

bution to the elastic constants. One can in fact make an approximate estimate for this case by regarding the interstitial ion as a rigid and incompressible sphere of radius  $\delta/2$  and volume  $2\pi/3$  of the atomic volume. If 1 percent of interstitial ions is represented by  $2\pi/3$  percent by volume of incompressible spheres inserted in an isotropic elastic continuum, a calculation by Bruggeman<sup>2</sup> shows that the bulk modulus is increased by  $2\pi(3K+4G)/9K \approx 3.8$  percent. On a similar approximation, 1 percent of vacant lattice sites, representing 1 percent by volume of spherical holes, reduces the bulk modulus by  $(3K+4G)/4G \approx 2.3$  percent. Here  $K$  is the bulk modulus and  $G$  the rigidity of the matrix, and the numerical values are taken for Poisson's ratio  $1/3$ ,  $K=5G/3$ .

<sup>1</sup>G. J. Dienes, Phys. Rev. **86**, 228 (1952).

<sup>2</sup>D. A. G. Bruggeman, Ann. Physik **29**, 160 (1937).

### Effect of Radiation on Elastic Constants\*

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THE writer pointed out in his original paper<sup>1</sup> that the major approximation of the theory comes from replacing the distorted lattice, which is locally inhomogeneous, by an approximately equivalent homogeneous material. No attempt was made to evaluate the influence of this inhomogeneity except in so far as an average was taken whereby the interactions ascribable to the vacancies and interstitials were smeared over the whole crystal. Nabarro<sup>2</sup> points out some of the weaknesses of this approximation and proposes an estimate based on an average derived from the theory of elasticity of an isotropic continuum. The physical argument given by Nabarro is certainly correct, and the question of the validity of the various approximations centers around the problem of properly averaging the elastic constants in an inhomogeneous medium. This problem in elasticity theory does not appear to be completely solved. For example, Bruggeman<sup>3</sup> and Mackenzie<sup>4</sup> are not in good quantitative agreement concerning the shear moduli of an isotropic material containing holes.

Within the framework of Bruggeman's theory the writer would like to make the following comments concerning Nabarro's calculations:

(1) The writer has underestimated in the original paper the decrease in elastic moduli caused by vacant lattice sites, i.e., the moduli are decreased by more than a bulk effect, as pointed out by Nabarro (2.3 percent *vs* 1 percent).

(2) The writer believes that Nabarro underestimates the influence of the interstitials by underestimating the effective volume over which the elastic moduli are greatly increased. Next nearest neighbors of an interstitial are strongly influenced by the presence of the interstitial particularly after relaxation has occurred, and it seems reasonable to consider the effective volume to have a radius of  $\sqrt{3}\delta/2$ . The corresponding volume percent is  $2\sqrt{3}\pi$  of the atomic volume, and the increase in bulk modulus is 19.6 percent for one percent interstitials. As a more conservative estimate it may be assumed that the effective volume is half-way between this and Nabarro's estimate. The corresponding increase in the bulk modulus is 11.1 percent. Thus, the writer's original averaging process, which gave about 9 percent, seems very reasonable. The somewhat higher contribution of the vacancies should be subtracted out, of course, but the main conclusions of the original paper are not changed.

(3) It seems worthwhile to point out that even on the basis of elasticity theory the difference between the effects of interstitials and vacancies is quite large in the case of the shear moduli. Using Nabarro's effective volume and Bruggeman's equations the calculations give these results: 1 percent interstitials raises the shear modulus by 6.3 percent and 1 percent vacancies decreases the shear modulus by 1.5 percent. The difference would be correspondingly larger if the larger effective volume of the interstitials were

used in the calculation. Thus, in the case of the shear moduli, the writer's original estimates are quite conservative and the conclusions remain unchanged.

The writer is grateful to Dr. Nabarro for calling attention to some of the difficulties of the original analysis and for pointing out that an independent estimate can be made based on elasticity theory.

\* Under contract with the AEC.

<sup>1</sup>G. J. Dienes, Phys. Rev. **86**, 228 (1952).

<sup>2</sup>F. R. N. Nabarro, preceding letter, Phys. Rev. **87**, 665 (1952).

<sup>3</sup>D. A. G. Bruggeman, Ann. Physik **29**, 160 (1937).

<sup>4</sup>J. K. Mackenzie, Proc. Phys. Soc. (London) **B63**, 1 (1950).

### The Response of Sodium Iodide Crystals to High Energy Protons\*

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FOR the purpose of analyzing the energy spectrum of high energy protons, a scintillation counter particle spectrometer has been constructed employing thallium activated sodium iodide crystals, as described in the accompanying letter.<sup>1</sup> To make such a device useful for energy determinations, it is necessary to know the relationship between the size of the light flash produced in the crystal and the energy of the particle striking it. Previous studies<sup>2,3</sup> have indicated that this relationship is approximately linear for

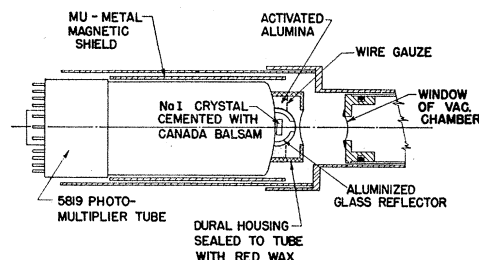


FIG. 1. Mounting of NaI crystal used for studying the response of NaI to high energy protons. Protons scattered by a platinum foil strike the crystal after passing through three aluminum windows and a short air path.

protons incident on sodium iodide. We have re-investigated this question with careful attention to the sources of error that may influence the result obtained.

The sodium iodide crystals employed in this experiment were cleaved from 200-gram ingots purchased from the Harshaw Chemical Company. The cleaving was accomplished in a large vacuum tight metal dry box equipped with airlocks. It was found that freshly cleaved surfaces of sodium iodide can be maintained indefinitely without deterioration in an atmosphere of nitrogen dried by circulation through activated alumina ( $Al_2O_3$ ).

The method of crystal mounting employed here is illustrated by Fig. 1. Two slots not shown on the diagram are provided for interposing aluminum foils between the exit window of the scattering chamber from which the protons originated and the aluminum foil cover of the dry box surrounding the crystal.

The energy of the external protons from the Princeton cyclotron was determined by measuring their range in aluminum. A differential detector consisting of a single layer of ZnS(Ag) crystals deposited on the face of a 5819 photomultiplier tube was used in this measurement. The average thickness of the crystals was estimated to be equivalent to 4.8 mg/cm<sup>2</sup> aluminum. The observed mean range of  $484 \pm 5$  mg/cm<sup>2</sup> deduced from an integral straggling curve can be interpreted in terms of Smith's range-energy curve or by considering the correction to this curve required by Hubbard and Mackenzie's recent experiment.<sup>6</sup> We have regarded the discrepancy between these two interpretations