter. Details of the transformation are given in the reference quoted and hence will not be discussed here. If the transformation is carried out on Eq. (3), the Dirac equation takes the form[6]

$$\left\{\beta mc^2+\frac{\beta}{2m}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2+e\varphi-\frac{e\hbar}{2mc}\beta\boldsymbol{\sigma}\cdot\mathbf{H}+\frac{e\hbar}{8m^2c^2}\left[\boldsymbol{\sigma}\cdot\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)\times\mathbf{E}-\boldsymbol{\sigma}\cdot\mathbf{E}\times\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)\right]-\frac{e\hbar^2}{8m^2c^2}\operatorname{div}\mathbf{E}+\cdots\right\}\psi=i\hbar\frac{\partial\psi}{\partial t},\quad (7)$$

where we have retained terms to order $(\hbar/mc)^2$ relative to the rest energy of the particle. It will be noted that the electromagnetic interaction is now expressed in a relatively complicated way but that one may immediately recognize the physical significance of the various terms:

(a) The term

$$e\varphi-(e\beta/2mc)[(\mathbf{p}-e\mathbf{A}/c)\cdot\mathbf{A}+\mathbf{A}\cdot(\mathbf{p}-e\mathbf{A}/c)]$$

represents the interaction of a point charge e with the electromagnetic field.

(b) The term $-(e\hbar/2mc)\beta\boldsymbol{\sigma}\cdot\mathbf{H}$ expresses the interaction of a magnetic moment of one Bohr magneton with the magnetic field.

(c) The term

$$(e\hbar/8m^2c^2)[\boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A}/c)\times\mathbf{E}-\boldsymbol{\sigma}\cdot\mathbf{E}\times(\mathbf{p}-e\mathbf{A}/c)]$$

represents the "spin-orbit" coupling associated with this magnetic moment; it arises from the fact that the motion of the magnetic moment gives rise to an electric moment for the particle which interacts with the electric field.

(d) The term $-(e\hbar^2/8m^2c^2)\operatorname{div}\mathbf{E}$ (the so-called "Darwin" term)[7] can be interpreted in the following way: if the charge of the particle is not concentrated at a point but is spread out over a small spherical volume, then the first-order correction to the point charge interaction is represented by this term. Since $\operatorname{div}\mathbf{E}=4\pi\rho$, where ρ is the density of charge giving rise to the external electromagnetic field, this term expresses a direct interaction of the Dirac particle with the charge distribution generating the external electromagnetic field.

(e) Higher order terms in the series which have not been written down describe interaction terms depending

[5] T. D. Newton and E. P. Wigner, Revs. Modern Phys. **21**, 400 (1949).

[6] The last section of the paper quoted in reference 2 contains a number of errors and misprints in its equations. The correct form of Eq. (36) of that reference is given in Eq. (7) of the present paper.

[7] C. G. Darwin, Proc. Roy. Soc. (London) **A118**, 654 (1928). The term we call the Darwin term is not identical with that introduced by Darwin in this reference. The difference is due to the fact that Darwin does not write his spin-orbit coupling term as a Hermitian operator.

on higher derivatives of the electromagnetic fields evaluated at the Newton-Wigner position of the particle. All such terms can be written as for (d) above directly in terms of the charge and current distributions which generate the external electromagnetic field. For example, the next term in the series is proportional to $\beta\boldsymbol{\sigma}\,\text{curl}\,\mathbf{j}$, where $\mathbf{j}$ is the current density generating the external electromagnetic field.

The direct physical interpretation of the various terms occurring in the Hamiltonian is possible since the wave function for the Dirac particle in a state of definite energy is just the usual Pauli wave function with well-known properties.

We can then interpret physically the interaction terms in our more general equation (4) by transforming it into the same representation. The methods given in the reference quoted above are directly applicable here, and we only quote the result:

$$\left\{\beta mc^2+\frac{\beta p^2}{2m}+\sum_{n=0}^{\infty}\left[\left\{\epsilon_n+\frac{\hbar}{2mc}\mu_{n-1}+\frac{1}{2}\left(\frac{\hbar}{2mc}\right)^2\epsilon_{n-1}+\cdots\right\}\Box^n\varphi-\left\{\mu_n+\frac{\hbar}{2mc}\epsilon_n+\frac{1}{2}\left(\frac{\hbar}{2mc}\right)^2\mu_{n-1}+\cdots\right\}\Box^n\beta\boldsymbol{\sigma}\cdot\mathbf{H}\right]\right\}\psi=i\hbar\frac{\partial\psi}{\partial t}. \quad (8)$$

In writing down this equation we have again indicated explicitly only those terms which are of no higher order than $(\hbar/mc)^2$ relative to the rest energy term. Also, in view of the fact that we have omitted terms nonlinear in the electromagnetic field and interaction terms which vanish for vanishing momentum of the particle in Eq. (4), we have likewise omitted such terms in Eq. (8). This explains the absence of the point-current term and the spin-orbit coupling term (for example) in this last equation.

We can now analyze the physical significance of the various interaction terms in Eq. (8). It will be noted that they fall into two general classes—electric interactions and magnetic interactions. In particular, we may note:

(a) The term $\epsilon_0\varphi$ represents the usual point charge interaction of the particle with the scalar potential which allows us immediately to identify it with the electric charge of the particle.

(b) The term $\{\mu_0+(\hbar/2mc)\epsilon_0\}\beta\boldsymbol{\sigma}\cdot\mathbf{H}$ represents the interaction of a magnetic moment of magnitude $\mu_0+(\hbar/2mc)\epsilon_0$ with the magnetic field. Since $(\hbar/2mc)\epsilon_0$ represents just the normal moment for the particle, μ_0 can immediately be identified with the anomalous or "intrinsic" magnetic moment of the particle.

(c) The term $\{\epsilon_1+(\hbar/2mc)\mu_0+\frac{1}{2}(\hbar/2mc)^2\epsilon_0\}\Box\varphi$ has just the form of a "Darwin" term ($\Box\varphi=-\text{div}\mathbf{E}=-4\pi\rho$) and so represents a direct interaction of the particle with the charge density of the sources of the electromagnetic field. It consists of three contributions, the part depending on $\frac{1}{2}(\hbar/2mc)^2\epsilon_0$, which is just the ordinary Darwin term, a contribution depending on $(\hbar/2mc)\mu_0$ and therefore arising from the anomalous magnetic moment, and an "intrinsic" part depending on ϵ_1. To have a name for this last quantity we shall call it the "intrinsic" Darwin coefficient for the particle. In the case of the neutron, it determines the "intrinsic" electron-neutron interaction, that is, the portion of this interaction not accounted for by the anomalous moment of the neutron.

(d) Further terms in the series have a character similar to the higher order terms described briefly in our earlier example. It will be noted that every coefficient ϵ_n or μ_n in the Dirac representation of the equation appears not only in the coefficient of the corresponding term in the Foldy-Wouthuysen representation but in the coefficients of all succeeding terms.

It is thus apparent that the same electromagnetic interaction which is described by the set of coefficients $\epsilon_0, \mu_0, \epsilon_1, \cdots$ in the Dirac representation is described by the alternate set of coefficients ϵ_0, $\mu_0+(\hbar/2mc)\epsilon_1$, $\epsilon_1+(\hbar/2mc)\mu_0+\frac{1}{2}(\hbar/2mc)^2\epsilon_0$, $\cdots$ in the Foldy-Wouthuysen representation. Although the latter coefficients have the more direct physical interpretation, it is presumed that the coefficients in the Dirac representation should be regarded as having the more fundamental significance. The important point, however, is that it is necessary to specify the representation of the Dirac equation to which one is referring in order to describe unambiguously the electromagnetic properties of a Dirac particle.

DISCUSSION

The development presented above may be considered to be a phenomenological description of certain electromagnetic properties of Dirac particles. In this sense it represents a framework into which may be fitted either empirical or theoretical values of the characterizing coefficients. Of course there is no binding reason why values of the coefficients determined empirically cannot be considered to be intrinsic properties of the particle which cannot be determined by any more fundamental theory, but this would represent a very unsatisfying conclusion. It is the general hope that the values are actually determined through the interactions of the particle with quantized fields; that the latter is very probably the true state of affairs is strongly suggested by the recent developments in the quantum electrodynamics of the electron. In spite of the fact that the rather unsatisfactory device of infinite renormalizations must be employed in this theory, one cannot ignore the fact that all observed electromagnetic properties of the electron find a correct quantitative prediction in terms of only two fundamental parameters, the observed charge and mass of this particle.

In view of the fact that we know at present more concerning the electromagnetic properties of the electron than any other particle, it is of interest to see if this knowledge can be fitted into the framework that we have constructed. Unfortunately, we encounter a rather unexpected difficulty. Examining the results of Schwinger,[8] for example, we find that there is no difficulty in identifying ϵ_0 immediately with the observed electric charge of the electron, $-|e|$, and further identifying the anomalous magnetic moment of the electron as

$$\mu_0 = -\frac{1}{2\pi}\frac{e^2}{\hbar c}\frac{|e|\hbar}{2mc} + \cdots$$

(where the dots represent further contributions of higher order in e). However, we encounter a problem with the intrinsic Darwin coefficient, for there is no finite quantity that can be identified with this coefficient. The quantity that would play this role in the expressions given by Schwinger is represented by a divergent integral. The "infrared catastrophe" involved in this result is known, however, to be canceled by an associated "infrared catastrophe" arising from the radiation of real photons by the electron in so far as the calculation of any result of physical significance is concerned. Why is our theory inadequate to deal with this problem? It is not difficult to find the answer when we examine the premises on which our description is derived. We have assumed that there exists some approximation in which the behavior of the particle can be described by our original Dirac equation, or at least that the elastic scattering part of the S-matrix may be computed from this equation. However, for an electron interacting with the quantized electromagnetic field whose quanta have zero rest mass, there exists no such elastic scattering part of the S-matrix. If the electron is scattered, however weakly, by an external electromagnetic field, it will of necessity radiate some low energy photons. This difficulty will therefore be present for any particle as well as for the electron when interaction with the quantized electromagnetic field is taken into account. We need not be too concerned over this failure of our formulation of the electromagnetic properties of Dirac particles, since the origin of the failure is well understood, and we have available in quantum electrodynamics the machinery for correcting this defect. In fact, an approximate repair of the theory may be made regarding a scattering in which the electron loses no more than some small part of its energy by radiation as an "elastic" scattering and then replacing the divergent integral which represents the intrinsic Darwin term by a finite integral by an appropriate modification of the lower limit in the original integral. On the other hand, we believe that we must not regard this inadequacy of our theory too lightly since it illustrates sufficient weakness in the foundation on which it rests to seriously limit its applicability.

We may expect that our theory may be more successful when coping with cases where the electromagnetic properties of the Dirac particle are determined principally by its interaction with a quantized field whose quanta have nonvanishing rest mass. Such a case is exemplified in the modification of the electromagnetic properties of nucleons through their interaction with a meson field. Since this problem is discussed in the following paper, we do not enter into details here but merely mention that recent calculations bear out this expectation, although calculated values are at great variance with the observed properties of nucleons. The important point, however, is that there is no difficulty in identifying in these calculations the characteristic coefficients which we have postulated in our formalism. This is sufficient to show that the division of the electron-neutron interaction into two contributions—one arising from the anomalous magnetic moment of the neutron and the other from an intrinsic Darwin coefficient—is justifiable in meson theory even though both of these contributions have a common origin.

Another point in regard to our description of the electromagnetic properties of Dirac particles worth some discussion is the question of the convergence of the series of interaction terms we have introduced into the Dirac equation. The convergence of the series depends, of course, on the nature of the external electromagnetic field and in particular on how rapidly it varies in space and time. One would expect that this series would be convergent for sufficiently slowly varying electromagnetic fields but that the convergence would cease when the fields varied appreciably over the charge and current distribution associated with the particle whose moments determine the values of our coefficients. It is reasonable to assume, however, that the values of the coefficients we have given may still determine the interaction with rapidly varying fields in the following manner: We consider the electromagnetic field to be expanded in a Fourier integral in space and time and deal with one Fourier component. The series of interaction terms then has the form of a power series expansion in the wave vector and frequency of this Fourier component and presumably this series will converge for sufficiently small values of these quantities. For larger values of the wave vector and frequency we may analytically continue the function represented by the series beyond the radius of convergence and assume that this represents the interaction for higher values of the wave vector and frequency. If the analytic continuation process can be extended over the whole complex plane for these variables, then the series of initial coefficients will characterize the interaction with electromagnetic fields of arbitrarily rapid variation.

We should mention also that if the coefficients ϵ_n and μ_n actually represent moments of the charge and

[8] J. Schwinger, Phys. Rev. **76**, 790 (1949).

current distribution associated with the particle, then these coefficients themselves can be finite only if this charge and current density ultimately fall off faster with distance from the particle than any inverse power of this distance. When this is not the case our representation obviously fails, for some coefficients in the series will then be infinite.

Our primary purpose in setting up the characterization of the electromagnetic properties of Dirac particles presented above has been to provide a framework for the interpretation of the experimental results on the electron-neutron interaction as given in the following paper. However, we regard our results as quite tentative and have emphasized the shortcomings of our characterization to indicate how urgently a more satisfactory characterization is needed.

APPENDIX

We consider the problem of constructing all possible Lorentz scalars formed from the Dirac matrices γ_μ and the four-vector A_μ of the electromagnetic potentials, and its derivatives, which are linear in A_μ. We employ a Lorentz gauge so that A_μ satisfies the equation

$$\partial A_\mu/\partial x_\mu=0. \tag{A-1}$$

We proceed by examining in succession invariants of the form

$$\gamma_\tau\cdots\gamma_\sigma\gamma_\nu\gamma_\mu\partial^n A_\mu/\partial x_\tau\cdots\partial x_\sigma\partial x_\nu.$$

The first of these containing one γ is $\gamma_\mu A_\mu$. The next containing two γ's is

$$\gamma_\nu\gamma_\mu\partial A_\mu/\partial x_\nu,$$

which we may transform by the use of the commutation relations of the γ's,

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2\delta_{\mu\nu},$$

in the following way:

$$\begin{aligned}\gamma_\nu\gamma_\mu\partial A_\mu\partial x_\nu&=\tfrac{1}{2}(\gamma_\nu\gamma_\mu-\gamma_\mu\gamma_\nu+2\delta_{\mu\nu})\partial A_\mu/\partial x_\nu\\&=\tfrac{1}{2}\gamma_\nu\gamma_\mu(\partial A_\mu/\partial x_\nu-\partial A_\nu/\partial x_\mu).\end{aligned}$$

With three γ's we may form and reduce the invariant

$$\begin{aligned}\gamma_\sigma\gamma_\nu\gamma_\mu\partial^2 A_\mu/\partial x_\sigma\partial x_\nu&\\&=\tfrac{1}{2}[\gamma_\sigma\gamma_\nu-\gamma_\nu\gamma_\sigma+2\delta_{\nu\sigma}]\gamma_\mu\partial^2 A_\mu/\partial x_\sigma\partial x_\nu\\&=\gamma_\mu\partial^2 A_\mu/\partial x_\nu\partial x_\nu=\gamma_\mu\Box A_\mu.\end{aligned}$$

By continuation of this process one readily finds that the most general invariants which may be formed belong to one of the two classes:

$$\gamma_\mu\Box^n A_\mu,\quad \gamma_\nu\gamma_\mu\Box^n(\partial A_\mu/\partial x_\nu-\partial A_\nu/\partial x_\mu),$$

where n is any non-negative integer.

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The Electron-Neutron Interaction*

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The known electromagnetic properties of nucleons, assuming that the electron-neutron interaction is fundamentally of electromagnetic origin, are fitted into the phenomenological framework developed in the preceding paper and the results compared with predictions of weak coupling meson theories. The detailed comparison shows that the intrinsic electron-neutron interaction is somewhat smaller than predicted and it is suggested that even in the more favorable cases, the rough agreement as to order of magnitude may be largely due to a fortuitious cancellation of different contributions, which may easily be upset when higher order effects are included in the theory. Even apart from the detailed calculations, it is indicated that the observed intrinsic electron-neutron interaction is considerably smaller than order-of-magnitude expectations from general meson-theoretical principles. The results emphasize the importance of more accurate experimental determinations of the electron-neutron interaction, since a smaller value of the intrinsic interaction will either pose a very stringent test for any meson theory or require a critical re-evaluation of our present ideas regarding nucleonic structure. Some phenomena related to the electron-neutron interaction and the possibility that the intrinsic interaction may be nonelectromagnetic in origin are briefly discussed.

RECENT measurements of the magnitude of the electron-neutron interaction by Hughes[1] and by Hamermesh, Ringo, and Wattenberg,[2] when combined with previous measurements by Fermi and Marshall,[3] and by Havens, Rabi, and Rainwater,[4] now yield an experimental value for this quantity with an accuracy of the order of ten percent. While there appear prospects for a considerably more accurate determination of this interaction in the near future, it appears appropriate, nevertheless, to examine the available results in the

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[1] D. J. Hughes, New York meeting of the American Physical Society [Phys. Rev. **86**, 606 (1952)].

[2] Hamermesh, Ringo, and Wattenberg, Phys. Rev. **85**, 483 (1952).

[3] E. Fermi and L. Marshall, Phys. Rev. **72**, 1139 (1947).

[4] Havens, Rainwater, and Rabi, Phys. Rev. **82**, 345 (1951).