Gamma-Ray Absorption Measurements*

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Gamma-ray absorption measurements have been made using a NaI energy selective scintillation counter with radioactive source gamma-rays of 0.411, 0.664, 1.33, and 2.62 Mev energy. A range of absorbers from hydrogen to uranium was used. The final absorption coefficient was determined to ±0.2 percent root-mean-square statistical accuracy. Additional absorption measurements were made with nuclear reaction gamma-rays at the energies 4.47, 6.13, and 17.6 Mev with the same absorbers. A statistical accuracy of ±2 percent was attained. A comparison of theory to experiment is presented and no radical differences are observed.

I. INTRODUCTION

MANY gamma-ray absorption measurements have been made with, in general, increasing accuracy.[1] The present experiment is based on the refinement of using an energy selective scintillation counter as detector.

The absorption of gamma-rays in matter is according to the well-known absorption law

$$I=I_0e^{-\mu x},$$

where μ is the absorption coefficient and x the distance of matter traversed. The primary ambiguity in an experiment to measure μ is the definition of the absorbed photon. Two absorption processes, Compton scattering and Rayleigh (or coherent) scattering, give rise to a fraction of "absorbed" photons that are only slightly modified in energy or direction. The means of discrimination between primary and absorbed photons is customarily made by making the solid angles of source to absorber and absorber to detector small. This conflicts with intensity requirements so that some compromise has always been necessary.

Geiger counters or ionization chambers are not energy selective, so that they give a composite absorption coefficient for those sources that are not monoenergetic; and furthermore, they require careful shielding from gamma-rays scattered from surrounding objects. The sodium iodide scintillation counter, on the other hand, permits the selection of the higher energy gamma-rays of a spectrum at relatively high efficiency. Because of these characteristics it seemed valuable to make absorption measurements with the scintillation counter using gamma-ray energies from 0.5 Mev to 17.6 Mev. The lower energy measurements would overlap the excellent two-crystal spectrometer measurements of Jones[2] in the x-ray region, and the high energy measurements at 17.6 Mev would duplicate the pair spectrometer work of Walker[3] using the lithium gamma-ray.

An absorption measurement is performed by observing the transmission ratio of an absorber placed between the source and the detector. It is well recognized that the accuracy of an absorption measurement is determined by the accuracy of the transmission ratio measurement divided by the number of mean free paths of the absorber. Thus for this experiment, if the transmission ratio was determined to a statistical accuracy of $\frac{1}{2}$ percent for an absorber of 3 mean free paths lengths, then the statistical accuracy of the absorption coefficient μ, so determined, would be ±0.17 percent provided the density and the thickness of the absorber were known to an accuracy better than this. Using radioactive sources of 10 millicuries and taking advantage of the high counting rate of 200 per second feasible with the NaI detector, this degree of statistical accuracy is attainable in times of the order of 10 minutes, so that counting times are not usually the limiting factor in the final accuracy. Instead, systematic errors in various features of the experiment become the limiting factor. For the low energy measurements it was endeavored to reduce the systematic errors to less than 0.2 percent and then determine μ to better than 0.2 percent statistical accuracy. The errors quoted in Tables I–VII are root-mean-square statistical errors. For the high energy measurements counting rate was the limiting factor for the quoted 2 percent accuracy. The experimental equipment is described first, followed by a description of the subsidiary measurements used to calibrate scattering and, in general, to substantiate the thesis that an absorption coefficient is being measured. A short description of the measurements and corrections for each gamma-ray with the subsequent results follows. The article is concluded by a comparision of these results to theory.

II. THE EXPERIMENTAL EQUIPMENT

A. The Detector

The detector for the low energy measurements was a 2-cm diameter by 2-cm long cylindrical crystal of NaI mounted on a 5819 photomultiplier. The differential pulse-height spectrum of Cs^{137} gamma-rays at 0.664 Mev showed a 10 percent full width at half-maximum of the photoelectric line. If a differential pulse-height

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[1] An excellent review of recent work is given in the article by C. M. Davisson and R. D. Evans, Phys. Rev. **81**, 404 (1951).
[2] M. T. Jones, Phys. Rev. **50**, 110 (1926).
[3] R. L. Walker, Phys. Rev. **76**, 527 (1936).

width of 10 percent was set on the photoelectric peak of the Cs^{137} gamma-rays, the stability of the counting rate was ±1 percent per hour.

The detector for the high energy measurements was a 4-cm diameter by 4-cm long cylindrical crystal. A larger crystal is a more efficient detector because it increases the probability for a given quantum to lose all its energy within the crystal. The crystal and mounting showed a 20 percent full width at half-maximum for the Cs^{137} photoelectric peak at 0.664 Mev.

B. Experimental Arrangement

The preliminary experimental arrangement, as shown by Fig. 1, was used to approximate the geometry of completely isolated source, detector, and absorbers, and to investigate the effects of collimation, solid angles, and differential bias settings. The linear distance between elements—source, absorber, detector—was varied according to the desired solid angle, the absorbers remaining half way between the source and detector. The alignment for Fig. 1 was estimated to be accurate

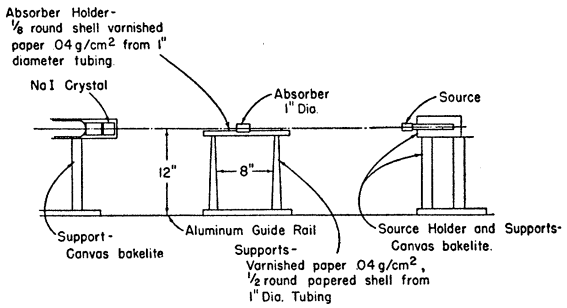

FIG. 1. Open geometry arrangement.

to within ±1 mm. Absorption measurements in copper were taken as a function of displacement of absorber from the central position, and showed that a misalignment of 1 cm could be tolerated with no change in the absorption coefficient. This was approximately the geometrical shadow.

The collimated geometry arrangement for the low energy measurements is shown in Fig. 2. The most critical solid angle from the standpoint of scattering is the largest solid angle, which in this case was the solid angle from source to absorber equalling 8×10^{-5} for a thin line source. This set-up was used as opposed to the open geometry one because of the need for health radiation shielding. The lead collimator was aligned by tracing the outside limit of the gamma-ray beam with a Geiger counter and placing the crystal at the center of the pattern.

The set-up of Fig. 3 was used for the high energy measurements to reduce background and to make use of the greater detection efficiency of the larger crystal.

C. Absorbers

The density of the absorbers was determined by measuring the volume and by weighing, and was esti-

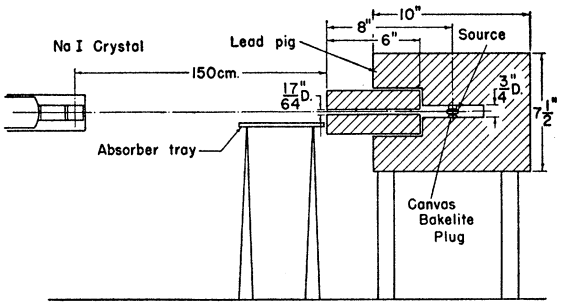

FIG. 2. Collimated geometry arrangement for low energy measurements.

mated to be accurate to 0.03 percent. The absorbers for the open geometry arrangement were 1-in. diameter disks except in the case of gold, platinum, polyethylene, and uranium, which were all approximately 4 square inches in cross section. The large size absorbers were always used in the collimated geometry arrangement. In order to check this and for the high energy measurements, absorbers of the remaining elements were duplicated in a 2 in.×2 in. square size from the same ingots from which the original 1-in. disks were made. The purity of the aluminum, carbon, copper, tin, bismuth, and lead was checked spectroscopically and chemically when necessary. The only element requiring corrections was the 2S aluminum which contained 0.6 percent iron plus approximately 0.1 percent copper. The correction for this was negligible for all but the 17.6-Mev measurements. The uranium was certified pure to 99.8 percent by the AEC. The carbon was pile graphite from Brookhaven, and the gold and platinum were certified to be better than 99.9 percent pure. The gold absorber showed a casting void in it and so was not used for any but the Au^{198} measurements where the void was missed by the gamma-ray beam, in which case a density of 19.52 g/cm³ was assumed.

D. The Sources

The ratios of the energies of all the gamma-rays used were checked to ±3 percent by observing the values of the pulse-height maximum from the scintillation counter. The actual energy value used in the calculations are from the National Bureau of Standards Circular 499 (Nuclear Data) for the radioactive sources, and from the review article "Energy Levels of Light Nuclei," by

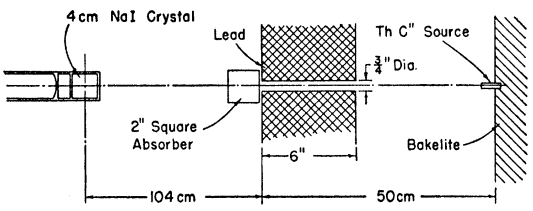

FIG. 3. Collimated geometry arrangement with 4-cm crystal for high energy measurements.

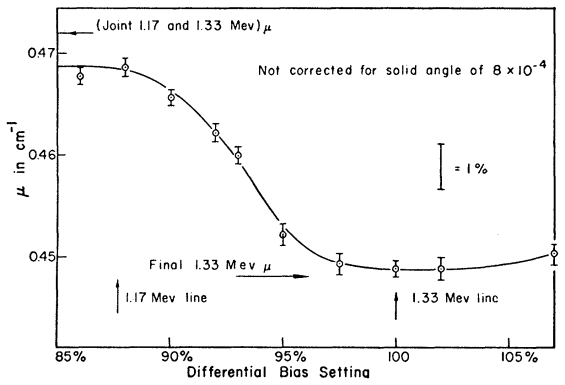

FIG. 4. Absorption coefficient in copper as a function of differential bias setting for Co^{60} source.

Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950), for the high energy gamma-rays.

III. EXPLORATORY MEASUREMENTS

A. Large Angle Scattering

Large angle scattering can give rise to two types of systematic error: (1) It can cause a high background that is slightly dependent upon the presence of the absorber. (2) Scattering within the source volume itself gives rise to secondary gamma-rays of degraded energy so that the effective gamma-ray energy of the source appears less than that of the primary emitted quanta.

The Klein-Nishina formula[4] for Compton-scattering predicts a unique relationship between the energy of a scattered photon and the angle of scattering. This energy dependence was qualitatively checked. For the set-up of Fig. 1 it was observed that no massive object outside a region defined by a maximum scattering angle of 25° would contribute detectable scattering to the detector for a differential bias setting of 80 percent to 100 percent of the 1.33-Mev gamma-ray of Co^{60}. The only mass inside this region for the set-up of Fig. 1 is the absorber supports and the surrounding air. The scattering from the absorber supports was observed to be negligible by observing the counting rate depending upon the presence or absence of the support when a lead attenuator of 10^{-6} attenuation was supported by a string from the above. The scattering from the surrounding air was calculated to be one order of magnitude less.

The problem of degradation of energy of the source by scattering within the source volume is determined by the total mass of the source and container. The change in absorption coefficient resulting from the change in effective energy of the source was observed by placing one mean free path of copper directly against the Co^{60} source and then using this combination source for an absorption measurement. For a 20 percent differential bias width set at the 1.33-Mev gamma-ray of Co^{60}, the observed change in Compton cross section was 0.5 percent. When the differential bias width was reduced to 10 percent width at pulse-height maximum, this change in absorption coefficient was reduced to approximately 0.1 percent. Both these corrections agree with calculations. Because of the high specific activity source used, and also because of the energy selectivity of the detector, no correction was made for the Compton degradation of effective energy of the sources.

To investigate further the properties of the experimental arrangement, an absorption coefficient in copper was taken as a function of the differential bias setting for the Co^{60} source. Figure 4 gives these results and shows a reasonable separation of the two absorption coefficients of the two gamma-rays, using the differential bias width of 10 percent. When the differential bias is set below the 1.17-Mev line, both gamma-rays are detected and the resulting absorption coefficient is a composite one. To get the true absorption coefficient for the 1.33-Mev gamma-ray alone, a correction must be made for the detection of the 1.17-Mev line. This correction was not made from this curve, but instead calibrated by using a Zn^{65} source at 1.12 Mev. This will be discussed under the absorption measurements with Co^{60}.

B. Small Angle Scattering

The effect of small angle scattering is that a photon from the source is scattered by the absorber through a small angle into the detector. The detection of these scattered photons gives a systematic error to the transmission ratio observed. For the Compton scattered photons the change in energy for small angles is too small to observe. The Rayleigh scattered photons have nearly identically the same energy as the primary ones. The relative magnitude of this effect is diminished by making the source-to-absorber and absorber-to-detector solid angles small. To show this, the observed absorption coefficient of Co^{60} 1.33-Mev gamma-rays in copper is plotted as a function of the mean solid angle (Fig. 5). This is perhaps the most significant experiment that indicates that an absorption coefficient is being measured. If effects of alignment, background scattering, and pile-up are negligible, then the functional behavior will essentially be determined by the "small angle" scattering from the absorber into the detector. Rayleigh scattering from copper will be negligible, so there is only Compton scattering to be considered. To see intuitively the different cross section behavior for small angles of approximately 4° and less, it can be pointed out that the momentum transfer from photon to electron is extremely small and so the Compton process approaches Thompson scattering which has a $(1+\cos^2\theta)$ differential cross section dependence.[4] Therefore, for small angles, the Compton differential cross section is essentially constant, so that the amount of scattering will be directly proportional to the solid angle. This

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, England, 1936).

behavior is observed experimentally as shown in Fig. 5 for copper, and the theoretical slope is plotted on the same curve. Figure 5 shows the same measurement for carbon and lead using the Co^{60} source. It is to be noted that the carbon slope is the same as the copper indicating that the scattering is the same from both carbon and copper as would be expected. The curve of absorption coefficient *versus* solid angle for lead, on the other hand, exhibits a behavior that can be explained in terms of Rayleigh scattering. The relative amount of the Compton scattered component from lead should be less than from copper and carbon because of the additional photoelectric absorption, but the Rayleigh component should give a strong peak of the order of 1 percent at very small angles. This behavior at small angles could not be determined accurately without a major experiment, and so the data was treated in the following manner.

IV. LOW ENERGY MEASUREMENTS

A. Method

For all but the Cs^{137} measurements the absorption coefficient was determined accurately at a very small angle. Then the Compton scattering and the Rayleigh scattering were corrected for in the treatment of the data. The Compton scattering correction was no larger than 0.1 percent and was determined both experimentally and theoretically. The Rayleigh scattering correction was less than 1.5 percent and was determined theoretically.[5] These corrections for the corresponding solid angles are given in the Table of Results (Tables I–VII). The absorption coefficient for copper determined with the arrangement of Fig. 2 agreed to within 0.1 percent of the extrapolated value in Fig. 5 for the open geometry arrangement. Absorption curves in copper for both arrangements were taken to an attenuation of 0.01 and showed logarithmic behavior to the statistical accuracy of ±1 percent for each point. The same was true for the lead absorption curve using the 2.62 Mev ThC″ source. All other measurements were made as a transmission ratio through approximately 3 mean free paths of absorber and the ratio determined to 0.5 percent root-mean-square statistical accuracy. The thickness of the absorbers was known to better than 0.1 percent. In all significant measurements the background counting rate was less than 5 percent of the lowest counting rate observed.

B. Results of Measurements with each Source

(a) *Au^{198} Source at 0.411 Mev*

The 50-mC Au^{198} source was obtained from Oak Ridge in the form of a gold foil, 0.5 cm×0.5 cm×0.001-in. thick. It was mounted 10 in. from the front of the collimator, back in the lead pig so that the solid angle of source to absorber remained 8×10^{-5}.

There exists in the gamma-ray spectrum of Au^{198} two gamma-rays of higher energy than the primary one at 0.411 Mev. They are at 0.67 Mev and at 1.09 Mev with relative intensities of 4.8×10^{-3} and 1.3×10^{-3}, respectively.[6] A qualitative check of these was made by taking oscilloscope pictures of the direct pulse pattern at increasing exposures. In order to determine the amount of 0.67-Mev gamma-ray included in an absorption measurement it is necessary to have an estimate of the relative detection efficiencies for the 0.411-Mev and 0.67-Mev gamma-rays in a 10 percent differential bias width set at 0.411 Mev. For an approximate estimate this can be considered composed of two parts: the ratio of the photoelectric cross section in NaI at 0.411 Mev and at 0.67 Mev, and the ratio of counting rates in a 6 percent channel width set at 0.411 Mev and at 0.67 Mev for a 0.67-Mev gamma-ray exclusively. The first ratio is, both experimentally and theoretically, 3:1; and the second ratio was determined experimentally from the differential pulse-height spectrum of Cs^{137} at 0.66-Mev gamma-ray and gives also 3:1. Therefore, the amount of 0.67-Mev gamma-ray detected in the 0.411-Mev absorption coefficient measurements was certainly less than 5×10^{-4} and still smaller for the 1.09-Mev gamma-ray so that no correction was needed.

(b) *Source: Cs^{137} at 0.663 Mev*

The Cs^{137} source was 1 mC in strength so that larger solid angles had to be used. This was done by using the

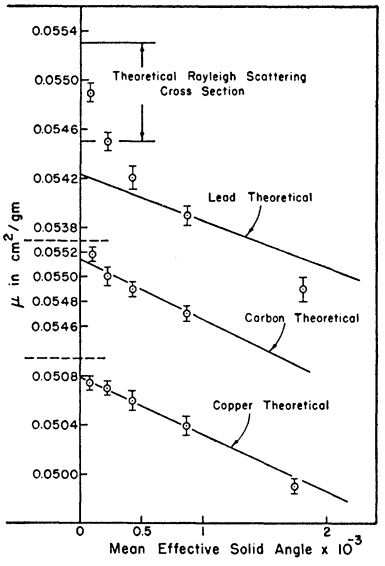

FIG. 5. Absorption coefficient in cm^2/g *versus* solid angle for Co^{60} source.

[5] The small angle dependence of Rayleigh scattering is given numerically by Debye (see reference 16). The asymptotic form of the large angle dependence is given by Franz (see reference 15). Together they determine a differential cross section that can be normalized to the total cross section given by Franz. For a given solid angle, absorber, and gamma-ray energy a fraction of Rayleigh scattering detected can be determined. In terms of total cross section this fraction gives the corrections listed in Tables I–IV.

[6] L. G. Elliot and J. L. Wolfson, Phys. Rev. **82**, 333 (1951).

TABLE I. Absorption measurements using the Au^{198} 0.411-Mev gamma-ray.

Absorber	Measured absorption coefficient, cm^2/g	Rayleigh scatter. correct.	Final absorption coefficient, cm^2/g
Carbon	0.0941±0.0002	···	0.0941±0.0002
Aluminum	0.0914±0.0002	0.0001	0.0915±0.0002
Copper	0.0909±0.00015	0.0003	0.0912±0.00015
Tin	0.1091±0.0002	0.00043	0.1095±0.0002
Platinum	0.1964±0.0004	0.00085	0.1973±0.0004
Gold	0.1981±0.0004	0.00088	0.1990±0.0004
Lead (round)	0.2125±0.0003	0.00084	0.2131±0.0003
Lead (square)	0.2126±0.0003	0.00084	
Bismuth	0.2164±0.0003	0.00084	0.2172±0.0003
Uranium	0.2666±0.0005	0.00092	0.2673±0.0005

open geometry of the preliminary experiments (Fig. 1), and scattering was determined by the method of plotting absorption coefficient as a function of solid angle. The curves for copper, tin, and lead are shown in Fig. 6. The scattering from copper and tin would be due entirely to Compton scattering, but from lead there would be 50 percent Rayleigh scattering. The relative amount of Compton scattering would be reduced in tin because of the photoelectric cross section, but Rayleigh scattering would still be too small to see, so that the expected slope of the scattering curve for tin would be flatter as demonstrated experimentally.

(c) *Source: Co^{60} at 1.33 Mev*

The Co^{60} source was 20 mC in strength enclosed in a Pyrex tube 2-mm inside diameter, 4-mm outside diameter, and 3-cm long. These dimensions excluded any possible Compton degradation of effective energy in the source. Using the geometry of Fig. 2, copper measurements were repeated that reproduced the extrapolated open geometry measurements to 0.1±0.1 percent. However, the absorption coefficient *versus* bias curve of Fig. 4 indicated that a fraction of the 1.17-Mev gamma-ray was being detected. A means was sought, therefore, for calibrating the amount of 1.17-Mev gamma-rays being detected and accurately controlling this throughout the experiment. Fortunately, Zn^{65} has a gamma-ray of 1.12 Mev that seemed ideal for the purpose. With the Zn^{65} source, the ratio of the counting rate in the 10 percent differential bias width was determined for the two settings of the center of the

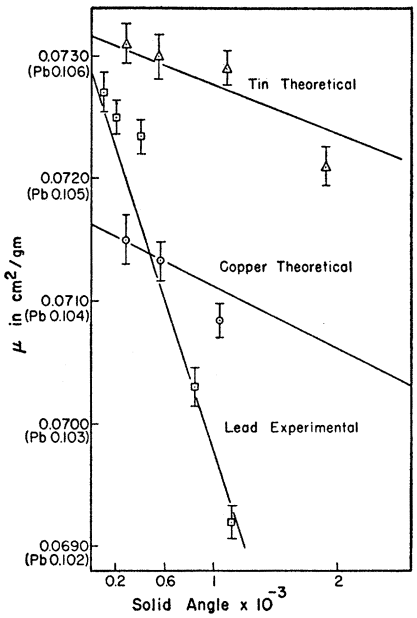

FIG. 6. Absorption coefficient in cm^2/g *versus* solid angle for Cs^{137} source.

"window" at 1.28 Mev and 1.12 Mev. That is, the ratio of counting rate was determined for the differential bias window set on the Zn^{65} photoelectric peak, and for off the photoelectric peak on the high energy side by an amount corresponding to the 1.33–1.17 Mev difference. In this way it was determined that 6.5 percent of the 1.17 Mev gamma-ray was being detected when the window was set on the 1.33-Mev photoelectric peak. This in turn gives rise to an absorption coefficient correction of 0.4 percent to carbon increasing to 0.8 percent correction to uranium. The setting of the differential bias width was checked frequently throughout the experiment by counting the Zn^{65} source in a standard position. The correction is estimated to be accurate to 10 percent which is smaller than the statistical accuracy.

(d) *Th″C Source at 2.62 Mev*

A 72 mC (radium equivalent) Ms Th_1 source was used of the same dimensions as the Co^{60} source, and in the identical geometry. The major difficulty in using

TABLE II. Absorption measurements using Cs^{137} 0.664-Mev gamma-ray.

Absorber	Mean effect. solid angle	Measured absorption coefficient cm^2/g	Compton scatter. correct.	Rayleigh scatter. correct.	Final absorption coefficient cm^2/g
Carbon	4.5×10^{-4}	0.07657±0.0001	0.00046	···	0.0770±0.0001
Aluminum	4.5×10^{-4}	0.07373±0.0001	0.00044	0.00007	0.0742±0.0001
Copper	1×10^{-4}*	0.0717 ±0.0001	···	0.0004	0.0721±0.0001
Tin	1×10^{-4}*	0.0732 ±0.0001	···	0.0008	0.0740±0.0001
Platinum	1×10^{-4}	0.0997 ±0.0002	···	0.0014	0.1011±0.0002
Lead (round)	1×10^{-4}*	0.1057 ±0.0002	···	0.0015	0.1072±0.0002
Lead (square)	1×10^{-4}	0.1056 ±0.0002	···	0.0015	
Bismuth	2×10^{-4}	0.1067 ±0.0003	···	0.0016	0.1083±0.0003
Uranium	1×10^{-4}	0.1241 ±0.0002	···	0.0017	0.1258±0.0002

* Extrapolated.

TABLE III. Absorption measurements using the Co^{60} 1.33-Mev gamma-ray.

Absorber	Measured absorption coefficient cm^2/g	Compton scatter. correct.	Rayleigh scatter. correct.	Correct. for 6.5% 1.17-Mev γ-detect.	Final absorption coefficient cm^2/g
Carbon	0.0552 ±0.0001	0.00005	···	−0.00022	0.0550 ±0.0001
Carbon[a]	0.0551 ±0.0001		···	−0.00022	
Aluminum	0.05338±0.0001	0.00005	0.00002	−0.00028	0.0532 ±0.0001
Copper	0.05074±0.00005	0.00005	0.00009	−0.00026	0.05062±0.0001
Copper[a]	0.05075±0.0001		0.00009	−0.00026	
Tin	0.04888±0.0001	0.00005	0.00017	−0.00030	0.0488 ±0.0001
Platinum	0.0537 ±0.0002	···	0.00033	−0.00038	0.0537 ±0.0002
Lead	0.05534±0.0001	···	0.00034	−0.00039	0.0553 ±0.0001
Bismuth	0.05578±0.0001	···	0.00035	−0.00039	0.0557 ±0.0001
Uranium	0.0587 ±0.0002	···	0.00039	−0.00047	0.0586 ±0.0001

[a] Extrapolared values.

Ms Th_1 as a source for absorption measurements is the large quantity of low energy gamma-rays. There are approximately 10 times as many gamma-rays below 1 Mev as above,[7] and these low energy gamma-rays will interact with the scintillation counter with approximately 10–20 times the efficiency for near maximum pulse size. Consequently pile-up becomes a limitation to the resolution for counting rates feasible in relation to long time electronic stability. The relative efficiency for a large pulse from a high energy gamma-ray is more favorable with a larger crystal, and so consequently these measurements were repeated using a 4-cm crystal in different geometry (see Fig. 3).

V. HIGH ENERGY MEASUREMENTS WITH GAMMA-RAYS OF ENERGY 4.47, 6.13, AND 17.6 MEV

The experimental arrangement of Fig. 3 was used with the ThC″ source replaced by the cyclotron target for the appropriate gamma-ray. The source to absorber distance was 36 cm, and absorber to detector was 57 cm. This gave an effective solid angle for all measurements of 5×10^{-4} for a source of 2-cm diameter.

The 4.47-Mev gamma-ray was obtained from the reaction[8] $N^{15}(p,\alpha)C^{12*}\gamma C^{12}$. 50 microamps of 1-Mev protons bombarded a thick 30 percent enriched N^{15} target on titanium.[9] The intensity was less than for the other targets so that measurements were made on only 4 absorbers as given in Table V. No other gamma-rays of higher energy were present to less than 10^{-3} as determined by counting in a 15 percent differential width set at 6.5 Mev. The gamma-ray intensity was monitored by a Geiger counter in 2 cm of lead.

The 6.13-Mev gamma-ray was obtained from the reaction $F^{19}(p,\alpha)O^{16*}\gamma O^{16}$. 50 microamps of 0.46-Mev protons bombarded a thick target of AlF_3. The intensity of higher energy gamma-rays as from $Al(p,\gamma)$ was less than 10^{-3}. From Walker's measurements[10] there is 5 percent of a 7.03-Mev gamma-ray from a thick target yield at 0.46-Mev proton energy. This gives a maximum correction of 0.25 percent to the results which is so much less than the root-mean-square statistical error of 2 percent that the correction has been neglected.

The 17.6-Mev gamma-ray was obtained from the reaction $Li(p,\gamma)$ by 50 microamps of 0.76-Mev protons bombarding a thick lithium target. No higher energy gamma-rays are likely to be present, and the 14.4-Mev gamma-ray in the $Li(p,\gamma)$ reaction was discriminated against by using a 10 percent differential bias width set at the maximum of the 17.6-Mev pulse-height distribution. It is estimated that no correction need be made for the detection of the 14.4-Mev gamma-ray. The fact that these measurements, when corrected for Compton scattering, agree within the statistical accuracy with the more accurate measurements of Walker[3]

TABLE IV. Absorption measurements using the ThC″ 2.62-Mev gamma-ray.

Absorber	Mean effect. solid angle	Measured absorption coefficient cm^2/g	Compton scatter. correct.	Rayleigh scatter. correct.	Final absorption coefficient cm^2/g
Polyethylene	1.5×10^{-4}	0.0436 ±0.00002	0.00005	···	0.0436 ±0.0002
Carbon	8×10^{-5}	0.03836±0.0001	0.00002	···	0.0384 ±0.0001
Carbon[a]	1.5×10^{-4}	0.0384 ±0.0002	0.00004	···	
Aluminum	8×10^{-5}	0.03774±0.0001	0.00002	···	0.0378 ±0.0001
Copper	8×10^{-5}	0.03745±0.0001	0.00002	0.00004	0.03755±0.0001
Copper[a]	1.5×10^{-4}	0.03755±0.00015	0.00004	0.00004	
Tin	8×10^{-5}	0.03745±0.0001	0.00002	0.00005	0.03745±0.0001
Tin[a]	1.5×10^{-4}	0.03725±0.0002	0.00004	0.00008	
Lead	8×10^{-5}	0.0419 ±0.0001	···	0.00015	0.04205±0.0001
Bismuth	8×10^{-5}	0.0425 ±0.0001	···	0.00015	0.04265±0.0001
Uranium	8×10^{-5}	0.0445 ±0.0004	···	0.00017	0.0447 ±0.0004

[a] These measurements were made with the 4-cm crystal set-up in Fig. 6.

[7] *Nuclear Data*, National Bureau of Standards Circular No. 499 (1950).
[8] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).
[9] The enriched N^{15} was obtained from the Eastman Kodak Company.
[10] R. L. Walker and B. D. McDaniel, Phys. Rev. **74**, 315 (1948).

using the pair spectrometer, gave faith in the method for the measurements at 6.13 and 4.47 Mev.

The Compton scattering correction was determined by making accurate absorption measurements with the ThC″ source in place of the cyclotron target and comparing these measurements with the ones made earlier at a much smaller angle. This showed a Compton scattering correction of 0.8 percent which agrees with calculations from the Klein-Nishina[4] formula. This interpolates to a 1 percent correction to the Compton cross section for 4.47 and 6.13 Mev and a 1.4 percent correction at 17.6 Mev.

The results and corrections are displayed in Tables V, VI, VII.

VI. COMPARISON TO THE WORK OF OTHERS

In the low energy region of gamma-ray absorption measurements from 0.5 to 3 Mev, the best measurements of other workers, in the opinion of this author, are those of Davisson and Evans.[1] Since there is an excellent review and comparison of past experimental work in their article, it is only to be pointed out here the excellent agreement between an extrapolation of their values to those of this experiment at 1.33- and 2.62-Mev energy. This agreement is ±0.2 percent when a Rayleigh scattering correction is applied to their measurements for high Z.[11] It has already been pointed out that the Li(p,γ) measurements at 17.6 Mev are in excellent agreement with the more accurate measurements of Walker.[3]

TABLE V. Absorption measurements using $N^{15}(p,\gamma)$ at 4.47 Mev.

	Measured absorption coefficient cm²/g ±2% error	Compton scatter. correct.	Final absorption coefficient cm²/g ±2% error
Al(13)	0.0295	0.0003	0.0288
Cu(29)	0.0322	0.00025	0.0324
Pb(82)	0.0412	0.0001	0.0413
U(92)	0.0424	0.0001	0.0425

TABLE VI. Absorption measurements using Fl(p,γ) at 6.13 Mev.

	Measured absorption coefficient cm²/g ±2% error	Compton scatter. correct.	Final absorption coefficient cm²/g ±2% error
Polyethylene	0.0270	0.00027	0.0273
Be(4)	0.0201	0.0002	0.0203
Carbon(6)	0.0243	0.00025	0.0246
Al(13)	0.0259	0.00024	0.0261
Cu(29)	0.0308	0.0002	0.0310
Sn(50)	0.0358	0.0001	0.0359
Pb(82)	0.0440	···	0.0440
U(92)	0.0459	···	0.0459

TABLE VII. Absorption measurements using Li(p,γ) at 17.6-Mev gamma-rays. No correction for Rayleigh scattering.

Absorber	Measured absorption coefficient cm²/g	Compton scatter. correct.	Final absorption coefficient cm²/g
Polyethylene	0.0163±0.005	0.00017	0.0165±0.005
Carbon	0.0160±0.004	0.00015	0.0161±0.004
Aluminum[a]	0.0216±0.0005	0.0001	0.0216±0.0005[a]
Copper	0.0339±0.0007	···	0.0339±0.0007
Tin	0.0457±0.001	···	0.0457±0.001
Lead	0.0590±0.001	···	0.0590±0.001
Uranium	0.0624±0.001	···	0.0624±0.001

[a] Corrected for 0.73 percent Iron content.

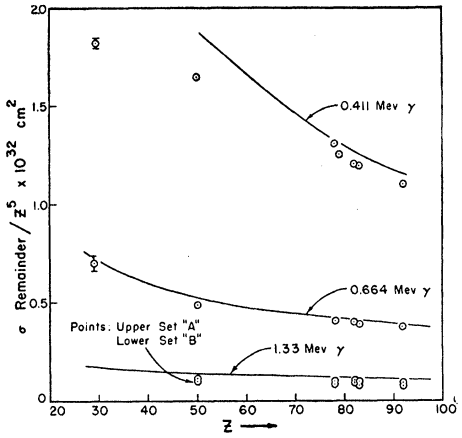

FIG. 7. Curves are the theoretical photoelectric cross section/Z^5 for the respective energies: 0.411-Mev, 0.664-Mev, and 1.33-Mev gamma-rays. The points represent the experimental cross section minus (the theoretical Compton, Rayleigh, and pair production cross sections).

VII. COMPARISON OF THE EXPERIMENTAL DATA WITH THEORY

The interaction of gamma-rays with matter can be classified under the following headings:

1. Photoelectric effect: calculated by Hulme *et al.*[12]
2. Compton scattering: Klein-Nishina formula.[4]
3. Pair production in the field of the nucleus using the exact calculations of Jaeger and Hulme,[13] and the Born approximation calculation of Bethe and Heitler.[4]
4. Pair production in the field of an electron using the calculations of Borsellino.[14]
5. Rayleigh (or coherent) scattering using the results of Franz[15] and Debye.[16]
6. Nuclear disintegration by the process (γ,p) or (γ,n).

The following effects will be neglected in this analysis as being of the same order of magnitude or smaller than the experimental errors.

1. Radiative corrections to the Klein-Nishina formula, approximately 0.2 percent for the ThC″ gamma-rays.[17]
2. Nuclear Thompson scattering.
3. Potential or Delbruck scattering.
4. Nuclear resonance scattering.

[11] This agreement is in a sense deceptive because it is estimated by this author that a correction of the order of 1 percent should be made to their Co^{60} and Zn^{65} measurements because of Compton degradation of effective energy by Compton scattering in their low specific activity sources (see Sec. III, part A).

[12] H. R. Hulme *et al.*, Proc. Roy. Soc. (London) **A149**, 131 (1935).

[13] H. R. Hulme and J. C. Jaeger, Proc. Roy. Soc. (London) **153**, 443 (1936).

[14] A. Borsellino, Helv. Phys. Acta **20**, 136 (1947).

[15] W. Franz, Z. Physik **98**, 314 (1935).

[16] P. Debye, Physik Z. **31**, 419 (1930).

[17] L. Brown, thesis, Cornell University, Ithaca, New York.

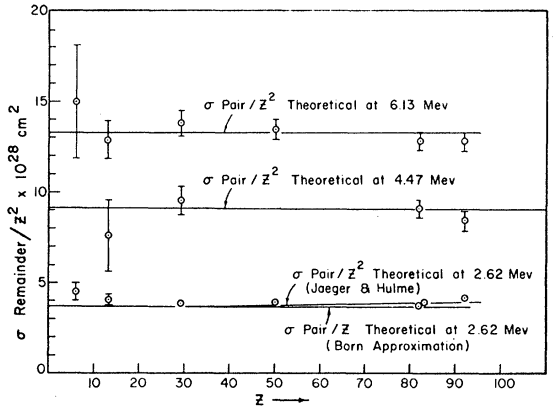

FIG. 8. Curves are the theoretical pair production cross section/Z^2 for the respective energies: 2.62-Mev, 4.46-Mev, and 6.13-Mev gamma-rays. The points represent the experimental cross section minus (the theoretical Compton, photoelectric, and Rayleigh cross section).

The following procedure will be used to analyze the results: It is observed experimentally that the low Z elements agree within the statistical accuracy with the theoretically predicted cross section. For low Z there is almost exclusively Compton scattering so that it will be assumed that the Klein-Nishina formula accurately predicts the Compton scattered component. The Compton cross section will be subtracted from the observed cross section and the remainder analyzed for its Z dependence. Rayleigh scattering will be treated as a small correction determined theoretically. The experiments of Storruste[18] establish the validity of the theory sufficiently for this purpose. No correction for the radiative correction to Compton scattering has been made. Brown[17] estimates that at 2.62 Mev this correction to the Compton cross section is approximately 0.2 percent. This is just the size of the experimental error. At lower energies the effect is smaller and so unobserved, and at higher energies the experimental error is so much greater that the radiative correction is still unobservable.

For gamma-ray energies below the pair production threshold of $2mc^2$, the remainder part of the experimental cross section is expected to behave as Z^5. The Z^5 behavior of the remainder cross section for the 0.411, 0.664, and 1.33-Mev gamma-rays is plotted in Fig. 7; The theoretical curves for photoelectric cross section

TABLE VIII. Calculating table for Au^{198} measurements.

	Carbon(6)	Aluminum(13)	Copper(29)	Tin(50)	Platinum(78)	Gold(79)	Lead(82)	Bismuth(83)	Uranium(92)
$(\sigma/\text{atom})\times10^{24}$ (measured)	1.876±0.004	4.0974±0.008	9.627±0.015	21.60±0.04	63.95±0.12	65.15±0.13	73.39±0.1	75.37±0.1	105.71±0.2
$\sigma_{\text{Compton}}\times10^{24}$	1.880	4.075	9.090	15.673	24.45	24.76	25.700	26.02	28.84
$\sigma_{\text{Rayleigh}}\times10^{24}$	0.0025	0.020	0.163	0.71	2.35	2.43	2.67	2.77	3.64
$(\sigma_{\text{remainder}}/Z^5)\times10^{32}$	···	···	1.822±0.08	1.66±0.01	1.282±0.004	1.231±0.004	1.212±0.003	1.181±0.003	1.110±0.003

TABLE IX. Calculating table for Cs^{137} measurements at 0.664 Mev.

	Carbon(6)	Aluminum(13)	Copper(29)	Tin(50)	Platinum(78)	Lead(82)	Bismuth(83)	Uranium(92)
$(\sigma/\text{atom})\times10^{24}$ measured	1.535±0.002	3.323±0.005	7.611±0.01	14.58±0.02	32.77±0.03	36.88±0.07	37.58±0.1	49.72±0.1
$\sigma_{\text{Compton}}\times10^{24}$	1.5321	3.3195	7.405	12.77	19.917	20.94	21.194	23.49
$\sigma_{\text{Rayleigh}}\times10^{24}$	0.00097	0.0076	0.064	0.275	0.90	1.03	1.07	1.40
$(\sigma_{\text{remainder}}/Z^5)\times10^{32}$		···	0.691±0.05	0.491±0.006	0.414±0.002	0.402±0.002	0.389±0.002	0.377±0.002

TABLE X. Calculating table for Co^{60} measurements at 1.33 Mev.

	Carbon(6)	Aluminum(13)	Copper(29)	Tin(50)	Platinum(78)	Lead(82)	Bismuth(83)	Uranium(92)
$(\sigma/\text{atom})\times10^{24}$ (measured)	1.097±0.002	2.382±0.005	5.344±0.01	9.618±0.02	17.407±0.07	19.026±0.04	19.34±0.04	23.17±0.04
$\sigma_{\text{Compton}}\times10^{24}$	1.097	2.3766	5.3015	9.1405	14.259	14.990	15.173	16.819
$\sigma_{\text{Rayleigh}}\times10^{24}$	0.00024	0.0019	0.016	0.069	0.228	0.260	0.270	0.352
σ_{pair}(J&H)$\times10^{24}$	···	···	0.019	0.094	0.31	0.36	0.37	0.52
$(\sigma_{\text{remainder}}/Z^5)\times20^{32}$	···	···	···	0.100±0.006	0.0904±0.003	0.0921±0.002	0.0895±0.002	0.0832±0.002
$\sigma_{\text{pair}}\times10^{24}$ (Born app.)	0	0	0.01	0.032	0.074	0.081	0.083	0.102
$(\sigma_{\text{remainder}}/Z^5)\times10^{32}$	···	···	···	0.120±0.006	0.099±0.003	0.0995±0.002	0.096±0.002	0.090±0.002

[18] A. Storruste, Proc. Phys. Soc. (London) **A63**, 1197 (1950).

TABLE XI. Calculating table for ThC″ measurements at 2.62 Mev.

	Polyethylene	Hydrogen(1)	Carbon(6)	Aluminum(13)	Copper(29)	Tin(50)	Lead(82)	Bismuth(83)	Uranium(92)
$(\sigma/\text{atom})\times10^{24}$ (measured)	1.0154±0.005	0.1248±0.005	0.7657±0.002	1.693±0.004	3.964±0.01	7.38±0.02	14.467±0.03	14.800±0.03	17.67±0.16
$\sigma_{\text{Compton}}\times10^{24}$		0.1249	0.7496	1.6242	3.6233	6.247	10.245	10.370	11.49
$\sigma_{\text{Rayleigh}}\times10^{24}$				0.0005	0.0042	0.018	0.0675	0.071	0.091
$\sigma_{\text{photoelectric}}\times10^{24}$			...	0.0003	0.013	0.164	1.51	1.60	2.50
$(\sigma_{\text{remainder}}/Z^2)\times10^{28}$			4.45±0.6	4.05±0.2	3.85±0.1	3.80±0.06	3.93±0.05	4.00±0.045	4.25±0.2

TABLE XII. Calculating table for $N^{15}(p,\gamma)$ at 4.47 Mev.

	Al	Cu	Pb	U
σ/atom $\times10^{24}$ cm² measured	1.29±2%	3.42±2%	14.2±2%	16.8±2%
σ_{Compton}	1.16	2.59	7.32	8.21
$\sigma_{\text{pair electron field}}$	0.0034	0.010	0.021	0.024
$\sigma_{\text{photoelectric}}$		0.0066	0.77	1.32
$\sigma_{\text{remainder}}/Z^2$ $\times10^{28}$	7.7±2	9.6±0.8	9.1±0.5	8.5±0.5

of Na^{24},[20] that the energy dependence of the Hulme calculations is not correct. If the theoretical photoelectric cross section at 2.62 Mev is accepted as being correct,[21] then the beta-spectrometer work would indicate that the photoelectric cross section at 1.33 Mev should be 15 percent lower than the theoretical value. This would bring the theory and the experimental data into approximate agreement for the Co^{60} gamma-ray.

Figure 8 shows the remainder cross section/Z^2 for 2.62, 4.47, and 6.13 Mev gamma-rays. The Compton

TABLE XIII. Calculating table for $Fl(p,\gamma)$ at 6.13 Mev.

	Polyethylene	H(1)	Be(4)	Carbon(6)	Al(13)	Cu(29)	Sn(50)	Pb(82)	Ur(92)
σ/atom $\times10^{24}$ cm² measured	0.635±2%	0.072±0.007	0.304±2%	0.491±2%	1.17±2%	3.27±2%	7.07±2%	15.1±2%	18.2±2%
σ_{Compton}		0.0722	0.289	0.433	0.939	2.095	3.61	5.92	6.54
$\sigma_{\text{pair electron field}}$		0.00053	0.0021	0.0032	0.007	0.0155	0.0267	0.0437	0.049
$\sigma_{\text{photoelectric}}$		...	...	...	...	0.004	0.06	0.54	0.93
$\sigma_{\text{remainder}}/Z^2$ $\times10^{28}$ cm²				15±3	12.8±1	13.8±0.8	13.5±0.64	12.8±0.5	12.7±0.5

TABLE XIV. Calculating table for $Li(p,\gamma)$ at 17.6 Mev.

	Polyethylene	Hydrogen(1)	Carbon(6)	Aluminum(13)	Copper(29)	Tin(50)	Lead(82)	Uranium(92)
$(\sigma/\text{atom})\times10^{24}$ cm² measured	0.385±0.01	0.035±0.007	0.315±0.008	0.970±0.02	3.58±0.07	9.02±0.2	20.3 ±0.4	24.65±0.5
Measurement by Walker: $(\sigma/\text{atom})\times10^{24}$ cm²		0.037 (extrapolated)	0.323±0.0045	0.972±0.01	3.62±0.02	8.96±0.1	20.56±0.12	25.3 ±0.2 (extrapolated

are taken from the calculations of Hulme *et al.*[12] using the rule

$$\sigma \text{ photo whole atom}/\sigma \text{ photo } K \text{ shell} = 5/4.$$

This relation is open to some doubt. However, exact calculations of the contribution of remaining shells gives a 10 percent lower cross section[19] which should lessen the observed agreement at high Z. In the case of the Cs^{137} gamma-ray this agreement is quite good, so that any change in the 5/4 rule would have to be compensated by some other correction. At 1.33 Mev there exists considerable disagreement of the photoelectric cross section with the experiment. There is some experimental evidence from beta-ray spectroscopy of the photoelectrons ejected by the two gamma rays

cross section is assumed to be known and the photoelectric effect and pair production in the field of the electron are treated as small corrections. The pair production cross section in the field of the nucleus shows, at 4.47 and 6.13 Mev, remarkable agreement with the Bethe-Heitler theory based on the Born approximation.[22] This is perhaps to be expected because at higher energies the theory is known to give too low a cross section, and at low energies the Jaeger and Hulme calculations predict a higher cross section. The cross over point where the Born approximation calculation is expected best to agree is near 4 Mev. This is

[19] Gladys White, National Bureau of Standards, private communication.

[20] Bishop, Collie, Halban, Hedgran, Seigbahn, Du Toit, and Wilson, Phys. Rev. **80**, 211 (1950).

[21] It agrees with the work of Latyshev, Revs. Modern Phys. **10**, 132 (1947).

[22] The theoretical pair production cross sections were based on the asymptotic formulas of P. V. C. Hough, Phys. Rev. **73**, 266 (1948).

confirmed within the statistical accuracy of the experiment.

The Li(p,γ) measurements agree within the statistical accuracy with the more accurate measurements of Walker.[3] The statistical error in his measurements was better than that of these measurements by a factor of 2, and his analysis is comprehensive. In view of this, no additional analysis of the Li(p,γ) measurements will be given, other than to say that they verify the high energy difference between experiment and the Born approximation theory.

It is with pleasure that I express my appreciation of the many suggestions and encouragement of Dr. Robert R. Wilson throughout the course of the experiment. Mr. James Draper spent considerable time operating the cyclotron for the high energy measurements.

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Carriers of Electricity in Metals Exhibiting Positive Hall Effects*

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The momentum associated with the electric current in metals has been measured for the first time under conditions in which they are known to have positive Hall coefficients. Such metals are of particular interest because their positive (anomalous) coefficients suggest the possibility of positive electric carriers. A coil of the wire under investigation was supported with its axis vertical as a torsional pendulum. The extremely small deflections resulting from the inertial effect of successive reversals of current were measured by a resonance method. A phototube and oscilloscope method for measuring both the amplitude and the phase of the small deflections is described.

In more than 100 determinations on Mo and Zn the sign of the charge-to-mass ratio was always found to be negative. For both Mo and Zn the mean value of e/m was found to be within 3 percent of the value for free electrons in slow motion. One set of determinations for Mo was made at liquid air temperatures. The direct mechanical method used to measure e/m is independent of any particular theory of metallic conduction. The results thus discriminate against theories for which current and momentum can have the same direction. In particular, the results rule out the possibility that positive Hall effects may be explained by positive carriers.

I. INTRODUCTION

IT is important to obtain a conclusive, experimental answer to the question whether negative electrons are the carriers of electricity in all types of metals.

On the one hand, the best conductors Ag, Cu, Au, and Al have Hall coefficients of sign and order of magnitude expected on the basis of the classical transverse force acting on negative electrons moving in a magnetic field. Furthermore, in the case of Ag, Cu, and Al direct measurements[1,2] of the momentum associated with the current have been found to be in accord with the hypothesis of negative electrons as carriers of electricity.

On the other hand many metals—including Mo, Zn, Cd, Pb, and Ni—have positive Hall coefficients. The transverse force acting on charged particles moving in a magnetic field could not apparently account for such coefficients unless the electric carriers had positive values of e/m. A modified theory based on the wave properties of electrons was proposed by Peierls[3] to account for positive Hall coefficients. However, since this theory is incomplete[4] and since the only certain answer must come from experiment, it is important to make direct measurements of e/m.

In 1930 the first successful measurements by the process adopted here, and first proposed by Maxwell, were made by one of us on e/m for the carriers in copper.[5] At that time an attempt was also made to apply this method, and later on, another and quite distinct method, to a metal having a positive Hall effect, namely, nickel; but experimental difficulties and the urgency of other investigations prevented completion of the work at that time.

In 1940 Kikoin and Gubar[6] published the details of what they described as a gyromagnetic experiment on superconducting Pb. This experiment (really an electron inertia experiment rather than a gyromagnetic one) showed that the electric carriers in superconducting Pb have a value of e/m nearly equal to that of free electrons. The Hall coefficient of Pb is positive at room temperature. However, Hall coefficients are known to change in magnitude and even in sign with change in temperature. Hence, no certain conclusions can be drawn from these measurements in the superconducting state.

The first measurements under conditions for which the Hall coefficient is known to be positive were made by the authors and the results were presented briefly

* Presented at the meeting of the American Physical Society at Los Angeles, California, December 28–30, 1950.

[1] S. J. Barnett, Revs. Modern Phys. **7**, 129 (1935).

[2] C. F. Kettering and G. G. Scott, Phys. Rev. **66**, 257 (1944).

[3] R. Peierls, Z. Physik **53**, 255 (1929).

[4] Certain considerations advanced by C. G. Darwin, Proc. Roy. Soc. (London) **A154**, 61 (1936), on the role of quantum-mechanical energy levels in conduction and electron-inertia phenomena likewise give no explanation of positive Hall coefficients.

[5] S. J. Barnett, Phil. Mag. **12**, 349 (1931).

[6] I. K. Kikoin and S. W. Gubar, J. Phys. (U.S.S.R.) **3**, 333 (1940).