fact, it would appear that the yields of the $(\gamma,p)$ and $(\gamma,pn)$ reactions are fairly constant over the range where the $(\gamma,n)$ yield increases appreciably. In these cases, it would appear that the increased Coulomb barrier causes a reduction in the yield just compensating the increased absorption cross section evidenced by the $(\gamma,n)$ yields. The $(\gamma,\alpha n)$ yields are also fairly constant in the mass range 23 to 92. The ratio of the $(\gamma,\alpha n)$ to the $(\gamma,n)$ yield for the same reacting nuclide given in the last column of Table III, shows a sharp decline at higher masses despite the fact that the calculated thresholds (see Table II) do not vary appreciably with increasing mass.

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# The Absorption of Slow $\pi^-$ Mesons by $He^3$

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A theoretical investigation of the absorption of $\pi^-$ mesons by $He^3$ has been carried out in the impulse approximation, using the operators of the weak coupling theory. A similar calculation of $\pi^-$ absorption in hydrogen and deuterium when compared with experiment fixes the constants in the theory. Consequently the branching ratios between the 6 competing absorption processes in $He^3$ can be predicted unambiguously. Comparison of these ratios and of the calculated $\gamma$-ray spectrum with experiment should provide a good test of the theory.

## I. INTRODUCTION

EXPERIMENTAL results[1,2] on the $\pi$-meson reactions with hydrogen and deuterium and their theoretical interpretation[3] have greatly extended our knowledge concerning $\pi$-mesons. It is now well established, in particular, that the charged $\pi$-meson possesses spin 0 and odd parity. We investigate theoretically in this paper the slow $\pi^-$ meson reactions in $He^3$ in an attempt to determine how much more can be learned about the meson-nucleon interaction and about the $He^3$ and $H^3$ nuclei. The method followed throughout is that of Tamor,[4] based on a "weak coupling" treatment of the meson-nucleon interaction and on a phenomenological description of the nuclear forces. The defects of the "weak coupling" approximation are well known; in many instances, however, it leads to results in reasonable agreement with experiment. For example, Tamor obtained a ratio 2.1 between the nonradiative and the $\gamma$-absorption of slow $\pi^-$ by deuterium on the basis of the $PS(PV)$ theory, in good agreement with the experimental result $2.4\pm0.4$. Therefore, the possibility exists that such "weak coupling" treatments are adequate to describe processes involving real mesons of moderate energy, in particular to describe the absorption processes of negative $\pi$-mesons by nuclei. Moreover, the treatment presented here may be viewed as an entirely phenomenological approach and many of the results would hold even if the "weak coupling" approximation were to break down completely; this point will be elaborated later on.

Three types of reactions involving slow mesons may take place in $He^3$:

| | | |
|---|---|---|
| (a) the pure absorption | $\pi^-+He^3\rightarrow 2n+p$ | $(p)$ |
| | $\pi^-+He^3\rightarrow n+d$ | $(d)$ |
| (b) the $\gamma$-absorption | $\pi^-+He^3\rightarrow 2n+p+\gamma$ | $(p\gamma)$ |
| | $\pi^-+He^3\rightarrow n+d+\gamma$ | $(d\gamma)$ |
| | $\pi^-+He^3\rightarrow H^3+\gamma$ | $(t\gamma)$ |
| (c) the $\pi^0$-absorption | $\pi^-+He^3\rightarrow H^3+\pi^0$. | $(t\pi^0)$ |

It can be shown that the absorption processes take place chiefly from the $K$ shell, or at least from an $s$ state of the mesic atom. Although the absorption from $p$ states is more favored than in the deuterium and hydrogen cases, it is still quite negligible.[5]

It is easy to see how the absorption in $He^3$ compares with the reactions in hydrogen and deuterium. Reactions $(t\gamma)$ and $(t\pi^0)$ are obviously the analogs of the hydrogen reactions:

$$\pi^-+p\rightarrow n+\gamma$$
$$\pi^-+p\rightarrow n+\pi^0.$$

[1] Panofsky, Aamodt, Hadley, and Phillips, Phys. Rev. **80**, 94 (1950); Aamodt, Hadley, and Panofsky, Phys. Rev. **80**, 282 (1950).

[2] Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. **81**, 652 (1951); Clark, Roberts, and Wilson, Phys. Rev. **83**, 649 (1951); Durbin, Loar, and Steinberger, Phys. Rev. **83**, 646 (1951).

[3] R. E. Marshak, Revs. Modern Phys. **23**, 137 (1951).

[4] S. Tamor, Phys. Rev. **82**, 598 (1951).

[5] The absorption rate from a $p$ state goes as $Z^6$ whereas the radiative transition rate from the $p$ to $s$ state goes as $Z^4$. Furthermore, the absorption is favored in $He^3$ since the $He^3$ nucleus contains a greater proportion of large momenta than the deuteron. But, contrary to the deuterium case, the operator $[\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\phi(0)]$ does not contribute to the $p$ absorption (same selection rule as references 8 and 16) so that the only contribution will come from the small recoil term $(\mu/M)(\boldsymbol{\sigma}\cdot\mathbf{P})[\mathbf{r}\cdot\boldsymbol{\nabla}\phi(0)]$ ($\phi(\mathbf{r})$ meson wave function in the $p$ state considered; $\boldsymbol{\sigma}$, $\mathbf{P}$, $\mathbf{r}$ spin, momentum, and position of the absorbing nucleus).

The energy released is 0.8 Mev larger in the $He^3$ case, leading to a larger available phase space for the outgoing quantum and to an increase in the transition rate, an effect which is more significant for the $\pi^0$ emission. On the other hand, since the proton, which absorbs the $\pi^-$ meson, is smeared around the center of the $He^3$ nucleus rather than concentrated at one point, the transition rates in $He^3$ are decreased by the presence of a form factor, an effect which is the more significant for the process involving the larger momentum transfer, namely, the $\gamma$-emission. Therefore, one expects the branching ratio $(w_{t\pi^0}/w_{t\gamma})$ to be larger than the corresponding ratio in hydrogen.

The pure absorption is very sensitive to the relative amount of high nucleon momenta in the $He^3$ nucleus. Since the $He^3$ nucleus contains a larger amount of high momenta than the deuteron, we expect the nonradiative absorption to be more dominant in the former case than in the latter case. The question then arises whether the pure absorption in $He^3$ has such a large probability that the other processes could not even be detected experimentally. Fortunately, the answer turns out to be in the negative.

Our calculations will confirm the above qualitative statements. Only the $PS(PV)$ theory will be considered; the $PS(PS)$ theory is not discussed because, as Brueckner[6] has pointed out, a phenomenological treatment of nuclear forces is not possible in the calculation of the nonradiative process.

Formally, each reaction is described by a matrix element

$$\mathfrak{M}=\langle\Psi_F|O|\Psi_0\rangle, \tag{1}$$

where $\Psi_F$ and $\Psi_0$ are the final and initial nuclear states. $O$ is an operator depending on the type of reaction considered; it has the form

$$O=\sum_i O^{(i)}\tau_-^i, \tag{2}$$

where $\tau_-^i$ transforms the $i$th nucleon from proton to neutron and $O^{(i)}$ is an operator acting on the dynamical variables of the $i$th nucleon only. Since the nucleons are treated nonrelativistically, it is necessary to find the nonrelativistic approximation to the operator $O$. The derivation of the specific form of $O$ is quite similar to Tamor's and will not be elaborated here. In the case of pure absorption, one gets

$$O^{(i)}=C_0(\boldsymbol{\sigma}^i\cdot\mathbf{p}^i), \tag{3}$$

where $\boldsymbol{\sigma}^i$ and $\mathbf{p}^i$ are the spin and momentum operators of the $i$th nucleon and where $C_0$ is related to the meson-nucleon coupling constant $f$, the meson rest mass $\mu$, and the value $\varphi(0)$ of the meson wave function at the origin by

$$C_0=if(2\pi/\mu)^{\frac{1}{2}}\varphi(0). \tag{4}$$

(We use throughout the units $h=c=$nuclear mass$=1$, unless otherwise stated.)

[6] K. K. Brueckner, Phys. Rev. 82, 598 (1951).

The $\pi^0$ absorption is treated as a second-order process where it is assumed that only one nucleon is involved in this charge-exchange reaction and that all binding effects can be neglected during the time of reaction ("impulse approximation"). This leads to

$$O_{\pi^0}{}^{(i)}=i(C_0/2)(f_N+f_P)(2\pi/\mu_0)^{\frac{1}{2}}\Delta_i(\mathbf{q}_0), \tag{5}$$

where $f_N$ and $f_P$ are the coupling constants of the $\pi^0$ to the neutron and the proton, respectively, $\mu_0$ and $\mathbf{q}_0$ the rest mass and momentum of the $\pi^0$, and $\Delta_i(\mathbf{q}_0)$ an operator determining the momentum transfer $(-\mathbf{q}_0)$ to the $i$th nucleon.

The $\gamma$-absorption is also treated in the impulse approximation. Up to order $(v/c)$, the only significant contribution comes from the triple interaction imposed by the gradient coupling in the $\pi$ meson-nucleon interaction. If the photon emitted possesses momentum $\mathbf{k}_\gamma$ and polarization $\boldsymbol{\varepsilon}$, one gets

$$O_\gamma{}^{(i)}=-iC_0e(2\pi/k_\gamma\mu^2)^{\frac{1}{2}}(\boldsymbol{\sigma}^i\cdot\boldsymbol{\varepsilon})\Delta_i(\mathbf{k}_\gamma). \tag{6}$$

## II. THE GROUND STATE OF $He^3$ AND $H^3$

The subsequent calculations imply the knowledge of the wave function of $He^3$ and, in some cases, of $H^3$. If one assumes the nuclear forces to be charge independent, the ground state wave functions of these two mirror nuclei ought to be identical (the Coulomb effect in $He^3$ is, at any rate, a small perturbation). Furthermore, since both nuclei possess spin $\frac{1}{2}$ and since their magnetic moments are very close to the magnetic moments of the proton and the neutron, respectively, it follows[7] that the wave function is chiefly $^2S$, completely symmetrical in the space coordinates. This fact is further supported[8] by the very small capture cross section of thermal neutrons by deuterium. An immediate consequence is that the wave function of $He^3$, say, is practically insensitive to the exchange character of the nuclear forces and to the nature of the triplet $p-p$ force.

In order to determine the $He^3$ wave function, it is necessary to know the nuclear potential and to solve the Schrödinger equation. Unfortunately, as is well known, the choice of a nuclear potential which fits the low energy two-nucleon data is not very unique. In fact, as long as the reactions considered involve small momentum transfers, the choice of a particular potential shape is of little importance. But, in the case of pure absorption of slow $\pi^-$ mesons, the relative amounts of high momenta in $He^3$ are of paramount importance (the average wavelength of the recoil nucleon is about half the range of nuclear forces) and potential shapes which differ widely for large momentum transfers may yield widely different transition rates. We have taken the Yukawa shape because it leads to reasonable agreement with the experimental results on high energy $n-p$ scattering,[9] in contradistinction to the "short-tail" potentials. The same shape is chosen for the

[7] F. Villars, Helv. Phys. Acta 20, 476 (1947).
[8] L. I. Schiff, Phys. Rev. 52, 242 (1937).
[9] R. S. Christian and E. W. Hart, Phys. Rev. 77, 441 (1950).

singlet $p-p$ force in conformity with the charge independence assumption for reasons of simplicity, although the high energy $p-p$ scattering experiments supply practically no information[10] concerning the singlet $p-p$ force.

We have then solved the eigenvalue problem by the variational method in momentum space. If $K$ is the kinetic energy of the nucleons in the center of mass system, $E$ the binding energy of tritium, $V=\lambda V'$ the nuclear potential ($V'$ is an integral operator, $\lambda$ a depth parameter), the wave function $\Psi$ satisfies the integral equation

$$(K+E)\Psi=\lambda V'\Psi, \tag{7}$$

where $\lambda$ is the eigenvalue parameter. Then the variational procedure consists in minimizing

$$\lambda_0=\langle\Psi_0|(K+E)|\Psi_0\rangle/\langle\Psi_0|V'|\Psi_0\rangle \tag{8}$$

with respect to the variational parameters entering the trial function $\Psi_0$. Svartholm has actually considered this problem in detail;[11] he obtains a further improvement of the variational result by applying Kellogg's method of iterated functions, which leads to a set of values $\lambda_0$, $\lambda_{\frac{1}{2}}$, $\lambda_1\cdots$ and a set of iterated functions $\Psi_0$, $\Psi_1$, $\cdots$ converging toward the exact eigenvalue $\lambda$ and the exact eigenfunction $\Psi$, respectively. In the case of the Yukawa potential, the potential depth turns out to be, according to Svartholm's estimate, 5 percent smaller than the depth necessary to fit all other low energy data, a sufficiently close fit to justify our neglect of noncentral forces. It is in fact possible to guess a better trial function than the one used by Svartholm and to derive by means of a pure variational procedure a wave function which is at the same time close enough to the exact eigenfunction and simple enough to make calculation easy; the deviation of the $\lambda_0$ to which it leads from the correct $\lambda$ estimated by Svartholm provides a test of how good an approximation to the exact eigenfunction this trial function is.

Before reporting our variational calculation, we give a survey of the notation employed in handling the three-nucleon system. Call $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ the position vectors of the three nucleons, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ their momenta. The motion of the center of mass is separated by introducing the new coordinates $\mathbf{X}$, $\mathbf{x}$, $\boldsymbol{\xi}$ and their conjugate momenta $\mathbf{P}$, $\mathbf{p}$, $\boldsymbol{\omega}$:

$$\begin{aligned} &\mathbf{X}=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3) && \mathbf{P}=\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3\\ &\mathbf{x}=\mathbf{r}_3-\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2) && \mathbf{p}=\tfrac{2}{3}\mathbf{p}_3-\tfrac{1}{3}(\mathbf{p}_1+\mathbf{p}_2)\\ &\boldsymbol{\xi}=\mathbf{r}_1-\mathbf{r}_2 && \boldsymbol{\omega}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_2). \end{aligned} \tag{9}$$

Then the kinetic energy in the center-of-mass system is

$$K=\tfrac{3}{4}p^2+\omega^2. \tag{10}$$

As for the spin, one gets one quartet (totally symmetrical under permutation of particles) and two doublets (one symmetric under permutation (12), the other antisymmetric). For example, designating by $\alpha_i$ the spin functions where the $i$th particle has spin down and the two others spin up, the 3 spin functions for $S_z=\frac{1}{2}$ are

$$\begin{aligned} &\text{if } S=\tfrac{3}{2} && \chi=(\sqrt{3}/3)(\alpha_1+\alpha_2+\alpha_3),\\ &\text{if } S=\tfrac{1}{2} && \chi'=(\sqrt{6}/6)(2\alpha_3-\alpha_1-\alpha_2)\\ & && \chi''=(\sqrt{2}/2)(\alpha_2-\alpha_1). \end{aligned} \tag{11}$$

The isotopic spin functions are treated in the same way. We denote by $\zeta$, $\zeta'$, $\zeta''$ the corresponding isotopic spin functions for $T_z=\frac{1}{2}$ (2 protons+1 neutron).

Then the $^2S$ space symmetrical wave function for $He^3$ has necessarily the form

$$\Psi_0=(\sqrt{2}/2)(\chi'\zeta''-\chi''\zeta')\Phi(\mathbf{p},\boldsymbol{\omega}), \tag{12}$$

where $\Phi$ has to be rotation and permutation invariant. The trial function chosen is[12]

$$\Phi=N(E+K)^{-1}(bE+K)^{-2}, \tag{13}$$

in which $b$ is the variational parameter and $N$ the normalization constant (see Appendix). The depth and range of the singlet and triplet Yukawa potentials are determined by the low energy data, using the theory of the effective range;[13] the following expression for the potential operator between nucleons 1 and 2 is obtained:

$$V_{12}=\lambda\frac{\kappa}{2\pi^2}[a+b(\boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2)+c(\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2)+d(\boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2)(\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2)]\frac{\delta(\mathbf{p}'-\mathbf{p})}{\kappa^2+(\boldsymbol{\omega}'-\boldsymbol{\omega})^2} \tag{14}$$

with

$$\begin{aligned} &1/\kappa=1.18\times10^{-13}\text{ cm}, && a+b-3c-3d=1.18,\\ &a-3b+c-3d=0.82, && \lambda=1.9_1. \end{aligned} \tag{14'}$$

All quantities, except $\lambda$, have been used with the above numerical values in our variational calculation. The calculation of $\lambda_0$ defined as a function of $b$ by Eq. (8) has been performed analytically and leads to an expression which involves elementary and elliptic functions; its sight will be spared the reader. Then $\lambda_0$ has been minimized numerically. For $b_{\min}=8$, one gets $\lambda_0=1.88$, to be compared with the depth values obtained by Svartholm:

$$\lambda_0=2.12,\quad \lambda_{\frac{1}{2}}=1.90,\quad \lambda_{\text{"correct"}}\simeq1.81.$$

The wave function defined by (12) and (13) with $b=8$ has been adopted in all subsequent calculations.

In the following we shall also need the deuteron wave function in momentum space. We shall use the

[10] R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1950). It should be noted also that the choice of a correct $n-p$ force is more important than the choice of the $p-p$ force since there are 2 $n-p$ bonds and only 1 $p-p$ bond in $He^3$.

[11] N. Svartholm, thesis, Lund (1945).

[12] The author wishes to thank Dr. E. Salpeter and Dr. F. Cole for an interesting discussion in connection with this choice.

[13] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949); J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950); H. A. Bethe, Phys. Rev. **76**, 38 (1949). The numbers hereafter correspond to a common "intrinsic range" $b_s=b_t=2.5\times10^{-13}$ cm and to the following "well depth parameters:" $S_s=0.93$, $S_t=1.36$.

Hulthen wave function

$$\Phi_d(\boldsymbol{\omega})=N_d[(\gamma^2+\omega^2)^{-1}-(\alpha^2+\omega^2)^{-1}], \quad (15)$$

with the normalization

$$N_d=[\alpha\gamma(\alpha+\gamma)/\pi^2(\alpha-\gamma)^2]^{\frac{1}{2}}.$$

Using the numerical values (14), one obtains in our units

$$\gamma=0.0485, \qquad \alpha=0.326.$$

## III. CALCULATION OF THE VARIOUS PROCESSES

Since $\Psi_0$ and $O$ [see Eqs. (12) and (2)] have the proper symmetry character, the exclusion principle is automatically taken into account without the necessity of antisymmetrizing the final wave function. Then it is easy to calculate the isotopic spin matrix elements and one is led to the general formula for the transition probabilities:

$$w=2\pi(1/dE)\sum|M|^2 \quad (16)$$

with

$$M=\langle\mathfrak{F}|[1-(23)]O^{(2)}|\Phi\chi'\rangle. \quad (17)$$

The spin and space function $\mathfrak{F}$ represents a final state where particle 1 is a proton and particles 2 and 3 are neutrons. The sum $\sum$ has to be carried out over the final states lying in the band of phase space allowed by the energy conservation law.

### A. The Pure Absorption

We calculate first the transition rate $w_p$ of reaction $(p)$. The operator $O^{(2)}$ to substitute in expression (17) is given by Eq. (3), and the final state is represented by plane waves. The energy shared by the three outgoing nucleons is 131 Mev (we take $E_{\mathrm{He}^3}=7.65$ Mev for the binding energy, 1.30 Mev for the neutron-proton mass difference, and $\mu=140$ Mev for the rest mass of the $\pi^-$ meson); since the density in phase space favors an equal sharing of this energy among the three nucleons, the relative momenta involved are rather high and the distortion due to nuclear interaction is small. Although it is difficult to estimate the error made by neglecting this distortion, the probability of transition thereby obtained should be, at any rate, of the correct order of magnitude. Then, in the momentum representation, the final wave function is merely $\mathfrak{F}=(2\pi)^3\delta(\mathbf{p}-\mathbf{p}_0)\times\delta(\boldsymbol{\omega}-\boldsymbol{\omega}_0)\chi_f$, where the momenta $\mathbf{p}_0$ and $\boldsymbol{\omega}_0$ are related to the kinetic energy $K_0$ by

$$K_0=\tfrac{3}{4}p_0^2+\omega_0^2,$$

and where $\chi_f$ is the final spin function. The (momentum) space integration involved in $M$ is trivial. It is convenient to express $[1-(23)]$ as a bilinear expression in the symmetrizing and antisymmetrizing operators acting on space only and on spin only. This permits one to perform the sums over spin by the standard spur method. One is left with a straightforward integration leading to

$$w_p=(40\pi^4/9\sqrt{3})|C_0|^2K_0^3\Phi^2(K_0) =1.6_2|C_0|^2. \quad (18)$$

In reaction $(d)$, the relative kinetic energy of the outgoing neutron with respect to the deuterium is 133.2 Mev. If we neglect the $(n, d)$ interaction, the final wave function assumes the form

$$\mathfrak{F}=(2\pi)^{\frac{3}{2}}\delta(\mathbf{p}-\mathbf{p}_0)\Phi_d(\boldsymbol{\omega})\chi_f,$$

where $\Phi_d(\boldsymbol{\omega})$ is the "momentum space" part of the deuteron wave function, $\mathbf{p}_0$ the momentum of the free neutron with respect to the c.m. of the deuteron, $\chi_f$ any spin function formed with a triplet in 1 and 2. The wave function $\mathfrak{F}$ corresponds to neutron 3 being free and neutron 2 being bound. The other alternative is taken into account by multiplying the final result by a factor 2. One finally obtains

$$w_d=16\pi^2|C_0|^2p_0^2B^2(\mathbf{p}_0, 0), \quad (19)$$

with

$$B(\mathbf{P}, \mathbf{Q})=\int\Phi(\mathbf{P}, \boldsymbol{\omega}-\mathbf{Q})\Phi_d(\boldsymbol{\omega})d\boldsymbol{\omega}. \quad (20)$$

Taking for $\Phi$ and $\Phi_d$ expressions (13) and (15), one can perform the analytic integration in $B$ (see Appendix) and one finds

$$w_d=0.6_2|C_0|^2. \quad (21)$$

### B. The Reactions Leading to Tritium

Considering first reaction $(t\pi^0)$, we use the operator $O_{\pi^0}{}^{(2)}$ from Eq. (5). Taking 5 Mev for the mass difference between the charged and neutral mesons[14] and 0.75 Mev for the difference in binding energy between $He^3$ and $H^3$, one obtains 4.3 Mev for the kinetic energy of the outgoing $\pi^0$ meson. Accordingly, the wavelength of the $\pi^0$ meson is large compared with the radius of the $He^3$ nucleus so that the momentum transfer $\mathbf{q}_0$ may be neglected and $O_{\pi^0}{}^{(2)}$ reduces to a constant times $\delta$-functions imposing conservation of momentum. Then the space part of the matrix element is just 1 on account of the identity of the $H^3$ and $He^3$ wave functions, and a straightforward calculation leads to the transition rate

$$w_{t\pi^0}=|C_0|^2(f_N+f_P)^2(q_0/2). \quad (22)$$

The operator $O_\gamma{}^{(2)}$ from which the transition rate $w_{t\gamma}$ for reaction $(t\gamma)$ can be calculated is given by Eq. (6). The outgoing $\gamma$-ray has 136 Mev, and the momentum transfer $k_{t\gamma}$ can no longer be neglected. The matrix element $M$ can be expressed with the help of the function

$$A(\mathbf{U})=\int\Phi(\mathbf{p}, \boldsymbol{\omega})\Phi(\mathbf{p}, \boldsymbol{\omega}+\mathbf{U})d\mathbf{p}d\boldsymbol{\omega}, \quad (23)$$

whose analytic expression is given in the Appendix. One gets

$$w_{t\gamma}=|C_0|^2(4e^2/\mu^2)(k_{t\gamma}/(1+\tfrac{1}{3}k_{t\gamma}))A^2(k_{t\gamma}\sqrt{3}/3). \quad (24)$$

[14] Recent experiments point to a somewhat greater value. This change practically does not affect Eq. (26) below; it leads to a slight shift toward higher energy of the $\gamma$-ray square pulse produced in the $\pi^0$ decay.

In our units,

$$k_{t\gamma}=0.145,\ A^2(k_{t\gamma}\sqrt{3}/3)=0.78_1,$$
$$w_{t\gamma}=0.14_2|C_0|^2. \quad (25)$$

and

Expressions (22) and (24) should be compared with the expression for the corresponding reactions in hydrogen;[15] one obtains

$$(w_{t\pi^0}/w_{t\gamma})=(w_{\pi^0}{}^{\mathrm{H}}/w_{\gamma}{}^{\mathrm{H}})(q_0/q_0{}^{\mathrm{H}})(k_{\gamma}{}^{\mathrm{H}}/k_{t\gamma})\times((1+\tfrac{1}{3}k_{t\gamma})/(1+k_{\gamma}{}^{\mathrm{H}}))A^{-2}(k_{t\gamma}\sqrt{3}/3) \quad (26)$$
$$=1.3(w_{\pi^0}{}^{\mathrm{H}}/w_{\gamma}{}^{\mathrm{H}}).$$

With the experimental value $1\pm0.2$ obtained in the hydrogen experiment, one deduces from (26) and (25)

$$w_{t\pi^0}=0.18_5|C_0|^2. \quad (27)$$

### C. The Other Reactions of $\gamma$-Absorption

Whereas reaction $(t\gamma)$ leads to a $\gamma$-line of 136 Mev, reactions $(d\gamma)$ and $(p\gamma)$ lead to a continuous spectrum extending from 0 to 130.2 Mev and from 0 to 128.1 Mev, respectively. In addition, the total $\gamma$-ray spectrum will include the "square pulse" 35.4 Mev wide from the $\pi^0$ meson decay.

It turns out that the direct production of $\gamma$-rays is practically confined to the emission of high energy photons; a situation quite similar was encountered in the radiative capture of $\pi^-$ mesons in deuterium. Firstly, the phase space factor slightly favors higher energy for the lighter outgoing particle. Secondly, and this is by far the main effect, the matrix elements leading to final states of the nuclear system with very low energy give the most important contribution due to the preponderance of low momenta in the wave function of $He^3$. This effect is greatly strengthened by the interaction between the nucleons in the final state, which results in the striking fact, demonstrated below, that reaction $(t\gamma)$ dominates over all the others.[16]

This strong dependence of the matrix element on the $\gamma$-ray energy enables one to calculate in a simple way the total transition probability $w_\gamma$ for $\gamma$-absorption. Call $w(\mathbf{u},\boldsymbol{\varepsilon})$ the probability for emission of a $\gamma$-ray with polarization $\boldsymbol{\varepsilon}$ in the direction $\mathbf{u}=\mathbf{k}_\gamma/k_\gamma$. By an obvious modification of formula (25),

$$w(\mathbf{u},\boldsymbol{\varepsilon})=2\pi\sum^{E_{\max}}|\langle\mathfrak{F}|[1-(23)]O_\gamma{}^{(2)}|\Phi\chi''\rangle|^2\rho, \quad (28)$$

where $\rho$ is the density of photon states,

$$\rho\equiv\rho(k_\gamma)=(2\pi)^{-3}k_\gamma{}^2(1+\tfrac{1}{3}k_\gamma)^{-1}. \quad (29)$$

The photon energy $k_\gamma$ is related to the energy $E$ of the nucleons by the conservation law,

$$E+\tfrac{1}{6}k_\gamma{}^2+k_\gamma=E_{\max}. \quad (30)$$

The summation in (28) is carried over all final states whose energy (kinetic plus binding) is compatible with relation (30). In this sum $\rho$ and $O_\gamma{}^{(2)}$ depend on $E$ through $k_\gamma$. However, since the matrix element is very much larger for large values of $k_\gamma$, little error is made by (1) substituting for $k_\gamma$ in $O_\gamma{}^{(2)}$ and $\rho$ a value $\bar{k}_\gamma$ close to its maximum; (2) extending the sum to nuclear states of any energy, even those forbidden by the conservation law. The sum is then performed easily by means of the closure property:

$$w(\mathbf{u},\boldsymbol{\varepsilon})=2\pi\rho(\bar{k}_\gamma)\langle\Phi\chi''|O_\gamma{}^{(2)\dagger}(\bar{k}_\gamma)\times[1-(23)]^2O_\gamma{}^{(2)}(\bar{k}_\gamma)|\Phi\chi''\rangle \quad (31)$$
$$=|C_0|^2(e^2/\mu^2)(\bar{k}_\gamma/2\pi(1+\tfrac{1}{3}\bar{k}_\gamma))(2-A(\bar{k}_\gamma)). \quad (32)$$

Finally, the total probability is obtained by summing over photon polarizations and directions of propagation:

$$w_\gamma=|C_0|^2(4e^2/\mu^2)(\bar{k}_\gamma/(1+\tfrac{1}{3}\bar{k}_\gamma))(2-A(\bar{k}_\gamma)). \quad (33)$$

In (33), $w_\gamma$ is not very sensitive to the value of $\bar{k}_\gamma$. Taking $\bar{k}_\gamma=0.138$, which corresponds to the end point of the continuous spectrum, one obtains

$$w_\gamma=0.22_3|C_0|^2. \quad (34)$$

Further information can be obtained as follows: We assume the nuclear forces to be central and charge independent; then, the final states can be classified according to their total isotopic spin $T$ and their total spin $S$. It is possible, in particular, to split the functional space into three mutually exclusive subspaces $(A)$, $(B)$, and $(C)$ as indicated in Table I: column 1 gives the values of $T$ and $S$ defining each subspace, column 2 gives equivalent definitions using the transformation properties of the wave function under permutation instead of the isotopic spin formalism, and column 3 lists the $\gamma$-absorption allowed in each case. The transition rates $w_A$, $w_B$, $w_C$ for $\gamma$-absorptions leading to final states belonging to subspaces $(A)$, $(B)$, and $(C)$, respectively, may be calculated by using the same method as for the total probability $w_\gamma$. The same approximations (1) and (2) are made; however, one restricts the sum to the final states belonging to the subspaces considered. Alternatively, one can sum over the

TABLE I. Classification of the final states of the nucleons in the case of gamma-absorption.

| | 1 | 2 | 3 |
|---|---|---|---|
| $(A)$ | $T=\frac{3}{2}$ | antisymmetric | $(p\gamma)$ |
| $(B)$ | $T=\frac{1}{2}$ $S=\frac{3}{2}$ | symmetric in spin, not antisymmetric in space | $(p\gamma)$ $(d\gamma)$ |
| $(C)$ | $T=\frac{1}{2}$ $S=\frac{1}{2}$ | all the rest | $(p\gamma)$ $(d\gamma)$ $(t\gamma)$ |

[15] R. E. Marshak and A. S. Wightman, Phys. Rev. **76**, 114 (1949).

[16] One must also note the occurrence of a selection rule which has the effect of cutting out the soft part of the $\gamma$-ray spectrum. For low photon momenta $k_\gamma$, the momentum transfer occurring in operator $O_\gamma{}^{(2)}$ may be neglected so that $O_\gamma{}^{(2)}$ does not contain operators acting on space any longer and all space integrals entering the matrix element vanish on account of either symmetry or orthogonality. This selection rule is the same as the selection rule which forbids the magnetic $(n,d)$ photocapture (see reference 8). If the $\pi$-meson were either scalar or pseudovector, the pure absorption would be forbidden by the operation of the same selection rule; this situation should be compared with the selection rules found for the tritium problem [see Messiah, Caianiello, and Basri, Phys. Rev. **83**, 652 (1951)], which are based on symmetry properties only.

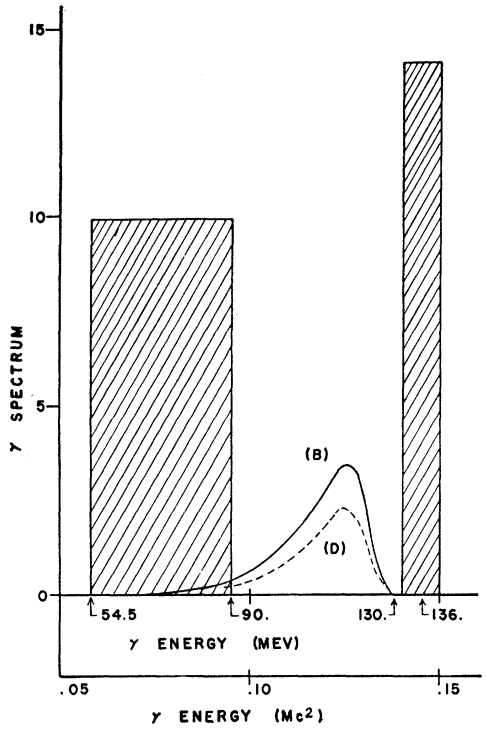


FIG. 1. The $\gamma$-ray spectrum. For convenience, the 136-Mev line has been replaced by a rectangular area proportional to its intensity. The solid curve ($B$) represents the estimated "background" from reactions ($p\gamma$) and ($d\gamma$). The dotted curve ($D$) represents the spectrum from the reaction ($d\gamma$) in the plane wave approximation.

complete set of final states (by means of the closure property) provided the proper projection operator has been inserted. The operator form of this projection operator is readily inferred from Table I, column 2. One finally obtains

$$w_A=|C_0|^2(4e^2/\mu^2)(\bar{k}_\gamma/(1+\tfrac{1}{3}\bar{k}_\gamma))\tfrac{1}{3}(1-A(\bar{k}_\gamma)), \quad (35a)$$

$$w_B=|C_0|^2(4e^2/\mu^2)(\bar{k}_\gamma/(1+\tfrac{1}{3}\bar{k}_\gamma))(4/3)(1-A(\bar{k}_\gamma)), \quad (35b)$$

$$w_C=|C_0|^2(4e^2/\mu^2)(\bar{k}_\gamma/(1+\tfrac{1}{3}\bar{k}_\gamma))\tfrac{1}{3}(1+2A(\bar{k}_\gamma)). \quad (35c)$$

Choosing $\bar{k}_\gamma=0.130$ for transitions ($A$) and ($B$) and $\bar{k}_\gamma=k_{t\gamma}=0.145$ for transition ($C$) (all choices justified by what follows), one gets

$$w_A=0.015|C_0|^2, \quad (36a)$$

$$w_B=0.060|C_0|^2, \quad (36b)$$

$$w_C=0.146|C_0|^2. \quad (36c)$$

Comparing the numerical values (36c) and (25) for $w_C$ and $w_{t\gamma}$, respectively, we obtain the important result that practically all transitions ($C$) lead to $H^3$. In the same way, one may infer that practically all transitions ($B$) are ($d\gamma$), a situation which will be made evident in the next paragraph. Therefore, the ratios $w_{t\gamma}:w_{d\gamma}:w_{p\gamma}$ go as $w_C:w_B:w_A$, that is, 9:4:1.

### D. The $\gamma$-Spectrum

The results of the last paragraph show that the $\gamma$-spectrum consists chiefly of the square pulse from $\pi^0$ decay and of the high energy $\gamma$-line from reaction ($t\gamma$). The continuous spectrum from reactions ($d\gamma$) and ($p\gamma$) appear as a rather small "background." An estimate of this "background" spectrum is given now.

Consider first reaction ($d\gamma$). Since its contribution to transitions ($C$) is negligible, we shall merely consider transitions ($B$), i.e., absorption reactions where the nucleons come off in a quartet ($n, d$) state; as a first orientation, we neglect the ($n, d$) interaction. Designating by $\mathbf{p}_0$ the momentum of the neutron relative to the deuteron, and by $u$ the cosine of the angle between $\mathbf{p}_0$ and $\mathbf{k}_\gamma$, the following formula obtains for the energy spectrum of the $\gamma$-rays:

$$P_q=|C_0|^2(64\pi/9)(e^2/\mu^2)k_\gamma p_0\int_{-1}^{+1}|B(\mathbf{p}_0-\tfrac{2}{3}\mathbf{k}_\gamma,0) -B(\mathbf{p}_0+\tfrac{1}{3}\mathbf{k}_\gamma,\tfrac{1}{2}\mathbf{k}_\gamma)|^2du. \quad (37)$$

The function $P_q$ *vs* $k_\gamma$ given by (37) has been plotted in Fig. 1 (broken curve $D$). It leads to a total probability for transition to quartet ($n, d$) states:

$$\int_0^{k_\gamma^{\max}}P_q dk_\gamma=0.05|C_0|^2.$$

One may expect the existence of an interaction between $n$ and $d$ to modify these results in two ways: (a) increase the total transition rate up to about its maximum value, and (b) shift slightly the maximum of curve ($D$) toward the upper end of the spectrum. In fact, the matrix elements for transitions to the most energetic photons must be small anyway because the $S$ part of the neutron wave function is suppressed by the operation of the Pauli principle (since the spin function is totally symmetric); therefore, for very slow neutrons, the neutron wave function is very small inside the $He^3$ nucleus, an effect which holds so long as the neutron wavelength is large compared with the radius of the deuterium and which consequently affects a band of about 4 Mev below the upper limit of the $\gamma$-ray spectrum.

As for reaction ($p\gamma$), it goes chiefly through transitions ($A$), leading to a continuous spectrum whose shape should not differ very much from the shape obtained in reaction ($d\gamma$), whereas its intensity is 4 times less. An estimate of the total "background" spectrum is shown on Fig. 1 [solid curve ($B$)] together with the square pulse from $\pi^0$ decay and the high energy $\gamma$-line from reaction ($t\gamma$).

## IV. DISCUSSION OF RESULTS AND CONCLUSION

In the framework of the "weak coupling" theory, the correctness of our calculation depends on whether (a) one can neglect higher order relativistic effects in

the motion of the nucleons, (b) the use of the "impulse approximation" is justified, (c) the nuclear interaction has been correctly taken into account.

By estimating the nuclear velocities involved, one obtains an order of magnitude for the relativistic corrections of 10 percent in the case of pure absorption and of 4 percent in the case of $\gamma$ and $\pi^0$ absorption.

The "impulse approximation" permits one to relate the radiative processes ($\gamma$ or $\pi^0$) to the corresponding radiative absorptions of $\pi^-$ mesons by free protons. Although its criteria of validity are not quite clear, the impulse approximation involves the average internuclear distance in $He^3$ ($2.25\times10^{-13}$ cm)[17] and is expected to hold if the average internuclear distance is greater than $c$ times the time $t$ during which the reaction takes place.[18] Taking $t=\hbar/\Delta E$ where $\Delta E$ is the energy denominator entering the second-order perturbation, one gets $ct\simeq10^{-14}$ cm for the two-step processes which go through intermediate states of negative energy; it turns out that, in all cases the chief contribution comes either from such two-step processes or from first-order processes (pure absorption, triple term in $\gamma$-absorption).

The fact that the wave functions that we have used to describe the nucleons in their initial and final states are approximate wave functions affects the probability of transitions that we have calculated in different ways. The transition probability $w_{t\pi^0}$ is practically unaffected. Also, little error can be made in the calculation of $w_\gamma$ since it involves the calculation of a form factor with a rather low momentum transfer and the same holds true for the calculation of $w_A$, $w_B$, $w_C$. On the other hand, because of considerable uncertainties in the nuclear potential for high momentum transfers, the result obtained for pure absorption must not be taken as giving more than an order of magnitude. At the same time, to take, as we did, plane waves to describe the final state undoubtedly constitutes a rather crude approximation, a fact well exhibited by our result that about $\frac{1}{3}$ of the pure absorption leads to deuterium. It is clear, in addition, that the relative amount of deuterium formation should depend on the correlation between the coordinates of the nucleons in $He^3$ and in deuterium; in this respect, by choosing a wave function for $He^3$ depending on $\mathbf{p}$ and $\boldsymbol{\omega}$ through the kinetic energy $K$ only, a choice made essentially for reason of simplicity, we have somehow fixed this correlation in a way which may be very different from the correlation given by the correct wave function.

Having made all of these qualifications, we summarize our results. About 80 percent of the slow $\pi^-$ absorption events in $He^3$ should be pure absorption events, $\frac{1}{3}$ of them leading to deuterons. The four other possible reactions take place in the following order of importance: $(t\pi^0)$, $(t\gamma)$, $(d\gamma)$, $(p\gamma)$. $(t\pi^0)$ is 1.3 times more probable than $(t\gamma)$ (this number would be changed if the $\pi^-$–$\pi^0$ mass difference and the branching ratio $(w_{\pi^0}/w_\gamma)_H$ are changed). The branching ratios $w_{t\gamma}:w_{d\gamma}:w_{p\gamma}$ between the various $\gamma$-absorption reactions go as 9:4:1.

So far, the results have been presented as predictions of "weak coupling theory," from which the operators $O$, $O_\gamma$, $O_{\pi^0}$ have been derived. One could as well consider these operators as phenomenological operators describing the three possible types of absorption. They are in fact the simplest among all the pseudoscalar operators which can be formed. Consider, for example, the $\gamma$-absorption: If we assume that the impulse approximation holds, $O_\gamma=\sum_i\tau_-{}^iO_\gamma{}^{(i)}$, and $O_\gamma{}^{(i)}$ depends on the dynamical variables of only one nucleon, namely, the $i$th nucleon which absorbs the meson. In complete generality this operator may assume three possible forms:

$$C_\gamma(\boldsymbol{\sigma}^i\cdot\boldsymbol{\varepsilon})\quad C_\gamma(\boldsymbol{\varepsilon}\cdot(\mathbf{p}^i\times\mathbf{k}_\gamma))\quad C_\gamma(\boldsymbol{\varepsilon}\cdot\mathbf{p}^i)(\boldsymbol{\sigma}^i\cdot\mathbf{k}_\gamma)$$

(the operator $\Delta_i(\mathbf{k}_\gamma)$ which insures momentum conservation has been omitted); the factors $C_\gamma$ are scalars depending on the nucleon and photon momenta $\mathbf{p}^i$, $\mathbf{k}_\gamma$. Linear combinations of the three forms could occur. The first form is the only one which leads to a nonvanishing probability for reaction $(t\gamma)$. The prediction that we obtained concerning the ratios $w_{t\gamma}:w_{d\gamma}:w_{p\gamma}$ follows from the particularly simple form assumed for $O_\gamma$, even if the scalar $C_\gamma$ is not related to the $\pi^-$ meson-nucleon coupling constant in the way shown in Eq. (6).

A similar statement can be made regarding our predictions for $(w_{t\pi^0}/w_{t\gamma})$. Even if the "orthodox" weak coupling theory is wrong, this prediction must be correct, so long as the $\gamma$-absorption and the $\pi^0$ absorption go through operators of the form $(\boldsymbol{\sigma}^i\cdot\boldsymbol{\varepsilon})$ and 1, respectively, whose ratio can be derived from the hydrogen experiment.

At present, it seems hard to reconcile on the basis of weak coupling theory the data on slow $\pi^-$ meson absorption in hydrogen and deuterium, on the one hand, and the other experimental results involving low energy mesons, on the other. In view of this fact, it is of interest to know if the slow $\pi^-$ meson absorption in (light) nuclei are capable of the common phenomenological interpretation suggested by weak coupling theory. Experiments with $He^3$, particularly the study of the outgoing $\gamma$-ray spectrum, would throw light on this matter.

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[17] This number has been calculated with the tritium wave function in configuration space obtained by R. L. Pease and H. Feshbach [Phys. Rev. 81, 143 (1951) and private communication of Dr. Pease].

[18] G. F. Chew and C. C. Wick, Phys. Rev. 85, 636 (1952).

## APPENDIX. ANALYTIC EXPRESSIONS OF INTEGRALS USED IN THE TEXT

A straightforward integration gives the following expression for the normalization constant $N$:

$$N=\frac{1}{2}\left(\frac{E\sqrt{3}}{\pi}\right)^{\frac{3}{2}}\left(\frac{b^2+10b+1}{3b(b-1)^4}-\frac{2(b+1)\ln b}{(b-1)^5}\right)^{-\frac{1}{2}}.$$

The overlap integral $A(\mathbf{U})$ defined by (23) is equal to the following expression:

$$A(\mathbf{U})=(32\pi^3N^2/3\sqrt{3}E^3(b-1)^4)$$
$$\times\left[\frac{(b-1)^2}{12v}-\frac{(v+\frac{2}{3}(b-1))(b-1)^2\ln b}{8v^2}\right.$$
$$+\frac{v+4}{12}\left(1+\frac{4}{v}\right)^{\frac{1}{2}}\tanh^{-1}\left(1+\frac{4}{v}\right)^{-\frac{1}{2}}$$
$$+\left(\frac{v+4b}{12}-\frac{1}{2}(b-1)-\frac{b(b-1)^2}{v(v+4b)}\right)\left(1+\frac{4b}{v}\right)^{\frac{1}{2}}$$
$$\times\tanh^{-1}\left(1+\frac{4b}{v}\right)^{-\frac{1}{2}}+\frac{(2(b-1)^2+(b-5)v-v^2)}{12v^2}$$
$$\times(v^2+2(b+1)v+(b-1)^2)^{\frac{1}{2}}$$
$$\left.\times\tanh^{-1}\frac{(v^2+2(b+1)v+(b-1)^2)^{\frac{1}{2}}}{v+b+1}\right],$$

where $v=\mathbf{U}^2/E$.

Finally $B(\mathbf{P},\mathbf{Q})$ defined by (20) is equal to

$$B(\mathbf{P},\mathbf{Q})=(2NN_d\pi^2/E^2(b-1)^2)$$
$$\times\left[\frac{1}{Q}\left(\tan^{-1}\frac{(\eta-\epsilon)Q}{(\gamma+\eta)(\gamma+\epsilon)+Q^2}-\tan^{-1}\frac{(\eta-\epsilon)Q}{(\alpha+\eta)(\alpha+\epsilon)+Q^2}\right)\right.$$
$$\left.+\frac{(b-1)E}{2\eta}\left(\frac{1}{(\alpha+\eta)^2+Q^2}-\frac{1}{(\gamma+\eta)^2+Q^2}\right)\right],$$

where

$$\eta=(bE+\tfrac{3}{4}P^2)^{\frac{1}{2}},\quad \epsilon=(E+\tfrac{3}{4}P^2)^{\frac{1}{2}}.$$

In particular, when $Q=0$,

$$B(\mathbf{P},0)=(2NN_d\pi^2/E^2(b-1)^2)$$
$$\times[(\eta-\epsilon)((\gamma+\eta)^{-1}(\gamma+\epsilon)^{-1}-(\alpha+\eta)^{-1}(\alpha+\epsilon)^{-1})$$
$$+((b-1)E/2\eta)((\alpha+\eta)^{-2}-(\gamma+\eta)^{-2})].$$

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# Internal Conversion of Gamma-Ray Transitions in the $L$-Subshells*†

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A number of internally converted $\gamma$-transitions have been investigated with high resolution $\beta$-spectrographs and the relative intensities of conversion electron lines from the three $L$-subshells obtained. Generalizations are made for $L$-conversion as related to multipole order. The magnetic transitions investigated are converted in the $L_{\text{I}}$- and $L_{\text{III}}$-shells, the ratio $L_{\text{III}}/L_{\text{I}}$ increasing with increasing multipole order. Less can be said about electric transitions; but $L$-conversion takes place mainly in the $L_{\text{II}}$- and $L_{\text{III}}$-shells for the transitions studied.

## INTRODUCTION

THE improved resolution with which internal conversion electron spectra are being examined and the eventual availability of accurate theoretical values for the $L$-subshell conversion coefficients make it of interest to determine what can be learned about $\gamma$-ray transitions from $L$-subshell internal conversion electrons. There are cases where a $K/L$ ratio or $K$-shell internal conversion coefficient is not obtainable with sufficient accuracy or may not give a unique assignment of multipole order by comparison with the theoretical values.[1] Particularly, if the $\gamma$-transition is of too low an energy to convert in the $K$-shell, some other criterion would be needed.

In the medium and heavy elements it is usually not difficult to compare the intensities of the electron lines from the internal conversion of the $L_{\text{I}}$-, $L_{\text{II}}$-, and $L_{\text{III}}$-shells. In certain cases, even electrons from different $M$-subshells have been resolved by us. In the course of experiments, data have been obtained for a number of internally converted transitions which have been classified as to multipole order by the analysis of Goldhaber and Sunyar.[2] It has become apparent that the relative conversion in the various subshells indeed depends upon the multipolarity of the $\gamma$-transition.

Comparison may be made with certain available theoretical calculations, in addition to the $K$-shell values of Rose *et al.*[1] for energies above 150 kev. Gell-

* A preliminary report of this work was given at the Chicago American Physical Society meeting in October, 1951, J. W. Mihelich and E. L. Church, Phys. Rev. **85**, 733 (1952).

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[1] Rose, Goertzel, Harr, Spinrad, and Strong, Phys. Rev. **83**, 79 (1951).

[2] M. Goldhaber and A. W. Sunyar Phys. Rev. **83**, 906 (1951).