APPENDIX. ANALYTIC EXPRESSIONS OF INTEGRALS USED IN THE TEXT

A straightforward integration gives the following expression for the normalization constant N:

$$N=\frac{1}{2}\left(\frac{E\sqrt{3}}{\pi}\right)^{\frac{3}{4}}\left(\frac{b^2+10b+1}{3b(b-1)^4}-\frac{2(b+1)\ln b}{(b-1)^5}\right)^{-\frac{1}{2}}.$$

The overlap integral $A(\mathbf{U})$ defined by (23) is equal to the following expression:

$$A(\mathbf{U})=(32\pi^3N^2/3\sqrt{3}E^3(b-1)^4)$$
$$\times\left[\frac{(b-1)^2}{12v}-\frac{(v+\frac{2}{3}(b-1))(b-1)^2\ln b}{8v^2}\right.$$
$$+\frac{v+4}{12}\left(1+\frac{4}{v}\right)^{\frac{1}{2}}\tanh^{-1}\left(1+\frac{4}{v}\right)^{-\frac{1}{2}}$$
$$+\left(\frac{v+4b}{12}-\frac{1}{2}(b-1)-\frac{b(b-1)^2}{v(v+4b)}\right)\left(1+\frac{4b}{v}\right)^{\frac{1}{2}}$$
$$\times\tanh^{-1}\left(1+\frac{4b}{v}\right)^{-\frac{1}{2}}+\frac{(2(b-1)^2+(b-5)v-v^2)}{12v^2}$$
$$\times(v^2+2(b+1)v+(b-1)^2)^{\frac{1}{2}}$$
$$\left.\times\tanh^{-1}\frac{(v^2+2(b+1)v+(b-1)^2)^{\frac{1}{2}}}{v+b+1}\right],$$

where $v=\mathbf{U}^2/E$.

Finally $B(\mathbf{P},\mathbf{Q})$ defined by (20) is equal to

$$B(\mathbf{P},\mathbf{Q})=(2NN_d\pi^2/E^2(b-1)^2)$$
$$\times\left[\frac{1}{Q}\left(\tan^{-1}\frac{(\eta-\epsilon)Q}{(\gamma+\eta)(\gamma+\epsilon)+Q^2}-\tan^{-1}\frac{(\eta-\epsilon)Q}{(\alpha+\eta)(\alpha+\epsilon)+Q^2}\right)\right.$$
$$\left.+\frac{(b-1)E}{2\eta}\left(\frac{1}{(\alpha+\eta)^2+Q^2}-\frac{1}{(\gamma+\eta)^2+Q^2}\right)\right],$$

where

$$\eta=(bE+\tfrac{3}{4}P^2)^{\frac{1}{2}},\quad \epsilon=(E+\tfrac{3}{4}P^2)^{\frac{1}{2}}.$$

In particular, when $Q=0$,

$$B(\mathbf{P},0)=(2NN_d\pi^2/E^2(b-1)^2)$$
$$\times[(\eta-\epsilon)((\gamma+\eta)^{-1}(\gamma+\epsilon)^{-1}-(\alpha+\eta)^{-1}(\alpha+\epsilon)^{-1})$$
$$+((b-1)E/2\eta)((\alpha+\eta)^{-2}-(\gamma+\eta)^{-2})].$$

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Internal Conversion of Gamma-Ray Transitions in the L-Subshells*†

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A number of internally converted γ-transitions have been investigated with high resolution β-spectrographs and the relative intensities of conversion electron lines from the three L-subshells obtained. Generalizations are made for L-conversion as related to multipole order. The magnetic transitions investigated are converted in the L_I- and L_{III}-shells, the ratio L_{III}/L_I increasing with increasing multipole order. Less can be said about electric transitions; but L-conversion takes place mainly in the L_{II}- and L_{III}-shells for the transitions studied.

INTRODUCTION

THE improved resolution with which internal conversion electron spectra are being examined and the eventual availability of accurate theoretical values for the L-subshell conversion coefficients make it of interest to determine what can be learned about γ-ray transitions from L-subshell internal conversion electrons. There are cases where a K/L ratio or K-shell internal conversion coefficient is not obtainable with sufficient accuracy or may not give a unique assignment of multipole order by comparison with the theoretical values.[1] Particularly, if the γ-transition is of too low an energy to convert in the K-shell, some other criterion would be needed.

In the medium and heavy elements it is usually not difficult to compare the intensities of the electron lines from the internal conversion of the L_I-, L_{II}-, and L_{III}-shells. In certain cases, even electrons from different M-subshells have been resolved by us. In the course of experiments, data have been obtained for a number of internally converted transitions which have been classified as to multipole order by the analysis of Goldhaber and Sunyar.[2] It has become apparent that the relative conversion in the various subshells indeed depends upon the multipolarity of the γ-transition.

Comparison may be made with certain available theoretical calculations, in addition to the K-shell values of Rose *et al.*[1] for energies above 150 kev. Gell-

* A preliminary report of this work was given at the Chicago American Physical Society meeting in October, 1951, J. W. Mihelich and E. L. Church, Phys. Rev. **85**, 733 (1952).

† Research carried out at Brookhaven National Laboratory under the auspices of the AEC.

[1] Rose, Goertzel, Harr, Spinrad, and Strong, Phys. Rev. **83**, 79 (1951).

[2] M. Goldhaber and A. W. Sunyar Phys. Rev. **83**, 906 (1951).

man, Griffith, and Stanley[3] have calculated conversion coefficients (neglecting screening) for the L_{I}-shell for $Z=92$, 84, and 49 for electric and magnetic dipoles and electric quadrupole. Reitz[4] has calculated K-shell conversion coefficients for the same multipole orders and same energy range ($k\sim0.1$ to $k\sim0.5$). It is convenient to compare these two sets of calculations. In order to utilize these data, we have made interpolations of [$\log\alpha$ (or $\log\beta$) *vs* Z]. On this semilog plot, the points do not fall far from a straight line for both K- and L_{I}-shells for the $M1$ and $E1$ cases. For the $E2$ case, the points fall on a sharply curving line and here the interpolated values are very approximate. Comparisons were made of the ratio of the K-conversion coefficient of Reitz divided by the L_{I}-conversion coefficient of Gellman *et al.* with the experimental K/L_{total}[2,5] ratio for various multipole orders. For magnetic dipole transitions, the agreement is good implying that the L_{I}-coefficients of Gellman *et al.* are reasonably good if there is not much conversion in the L_{II}- and L_{III}-shells. The latter point is in accord with experiment (see below).

For electric dipole transitions, the theoretical K/L_{I} ratio varies between 6 and 9 for atomic numbers between $Z=40$ and $Z=80$ and transition energies of $k=0.1$ to $k=0.5$. As yet, there is no well-established electric dipole transition with which to compare these ratios. Hulme[6] has made calculations for an electric dipole transition of 362 kev and finds that $K/L_{\text{tot}}=\sim7$ and the ratio of L_{I}:L_{II}:L_{III}-conversion is 1:0.0086:0.049. It is of interest to note that for the 47-kev transition in RaD, Cranberg[7] finds the L_{I}:L_{II}:L_{III}-ratio to be 1.0:0.09:0.019. However, Cranberg gives the L-conversion coefficient as 16 by comparing the number of L-electrons to the total number of disintegrations. By referring to the coefficients of Gellman *et al.* the values of the L_{I}-conversion coefficient for magnetic dipole and electric dipole are >10.0 and ~0.25, respectively. From these data, a magnetic dipole is definitely indicated.

For electric quadrupole transitions $K/L_{I\,\text{theor}}$ is much larger than the experimental K/L_{total}, requiring a large amount of conversion in the L_{II}- and/or L_{III}-shells to be consistent. This is in agreement with our experiments on L_{I}:L_{II}:L_{III}-ratios. In addition, Goodrich, in a private communication to Bowe and Axel,[8] finds that when the formulas of Hebb and Nelson[9] are separated to show the L_{I}-contribution explicitly, an agreement within a factor of two with the value of Gellman is obtained.

Tralli and Lowen[10] have published results of calculations estimating the ratio of conversion in the L_{III}- and L_{I}-shells for magnetic multipole orders one to five. L_{II}-conversion is said to be less than 5 percent of the other two. In short, their curves indicate that the ratio $\beta L_{III}/\beta L_{I}$[11] increases with Z^2/E and increasing multipole order. Our experiments are in accord with this.

Table I lists some comparisons between available theoretical data and experimental values for $E1$, $M1$, and $E2$ transitions.

TABLE I. Comparison of theoretical (K/L_{I}) ratios with experimental (K/L_{tot}) ratios.

Multipole order	Nucleus	E (kev)	(K/L_{I}) theor	(K/L_{tot}) exp
$E1$		none known	6–9	...
$M1$	Xe^{131}	80	6.7	7.0
	Tc^{99}	140	~8.0	7.3
	Te^{121}	213	6.5	7.3
	Te^{123}	159	7.7	7.6, 8.2
	Te^{125}	35.4	~7.5	7.3
	Hg^{199}	209	6.2	5.4
$E2$	Cd^{111}	247	~8	5.12
	Er^{166}	80.8	~5	0.10
	Yb^{170}	84.8	~4.5	0.14
	Ta^{181}	134	8.0	0.5
	Os^{186}	137	7.5	0.6
	Hg^{199}	159	7.5	0.6
	Pb^{204}	374	~3.2	2.0

EXPERIMENTAL DATA

Permanent magnet photographic spectrographs of 20-cm maximum radius of curvature using x-ray film as a detector were employed. An instrument of this type is ideal for an investigation of this sort, due to the high resolution available, the integrating action of the film, and the permanence of the magnetic field. Magnets of various field strengths were employed, depending on the energy of the conversion electrons. For electrons of <90 kev, a 54-gauss magnet was used, giving an energy dispersion of ~0.4 kev/mm. The magnets were calibrated against the conversion electron lines of I^{131}, Au^{198},[12] and W^{187}.[13] The group of L-lines lie close together on the film; hence their relative energies may be determined with considerably more accuracy than their absolute energies. The magnitudes of the differences of the L-shell energy levels may be seen by referring to Fig. 1, where the x-ray absorption edge energies in kev are plotted against Z. These values are obtained from Siegbahn's book[14] by converting ν/R values to kev using up to date values of physical constants.[15] One sees that in the rare earth region, the L_{I}–L_{II} and L_{II}–L_{III} separations are about the same; this makes the

[3] Gellman, Griffith, and Stanley, Phys. Rev. **80**, 866 (1950).
[4] J. R. Reitz, Phys. Rev. **77**, 10 (1950).
[5] *Nuclear Data*, National Bureau of Standards Circular No. 499 (1950).
[6] H. R. Hulme, Proc. Roy. Soc. (London) **A138**, 643 (1932).
[7] L. Cranberg, Phys. Rev. **77**, 155 (1950),
[8] J. C. Bowe and P. Axel, Phys. Rev. **84**, 939 (1951).
[9] M. Hebb and E. Nelson, Phys. Rev. **58**, 486 (1940).
[10] N. Tralli and I. S. Lowen, Phys. Rev. **76**, 1541 (1949).
[11] Following the custom of abbreviation for α_K/α_L, etc., ratios, we shall designate the ratio of conversion coefficients in two subshells as L_I/L_{III}, etc.
[12] Lind, Brown, Klein, Muller, and DuMond, Phys. Rev. **75**, 1544 (1949).
[13] J. W. M. DuMond, 16th Quarterly Report California Institute of Technology NP 3142 (1951).
[14] M. Siegbahn, *Spektroskopie der Roentgenstrahlen* (Julius Springer, Berlin, 1931).
[15] Hill, Church, and Mihelich, *A Table of Critical X-Ray Absorption Energies*, privately circulated.

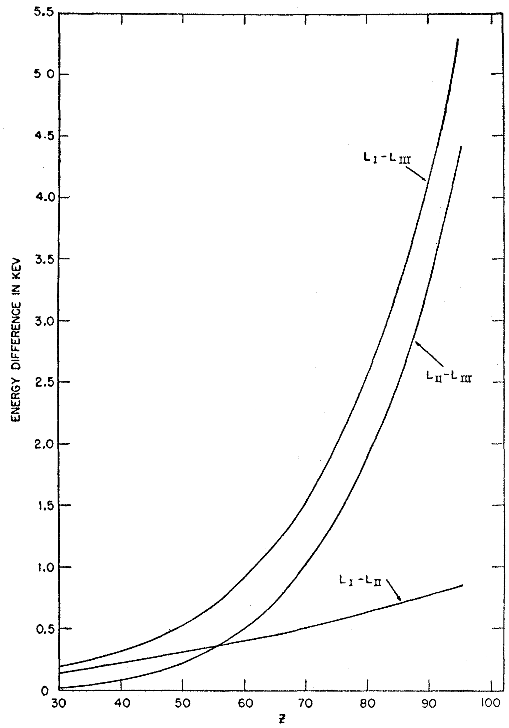

FIG. 1. Differences between L subshell x-ray critical absorption energies *vs* atomic number. The points on the curve correspond to the energy separation of conversion electrons from the designated shells.

assignment of a doublet L-line as $L_I L_{III}$ or $L_{II} L_{III}$ simple. In the region of high Z, where the L_I–L_{II} difference is small compared to L_{II}–L_{III} or L_I–L_{III}, there might be cases where a doublet could not uniquely be described as $L_I L_{III}$ or $L_{II} L_{III}$, if an instrument of marginal resolving power was being employed.

The position of the lines relative to one another can be determined to within ± 0.15 mm on a good photographic plate, which means an energy difference uncertainty (in a 54-gauss magnet) of the order of 0.06 kev. The consistency of the energy sums obtained shows that this accuracy is indeed obtained. In addition, the small difference in energy means that the response of the film is essentially linear over the L-electron group. In addition, source self-absorption effects are of the same magnitude for the different L-lines. Hence, photometric densitometry allows a measure of the electron line intensities. In certain cases, where Z is not high and the energy of the transition is large enough to require a higher field magnet, clear resolution of the lines is not obtained; however, a study of the line shape will often allow one to decide whether the L-line is complex and if so, whether or not it is an $L_I L_{III}$ or $L_{II} L_{III}$ doublet.

Sources generally were a thin layer of fine radioactive powder (activated in the Brookhaven reactor) on a Scotch tape strip about 15 mm$\times$0.4 mm. The sources had an average density of the order of 1.0 mg/cm^2.

The following is a discussion of the data obtained on the L-conversion of certain transitions of various multipole orders which are believed to be well established.

*M*1

For the several cases investigated (as well as those published by others), it is apparent that the L_I-conversion electron lines are far more intense than the other two. No L_{II}- or L_{III}-electrons were observed in any of these cases, and a conservative upper limit of L-conversion for L_{II} and L_{III} compared to L_I is 10 percent. Table II lists the data for the $M1$ (as well as the other multipole orders).‡

Caution is required in the assigning of an L-line where only one is visible, as the $K-L$ energy is large enough that errors due to small inaccuracies in field calibration and correction for film expansion may be sufficient to make a definite assignment as either L_I- or L_{II}-electrons uncertain.

*M*2

No L-conversion data have been obtained yet for this multipole transition. Of interest is the 150-kev transition in Lu^{177} following the 1.5-hr β^- activity of Yb^{177}. McGowan[16] has obtained a K/L ratio of 3 using a scintillation spectrometer; this datum along with the lifetime[16] of 1.3×10^{-7} makes the assignment of $M2$ likely. The activity obtained by thermal neutron ac-

TABLE II. Experimental intensities of L-conversion lines for various multipole order transitions.

Multipole order	Nucleus	E (kev)	Magnetic transitions L_I/L_{III}	Remarks
$M1$	Xe^{131}	80	large (>10)	L_{II} and L_{III} not observed
	Tc^{99}	140	large (>10)	L_{II} and L_{III} not observed
	Sn^{119}	24		L_{II} and L_{III} not observed
	Te^{123}	159	large (>10)	L_{II} and L_{III} not observed
$M3$	Br^{80m}	49	1.0	
$M4$	Te^{123m}	89	0.5	L_{II} if any, of low intensity
	Te^{125m}	109	~1 (estimated)	
	Sn^{117m}	155	~1	
Multipole order	**Nucleus**	**E (kev)**	**Electric transitions $L_I:L_{II}:L_{III}$**	**Remarks**
$E1$	See text			
$E2$	Er^{166}	80.8	$\gtrsim$0.1:0.7:1	L_I intensity estimated
	Yb^{170}	84.8	~0.1:0.8:1	L_I intensity estimated
	Hg^{198}	411	2.5:1	
	Hg^{199}	159	— :1.6:1	
$E3$	Dy^{165m}	109	— :1.5:1	Reinterpretation of Caldwell's data
	Cs^{134m}	127.6	— :1:1	Unresolved. Rough estimate of L_{II}/L_{III} ratio
$E4$	In^{114m}	191.6	...	Probably L_{II} and L_{III} but unresolved. Also, possibly weak L_I

‡ *Note added in proof:* Recent theoretical results of Gellman, Griffith, and Stanley (see reference 29) extended to the L_{II} and L_{III} shells obtain the following $L_I:L_{II}:L_{III}$ ratio for an $M1$ transition ($k=0.23$, $Z=84$): 1:0.081:0.0018. In addition, comparison may be made of K/L_{tot} ratios as obtained from the data of Reitz and Gellman *et al.* with the experimental values. For high atomic number, the theoretical K/L ratio for $M1$ transitions is about 4 or 5. Experimental data for the 286-kev transition in Tl^{203} indicate a K/L ratio of 4.8 for an $M1$ transition (see reference 2).

[16] F. K. McGowan, Oak Ridge National Laboratory Report No. 952 (1951), p. 104.

tivation is not high; attempts to obtain a measurable spectrum by inserting several fresh sources through a vacuum lock and integrating the exposure on a single photographic plate have so far been unsuccessful.

The number of examples in which the L-lines may be resolved is restricted by the energy resolution requirement that the transition energy not be too great compared to the shell excitation energies.

*M*3

A transition believed to be $M3$ is the 49-kev transition in the decay of Br^{80m}. A source was made from neutron irradiated potassium bromate by a Szilard-Chalmers separation and the conversion spectrum obtained.[17] The L-conversion is in the L_I- and L_{III}-shells, and the L_I/L_{III} ratio is 1.0 ± 0.25. The only other $M3$ transition for which L-lines have been resolved is that in the 19-sec Hf^{179m} (161 kev). Keller and Burson[18] report two L-lines which are either an L_IL_{III} or $L_{II}L_{III}$ pair.

*M*4

In agreement with Tralli and Lowen[10] who predict an increasing amount of L_{III}-conversion relative to L_I for increasing magnetic multipole order, $M4$ transitions are L-converted in the L_I- and L_{III}-shells. Several $M4$ spectra have been obtained with the L_I- and the L_{III}-lines resolved. In the case of the Te^{123} (88 kev) transition the L_{III}/L_I ratio is 2.0 ± 0.3; for this value of Z^2/E the Tralli and Lowen graph gives $\sim$1.8. Figure 2 is the graph of Tralli and Lowen with our experimental points on magnetic transitions. It is remarkable that the agreement is so good since the Pauli approximation they employed is known to be poor at the origin (nucleus).

*E*1

As stated previously, no experimental data on L-shell conversion are yet available for this multipole order.

*E*2

Electric quadrupole transitions are relatively common and considerable data have been obtained. As stated above, available theoretical estimates suggest that the L-conversion is mainly in the L_{II}- and L_{III}-shells. For several low energy ($\sim$85 kev) transitions in the rare earths ($Z\sim68$),[19] the ratio of the intensities of the L_{II}- and L_{III}-lines is about 0.8. The $E2$ 159-kev γ-ray in Hg^{199} has an L_{II}/L_{III} ratio of 1.6 while the 411-kev transition in Hg^{198} has an L_{II}/L_{III} ratio of 2.5.[20] If one considers the K/L_{total} ratios for these two transitions, along with their K-conversion coefficients and their L_{II}/L_{III} ratios one sees that the conversion coefficient of the L_{II}-shell decreases less rapidly with increasing transition energy than does that for the L_{III}-shell.

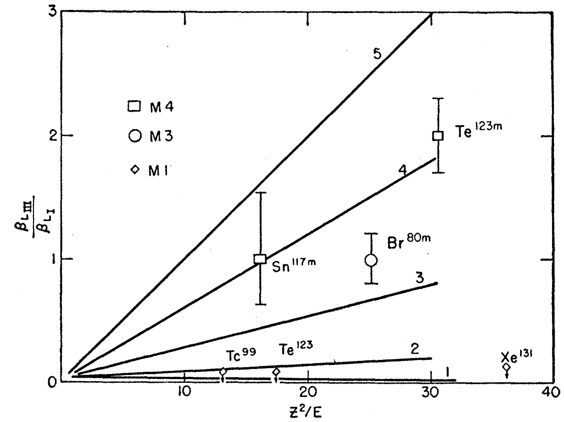

FIG. 2. The theoretical values of Tralli and Lowen for the ratio of conversion coefficients in the L_{III}- and L_I-shells for magnetic transitions of various multipole order. Their calculations were made in the Pauli approximation and are valid for low Z and low energy. Our experimental L_{III}/L_I ratios are shown for comparison.

Only an estimate is possible for the L_I-conversion. For the low energy transitions Er^{166} and Yb^{170}, a rough value of the intensity of the L_I-line as compared to the L_{II} is about or less than 0.1.[19]

It must be pointed out that these results are for $E2$ transitions occurring in high Z nuclei ($Z>65$).§

*E*3

Data on $E3$ transitions are meager, only isolated cases having been studied with sufficient resolution. One of the first was that of the 109-kev (1.25 min) isomeric transition in Dy^{165m} which was examined by Caldwell.[21] He was able to distinguish two L-lines which he labeled L_I and L_{III} with an intensity ratio of 1.5. However, if one adds x-ray absorption energies

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[17] Thanks are due Dr. Joan Welker of the Brookhaven Chemistry Department for performing the Szilard-Chalmers separation.

[18] H. Keller and S. B. Burson, Phys. Rev. **83**, 62 (1951).

[19] J. W. Mihelich and E. L. Church, Phys. Rev. **85**, 690 (1952); J. W. Mihelich, unpublished.

[20] R. D. Hill and J. W. Mihelich, Phys. Rev. **79**, 275 (1950). Here the L- lines of both transitions had been designated as L_I and L_{III}. However, the author has remeasured the 159-L conversion lines under better conditions and finds that they are L_{II} and L_{III}. The 411-L lines could not be sufficiently resolved. However, $L_{II}-L_{III}$ conversion electrons are assumed since this is a well established $E2$ transition and all evidence indicates $L_{II}-L_{III}$ conversion for this multipole order.

§ *Note added in proof:* The theoretical L ratio (see reference 29) for an $E1$ transition ($k=0.23$ and $Z=84$) is:

$$1:0.41:0.395.$$

For $E2$ transitions, when the energy is low ($k<0.25$) and atomic number large ($Z>60$), the theoretical values indicate that L_I conversion is less than 10 percent of L_{II} or L_{III} conversion. For low Z and large energy, L_I conversion becomes important. For $Z=49$ and $k=0.2$, the conversion coefficients for the three L-shells is about equal ($\sim$0.1). For $Z=49$ and $k=0.4$, the L ratio is:

$$1:0.4:0.45.$$

In Table I where comparison was made of the theoretical K/L_I ratio with the experimental K/L ratio, the values for Cd^{111} were $\sim$8 and 5.12, respectively. This was close agreement as compared with the case for $E2$ transitions in heavy elements. The results of Gellman *et al.* explain this behavior. The theoretical K/L ratio is $\sim$4.5.

[21] R. L. Caldwell, Phys. Rev. **78**, 407 (1950).

TABLE III. Estimate of L-subshell conversion coefficients for certain $E2$ transitions.

γ-transition	(K/L)	$\left(\frac{L_{II}}{L_{III}}\right)$	α_K	α_L total	α_{L_I}	$\alpha_{L_{II}}$	$\alpha_{L_{III}}$
84.8 kev (Yb^{170})	0.15[a,b]	0.8[b]	1.4[c,d]	9.3	~0.3	4.1	5.1
159 kev (Hg^{199})	0.8[a]	1.6[e]	0.03[f]	0.4	~0.04	0.246	0.154
411 kev (Hg^{198})	3.0[a]	2.5[e]	0.01[f]	0.01	~0.001	0.0072	0.0029

[a] See reference 5.
[b] See reference 19.
[c] See reference 3.
[d] See reference 4.
[e] See reference 20.
[f] See reference 1.

believed to be the best available at the present time,[15] the assignment L_{II} and L_{III} is preferable.

We have examined the (127.6±0.3 kev) $E3$ transition in Cs^{134m} and believe that the L-line is complex and, although complete resolution was not possible, that it is an $L_{II}L_{III}$ pair. The intensities of the two lines are of the same order of magnitude.

The 57.3-kev transition in Ir^{192m} (1.5 minutes) is believed to be either $E3$ or $M3$.[2] Caldwell[21] observed two L-conversion lines and reported them as L_I and L_{III}. However, an analysis of Caldwell's data, obtained with a thick source, shows that the energy difference of these two lines is not sufficiently well measured to definitely indicate whether the pair is L_I and L_{III} or L_{II} and L_{III}. The experimental energy difference is 1.9 kev; x-ray absorption energy differences for L_I–L_{III} and L_{II}–L_{III} are 2.2 and 1.6 kev, respectively. Goldhaber, Muehlhause, and Turkel[22] have shown by critical absorption the existence of L x-rays arising from the filling of holes in the L_{III}-shell. A clear-cut decision as to whether the lower energy L-line is L_I or L_{II} should indicate whether the transition is $M3$ or $E3$, respectively.

E4

One case has been examined, that of In^{114m},[23] which has been well established as an $E4$. The low atomic number and relatively high energy (191.5±0.5 kev) make a clear-cut decision difficult. It is believed that L-conversion takes place mainly in the L_{II}- and L_{III}-shells.

The internal conversion spectrum consists of three lines: K, L_{III}, and M_V; the experimental energy difference between the K- and L-lines is 24.33 kev. The energy difference expected if the L-line were L_{II} or L_{III} would be 24.00 or 24.20 kev, respectively. The L_{III} assignment is favored.

M1+E2 Mixtures

As pointed out by Goldhaber and Sunyar,[2] the only mixtures expected are those of $E2$ and $M1$. We have investigated certain transitions including the 70- and 104-kev γ-transitions in Eu^{153} following the β-decay of Sm^{153} (47 hr). Here the K/L ratios which we obtain ($K/L_{70}=3.5^{+2.5}_{-1.0}$; $K/L_{104}=6.5\pm1.0$) are lower than would be expected for magnetic dipoles. Lifetime considerations rule out $M2$ or $E3$ multipole orders. The L-electron spectra consist of L_I, L_{II}, L_{III}-lines in the intensity ratios 1, ~0.1, ~0.1, respectively. The presence of the L_{II}- and L_{III}-electrons to this amount may indicate an admixture of $E2$.‖ It is possible that accurate determinations of the L-line intensities may be of aid in deciding proportions of such mixtures. This could be another criterion for studying these interesting mixtures, in conjunction with K/L ratios, K-conversion coefficients, and angular correlation data, all of which are more or less sensitive to the degree of mixture.

DISCUSSION

Admittedly, the data presented are not complete in the disclosure of trends with energy and atomic number. What has been shown is the fact that here is a means of identifying or confirming multipole assignments.

In general, it appears that for magnetic multipoles, conversion takes place in L_I- and L_{III}-shells with the L_I/L_{III} ratio decreasing with increasing multipole order. For electric transitions where $Z>65$, the L_{II}- and L_{III}-shells are preferred. Not enough data is available to predict any variation of the L_{II}/L_{III} ratio with multipole order. For the two $E2$ transitions in Hg, it was pointed out that the L_{II}/L_{III} ratio decreased with decreasing energy.

If one takes the K/L_{total} ratios, the theoretical K-conversion coefficients, and our L-ratios, an estimate may be made of the absolute conversion coefficient of the L-subshells for certain multipole orders. For electric quadrupole transitions, values may be obtained for the L_{II}- and L_{III}-shells, and comparison made with the L_I-shell values of Gellman *et al.* These data for transitions of various energies are shown in Table III. These L_I-shell values are very approximate due to the difficulty of interpolation between Z values. However, the relative magnitudes of the coefficients of the three shells is consistent with the experimental data.¶

The two transitions in Hg are of particular interest since they are of quite different energy but occur in nuclei of the same atomic number. For these two points

TABLE IV. Estimates of L-subshell conversion coefficients for $M3$ and $M4$ transitions. L_{II}-conversion is assumed to be negligible.

Multipole order	γ transition	(K/L)	$\left(\frac{L_I}{L_{III}}\right)$	β_K	β_{total}	β_{L_I}	$\beta_{L_{III}}$
$M3$	49 kev (Br^{80m})	5.3[a]	1.0	100[b]	~20	~10	~10
$M4$	88.5 (Te^{123m})	0.68[a]	0.5	620[b]	910	303	607

[a] See reference 5.
[b] See reference 2.

[22] Goldhaber, Muehlhause, and Turkel, Phys. Rev. **71**, 372 (1947).
[23] R. M. Steffen, Phys. Rev. **83**, 166 (1951).

‖ *Note added in proof:*—The possibility of these transitions being $E1$ may not be definitely excluded.

¶ *Note added in proof:*—Interpolated values taken from the results of Gellman *et al.* (see reference 29) for $\alpha^2_{L_{II}}$ and $\alpha^2_{L_{III}}$, respectively, are: (84.8 kev) 1.35 and 1.35; (159 kev) 0.36 and 0.25. The experimental values appear to be larger than the theoretical coefficients for which screening had not been taken into account.

it would appear that conversion coefficients for all three L-shells fall off with increasing energy. One should expect this behavior to be rather regular.

For magnetic transitions, $M3$ and $M4$, estimates may be made for the coefficient for the L_I- and L_{III}-shells (Table IV). For $M1$ transitions, at least for the energies and atomic number of nuclei investigated, L-conversion is predominantly L_I.

Obviously, when a nucleus undergoes a transition of a given multipolarity, the probability of a given L-orbital electron being ejected is strongly dependent upon the l and j values of the electron since the two L_I-electrons are $s_{\frac{1}{2}}$, the two in the L_{II}-shell are $p_{\frac{1}{2}}$, and the four in the L_{III}-shell are $p_{\frac{3}{2}}$ electrons.

It is of interest to note that in the case of Os^{191} (15-day) there is a 42-kev transition (probably $E2$) which, of course, does not convert in the K-shell. The L-conversion occurs in the L_{II}- and L_{III}-shells, the ratio L_{II}/L_{III} being ~ 0.8. In addition, the following M-subshell electrons are resolved: M_{II}, M_{III}, and M_{IV} and/or M_V. The intensity ratio for these three lines is 0.8:1.0:0.2. The first three M-subshells have the same "l" and "j" values as the three L-shells, while M_{IV} and M_V are $d_{\frac{3}{2}}$ and $d_{\frac{5}{2}}$ electrons.

It is possible to assign multipolarity of transition from data on x-ray emission spectra of radioisotopes, particularly in the very heavy elements, since if a detector of sufficient resolution is employed, the relative number of L_I, L_{II}, and L_{III} holes may be determined. Barton, Robinson, and Perlman[24] have studied the L x-ray spectra of certain transuranic radioisotopes. In the case of plutonium x-rays arising from L-shell vacancies due to internal conversion of a 43-kev transition[25] following the α-decay of Cm^{242}, they observe that transitions to the L_{II}-level are relatively twice as abundant for the internal conversion spectrum as for the x-ray spectrum to be expected from external electron bombardment, after normalization of the data for L_{III}-transitions. In addition, certain transitions involving the L_I-level seen in moderate abundance in the electron bombardment source are missing (<20 percent). One is not able to tell what the increase in L_{III}-transitions is for the internal conversion case. If L_{II}- and L_{III}-levels have been depleted by the internal conversion process, an electric γ-transition is indicated.

This conclusion is strengthened by the results of Freedman, Jaffey, and Wagner[26] on the β-decay of Np^{238}, which leads to excited states of the same Pu^{238}. In addition to γ-rays of 1.03 and 0.983 Mev, four conversion lines of low energy are seen. Table V lists their conversion line energies and intensities along with their[26] assignments which indicate three transitions of low energy. Applying extrapolated[15] x-ray absorption energies, the four lines fit well the assignment L_{II}, L_{III}, M, and N for a 43-kev transition in Pu. The existence of L_{II}- and L_{III}-conversion in the reported relative intensities and the existence of electron-x-ray coincidences[26] indicate an $E2$ transition. It would appear that the 43-kev state is reached both by β-decay of Np^{238} and α-decay of Cm^{242}. The regularity of 2^+ first excited states of even-even nuclei also is in line with the $E2$ assignment.[27]

TABLE V. Conversion electron lines of Pu^{238}. Energies and intensities are from Barton *et al.* (see reference 24).

Electron energy kev	Relative intensity	Previous assignment	Our assignment
20.8	71	$43-L$	$43.1-L_{II}$
24.7	37	$47-L$	$42.8-L_{III}$
37.4	27	$42-M$	$43.0-M_{II}$
41.9	6.0	$47-M$	$43.0-N_{III}$

One is tempted to postulate the multipolarity of the low energy isomeric transition in Am^{241m}. O'Kelley, Barton, Crane, and Perlman[28] have studied the L x-rays arising from internal conversion and note that radiation due to L_{III}-holes is five times as prevalent as that due to L_{II}-holes. In the Pu^{238} case, which we postulated as $E2$, the L_{III}/L_{II} ratio was about one-half. Lifetime-energy considerations indicate the isomeric transition in Am^{241m} should be $E3$ or $M3$. By comparing the L_{II}/L_{III} ratio to those of other $E3$ transitions (of considerably smaller atomic number and greater energy) and extrapolating the Tralli and Lowen graph to a very high value of Z^2/E, the $M3$ assignment is favored. For large Z^2/E, L_I-conversion should be small. Possibly for magnetic transitions, L_{II}-conversion becomes more probable for high atomic number. For $E3$, more L_{II}-conversion than is observed would be expected.

Some further remarks may be made. Interpretation of angular correlation data involving L-conversion electrons will require knowledge of the kind of electrons being observed, that is, whether they are from the $p_{\frac{1}{2}}$, $p_{\frac{3}{2}}$, or $s_{\frac{1}{2}}$ levels. In addition, the determination of γ-ray transition energies from internal conversion lines may be made more accurately if one knows which L-subshell absorption energy to add to the energy of the L-conversion line (or lines).

After this paper was completed, theoretical values for the L_{II}- and L_{III}-shell conversion coefficients, neglecting screening, were published by Gellman, Griffith, and Stanley.[29] Our experimental results are in agreement with their theoretical ones.

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24 Barton, Robinson, and Perlman, Phys. Rev. **81**, 208 (1951).
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26 Freedman, Jaffey, and Wagner, Phys. Rev. **79**, 410 (1950).
27 G. Scharff-Goldhaber, Phys. Rev. **87**, 218 (1952).
28 O'Kelley, Barton, Crane, and Perlman, Phys. Rev. **80**, 293 (1950).
29 Gellman, Griffith, and Stanley, Phys. Rev. **85**, 944 (1952).