From the table it can be seen that the ratio of the experimental results to Rutherford's predictions is 0.82; they are, rather, closer to Massey's values, the ratio being 0.96 in the angular range 15° to 85°, thus confirming Massey's assumption that the scattering of positrons takes place on the interaction of the Dirac particle of positive charge with the field of the nucleus.

The difference between the theories for the scattering of electrons due to Mott and of positrons due to Massey lies in the sign of the spin interaction terms, and calculations show that it is small for light elements like nitrogen; consequently it would not be evident in cloud-chamber experiments, considering the statistical error involved. It should, however, easily be detectable in heavy elements such as lead, and experiments to observe it are in progress in this laboratory.

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Possibility of Obtaining Experimental Information Concerning the Meson-Meson Interaction*

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The observation of correlated pairs of mesons associated with high energy stars is suggestive of a strong attractive interaction between pairs of π-mesons. If such an interaction exists, it should appear in a relatively simple manner in the production of pairs of mesons by protons or γ-rays near the energetic threshold. The observation of a marked correlation in energy and angle between the mesons of a pair produced in a single event should lead to quantitative information concerning the meson-meson interaction.

DANYSZ, Lock, and Yekutieli[1] have recently reported evidence for marked correlation in the directions and energies of emission of π-meson pairs produced in stars by particles of cosmic-ray origin. This was interpreted as possibly implying the existence of a neutral particle which decays into a π^+ and a π^- meson with a Q value of the order of 2 to 5 Mev. It would appear, however, that these observations could also be interpreted on the basis of an assumed strong, attractive interaction between pairs of π-mesons.

Similar effects involving correlation in the motion of outgoing particles in a reaction have been observed in the production of π^+ mesons in proton-proton collisions[2] and in the absorption of stopped π^- mesons by deuterons.[3] In both cases it appeared possible to describe the phenomena[4,5] by treating the interaction of the outgoing particles separately from the mechanism of the primary event which initiated the reaction. These analyses indicated that such interactions can in a very striking manner control the relative angles of emission and energies of the outgoing particles—and that the effect may be expressed in terms of the phase shifts which would describe the scattering of the two correlated outgoing particles. Estimates of the effect of a strong attractive interaction between π-mesons indicate that this can account for the observations of Danysz *et al.*[1]

Regardless of whether this is the correct interpretation of the above phenomena,[1] it seems worth remarking that for the production of pairs of π-mesons near the energetic threshold any interaction between π-mesons should make itself felt relatively strongly (accelerators with sufficient energy to produce meson pairs by γ-rays and by protons are expected to be available in the near future). The possibility of interpreting an observed correlation in energy and angle in terms of a meson-meson interaction rests upon the argument that near the energetic threshold the primary production process will involve only the lowest order spherical harmonics for the meson angular variables. Thus a marked correlation between the mesons (i.e., involving an anomalously

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1 Danysz, Lock, and Yekutieli, Nature **169**, 364 (1952).
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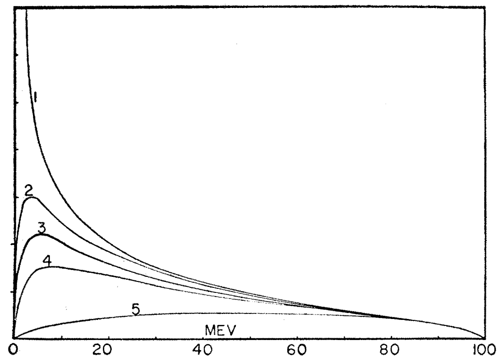

FIG. 1. Distribution in relative energy for a pair of mesons with a total available energy of 100 Mev. The curves labeled 1, 2, 3, 4 are for scattering lengths of ∞, 10.5×10^{-13} cm, 7.4×10^{-13} cm, 5.3×10^{-13} cm, respectively; the curve (5) gives the phase space dependence only. The vertical scale is arbitrary.

large number of meson pairs with only a few Mev of relative energy) might very well be interpretable in a fairly straightforward manner to give some quantitative information concerning the meson-meson interaction.

The question immediately arises as to the expected cross section for producing pairs of π-mesons with accelerators in the Bev range. For the production by protons the theory of multiple production of Fermi[6] is perhaps as reliable as any available estimates. This predicts that for protons whose energy (in the laboratory system) is between one and two Bev the cross section for the production of a pair of mesons varies from 10 to 50 percent of that for single meson production. The absolute cross section for the production of a pair would be of the order of that for single meson production by present accelerators. The cross section for producing a pair of mesons by a γ-ray has been estimated from pseudoscalar meson theory. Near the energetic threshold of 322 Mev the cross section has the approximate form

$$\sigma(\gamma\rightarrow\pi^+ + \pi^-) = 2(10)^{-29}(2\mu c^2/E)(E/\mu c^2-2)^3 \text{ cm}^2, \quad (1)$$

where μ is the meson rest mass and E is the total energy in the center-of-mass system. For γ-ray energies of 600 Mev and 1000 Mev (in the laboratory system) one would estimate from Eq. (1) a ratio for the pair cross section to the single meson cross section of about one-fourth and unity, respectively.

The estimate of the correlation to be expected for the relative motion of the mesons follows the treatment of references (4) and (5). For simplicity, let us assume that two π-mesons with small relative velocity interact primarily in the S state and that the S wave function describing their relative motion is

$$\psi_S = \sin(qr/\hbar+\delta)/(qr/\hbar) \quad (2)$$

outside the region of interaction. q is the relative momentum of the mesons and r is their relative coordinate.

[6] E. Fermi, Prog. Theoret. Phys. **5**, 570 (1950).

For low energies we write $\sin^2\delta$ in the familiar form

$$\sin^2\delta = q^2/(\alpha^2+q^2), \quad (3)$$

where $\hbar/\alpha$ is the "scattering length" for the meson-meson interaction.

Following the method of treatment given in the introduction to reference (5) and assuming that the region of space in which the mesons are produced has dimensions much less than the meson wavelength, we can expect that the transition matrix element T for the production of the pair of mesons is proportional to ψ_S [Eq. (2)], within the region of interaction; i.e.,[7]

$$T\sim\sin\delta/q.$$

We assume (for simplicity) that the remaining energy dependence comes only from the phase space available. Let us consider the entire process to take place in the center-of-mass coordinate system and idealize our model by supposing that two mesons are emitted from the "volume of primary interaction" with a total kinetic energy E_0 to be distributed between them. Then in non-relativistic approximation the fraction f of pairs having a kinetic energy of relative motion less than E is

$$f = (4/\pi)(E/E_0)^{\frac{1}{2}}. \quad (4)$$

In Eq. (4) it is assumed that $E\ll E_0$ and that α^2/μ is appreciably less than E. The corresponding fraction f_0 for the case with uncorrelated emission for the two mesons (i.e., no meson-meson interaction) is

$$f_0 = (16/3\pi)(E/E_0)^{\frac{3}{2}}, \quad (5)$$

again assuming $E\ll E_0$. The ratio f/f_0 then gives the extent to which the correlation of the pairs in relative energy (and therefore in angle) exceeds the statistical correlation. For instance, if we take $E=4$ Mev and $E_0=100$ Mev, we have $f=0.25$ and $f/f_0=20$. Correspondingly, a large fraction of the meson pairs produced in such collisions would be highly correlated in angle and in relative energy, in agreement with the qualitative observations of Danysz *et al.*[1] As E is decreased below the value of α^2/μ, the effect of the correlation for meson energies less than E rapidly diminishes. This is indicated in Fig. 1. where the spectrum of relative energy for the the mesons is plotted for several values of $\hbar/\alpha=\lambda$, the "scattering length."

More singular meson-meson interactions, which would lead for instance to large phase shifts for one of the higher angular momentum states at energies of a few Mev, could lead to much larger correlation effects.

The above estimates are meant to be only qualitative, since it is intended merely to point out that any observed correlation or lack of correlation in the emission of meson pairs should enable one to put some sort of limits on the properties of the meson-meson interaction. More detailed calculations can best be made after there is experimental information available for the properties of the multiple production cross sections.

[7] The authors intend to publish in the near future a formal treatment of problems of this type and of the similar problems considered in references (4) and (5).