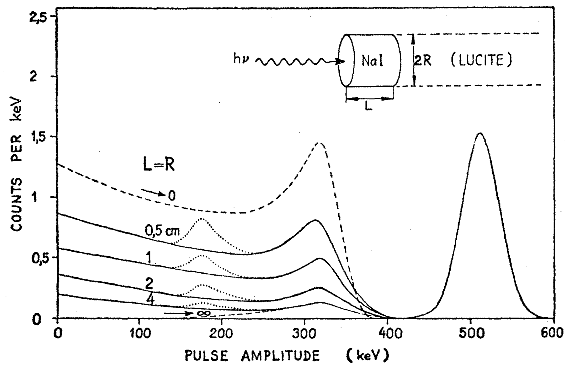

FIG. 1. Theoretical pulse amplitude distribution for 511 kev γ-rays. Ordinate gives absolute number of counts for ∞, 2213, 873, 358, 176, and 100 quanta incident on cylindrical NaI crystals of the different sizes indicated in the figure. Dotted lines show the backscattering effect from long light pipes.

for zero crystal size is included as a limit for cases of bad geometry. These curves might be useful for the measurement of weak γ-rays with energy <0.5 Mev in the presence of annihilation radiation.

The theoretical line shapes have been smeared out with a Gaussian distribution of very small width only, corresponding to the highest electron yield (about 1000 per Mev) reported by Hofstadter,[1] in order to show as much detail of the curves as can be expected with present day techniques. Our experimental data are in good agreement with the calculated curves, except that our electron yield is somewhat lower which causes an additional broadening. A fuller account of this work will appear in *Helvitica Physica Acta.*

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Narrow Angle Pairs of Particles from Nuclear Interactions

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AN experiment is in progress to study nuclear interactions in Ilford G-5 emulsions which were produced by 145- and 227-Mev negative pions from the Chicago cyclotron.[1] A number of plates 400 microns thick were examined both by following individual pion tracks and by using general scanning techniques. Out of an observed number of 800 nuclear interactions, two of these showed evidence for the emission of a pair of fast particles and will be described at this time. In one of these collisions, Fig. 1, track *a* was due to an incident negative 138±7 Mev pion of the 145-Mev pion beam of the Chicago cyclotron,[1] and track *b* has the same gap density as that of a 17±4-Mev proton. A narrow angle pair of tracks C_1 and C_2 (0.3 ± 0.06 degrees between tracks) were emitted at an angle of 100±2° with respect to the incident pion track. Multiple scattering and grain density measurements of these tracks (Pr_1) are summarized in Table I. In a region where the tracks are well separated, the grain density of both C_1 and C_2 corresponds to minimum ionization within a precision of 15 percent. It can then easily be shown in conjunction with the multiple scattering measurements that C_1 must have been due to a particle less massive than about 1/2 that of a muon, while C_2 can only be shown to be less massive than a muon.*

The second nuclear interaction observed is almost identical in appearance to Fig. 1 except that the angle between the pair, Pr_2 of Table I, is 6.3° or about 25 times larger than in the first case Pr_1. Again the tracks Pr_2, are of minimum ionization and, considering the multiple scattering, (Table I) the mass of both particles is less than that of muons.

The above features, while by no means conclusive, tend to indicate that Pr_1 and Pr_2 are probably electron pairs, in which case their total energies would be 201±39 and 205±25 Mev respectively. Electron pairs of such high energy and originating from nuclear collisions of cyclotron accelerated particles have not been reported so far. However, the following two well-known nuclear reactions of negative pions first studied at low energies by Panofsky[2] could provide the initial step in the production of high energy electron pairs:

$$\text{(i)}\quad \pi^- + P \rightarrow N + \gamma,$$

$$\text{(ii)}\quad \pi^- + P \rightarrow N + \pi^0.$$

Process (i), the inverse of the production of a negative pion (π^-) by a gamma-ray (γ), has been estimated by Fermi[3] to occur in about 1 percent of the negative pion-proton collisions. Both of the pion collisions leading to Pr_1 and Pr_2 were with carbon or heavier nuclei of the emulsion, which would mean that in these cases the upper limit for the frequency of process (i) would be 1 percent. The internal conversion of this gamma ray to a pair of electrons should occur with a probability of about 1 percent according to theory.[4] The experimentally obtained value of approximately one pair for each 350 collisions seems certainly much higher than the above expected value of the total probability of 1 out of 10,000. In process (ii), the so-called charge exchange scattering,[5] the neutral pion, π^0, will decay normally into two gamma-rays. However, a decay into two electrons and a gamma-ray can also be considered.[6] Decay into two gamma-rays would in general lead to electron pairs at distances of the order of a radiation unit (about 3 cm) from the center of the star and hence it is ruled out. Decay of a neutral pion into a gamma-ray and two electrons would result in a pair originating at a mean distance of the order

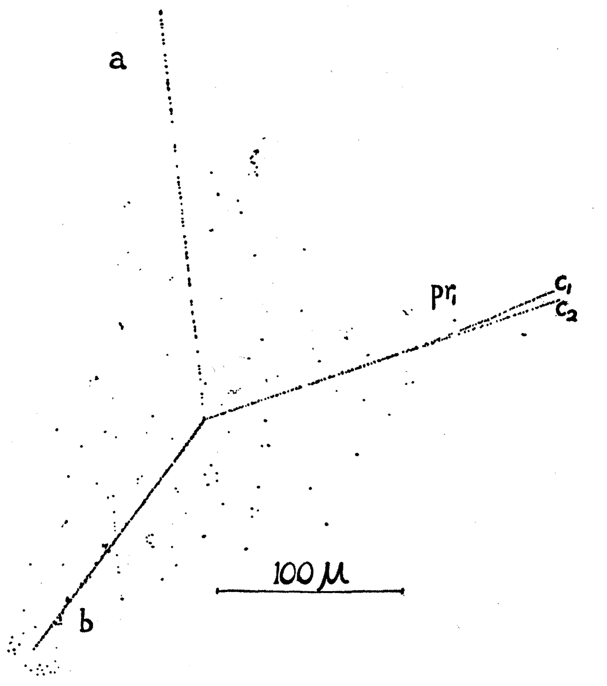

FIG. 1. Nuclear interaction produced by a 138-Mev negative pion, *a*, in which a pair of fast particles C_1 and C_2 appear to come directly from the center of the star.

TABLE I. Data regarding narrow angle pairs.

Event	Angle between tracks in degrees	Mean scattering angle in degrees per 100 microns Track C_1	Track C_2	Energy of electrons in Mev C_1	C_2	Total
Pr_1	0.30 ±0.06	0.78	0.14	32 ±8	173 ±25	205 ±39
Pr_2	6.3 ±0.3	0.21	0.30	118 ±24	83 ±17	201 ±25
Pr_3	2.4 ±0.5	1.25	2.78	20 ±3	9 ±2	29 ±3

of 4 microns from the star if the mean life is 10^{-14} sec or longer.[7] Examination of the two collisions described above indicates that Pr_1 occurred less than 1 micron and Pr_2 less than 1/2 micron from the centers of their respective stars, which, however, would not be incompatible with a mean life of the order of 10^{-15} sec.[8] The angle between the electrons would be small since in the above cases the total energy of the neutral pion is about equal to its momentum times velocity of light. Process (ii) occurs in about 3/4 of the pion-proton collisions, and this is again an upper limit for pion collisions in the emulsion. Since Dalitz[6] has calculated that 1 out of 80 neutral pions should decay into two electrons and a gamma-ray, it would be expected that at most about 1 pion collision out of 100 in the emulsion would be associated with a pair of fast electrons.

While electron pairs produced through process (i) would lead to close angular correlation of the tracks, the expected frequency is about 20 times less than measured. On the other hand, although through process (ii) the expected frequency of pairs is of the right order of magnitude, the mean life of the neutral pion would have to be of the order of 10^{-15} sec or less.

We wish to thank Professor H. L. Anderson and the cyclotron group for their assistance in this experiment and Professors E. Fermi and G. Wentzel for very stimulating discussions.

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[*] Professor Bernardini informed us that out of 89 stars produced by 110-Mev negative pions he found one emitted fast pair similar to that in Fig. 1. At the time of the writing of this note a third pair was found and its characteristics are given in Table I.

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The Validity of Born Expansions

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RECENTLY the nature of the Born expansions[1] for the case of a nonrelativistic particle scattered by a static potential has been clarified by Jost and Pais.[2] We have supplemented this work by establishing, for central potentials, estimates for the radii of convergence for various energy ranges and any angular momentum.

We consider the radial Schrödinger equation

$$\left(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}+k^2\right)\psi(r)=\lambda V(r)\psi(r), \tag{1}$$

where $\lim_{r\to 0}r|V(r)|<\infty$ and $\lim_{r\to\infty}r^2V(r)=0$. The various Born expansions of the solution of (1) differ by the choice of boundary conditions.[3] The following two are commonly used:

$$\psi(0)=0,\ \psi(r)\to\sin\left(kr-\frac{l\pi}{2}\right)+\tan\eta_l\cos\left(kr-\frac{l\pi}{2}\right)\quad\text{for}\quad r\to\infty; \tag{2}$$

$$\psi(0)=0,\ \psi(r)\to\sin\left(kr-\frac{l\pi}{2}\right)+\frac{S_l-1}{2i}\exp i\left(kr-\frac{l\pi}{2}\right)\quad\text{for}\quad r\to\infty. \tag{2'}$$

$S_l-1\equiv e^{2i\eta_l}-1$ occurs in the three-dimensional scattering amplitude; S_l is a scattering matrix element. The iteration of the integral equation equivalent to (1) and (2) leads to a power series in λ for $\tan\eta_l$, and similarly (1) and (2′) yield a series for S_l.

Expansion of $\tan\eta_l$:—For a given $V(r)$ let λ_T be that value of $|\lambda|$ up to which this expansion converges. One can then show from the integral equation that for *all* potentials

$$k=0:\quad \lambda_T\int_0^\infty r|V(r)|dr\geqslant 2l+1; \tag{3}$$

$$\text{All } k:\quad \lambda_T\int_0^\infty r|V(r)|dr\geqslant t_l;$$

$$\begin{array}{c||c|c|c|c|c|c|} l & 0 & 1 & 2 & 3 & & \text{large} \\ \hline t_l & 1 & 2.344 & 3.339 & 4.198 & & 1.157\cdot(2l+1)^{\frac{2}{3}} \end{array}\ . \tag{4}$$

These estimates are optimal in the sense that the right-hand sides cannot be replaced by larger numbers. The equality signs are approached as $V(r)\to\delta(r-a)$.

For any *fixed* potential (3) and (4) become very conservative for large l. The following asymptotic expression for large l is then useful. Let $r^2|V(r)|$ have its maximum value at r_0. Then

$$\lambda_T\sim\frac{1}{r_0^2|V(r_0)|}\left\{l(l+1)+\left(3-\frac{r_0^2V''(r_0)}{V(r_0)}\right)[l(l+1)]^{\frac{1}{2}}\right\} \tag{5}$$

to within terms of order $<[l(l+1)]^{\frac{1}{2}}$ which contain the energy dependence. At low energies, (5) has an error of only 10–15 percent for the usual potentials, even for $l=1$.

As for the behavior of λ_T at low energies, one can show that

$$l=0:\quad \left.\frac{\partial\lambda_T}{\partial(k^2)}\right|_{k=0}>0, \tag{6}$$

if $V(r)$ does not change sign (otherwise the inequality may go the other way!); and for all potentials

$$l\geqslant 1:\quad \left.\frac{\partial\lambda_T}{\partial(k^2)}\right|_{k=0}<0, \tag{7}$$

i.e., λ_T *decreases* as the centrifugal barrier is being overcome. For large l,

$$\left.\frac{\partial\lambda_T}{\partial(k^2)}\right|_{k=0}\to-\frac{1}{|V(r_0)|}. \tag{8}$$

At high energies and for any l

$$\lambda_T\left|\int_0^\infty V(r)dr\right|=\pi k+O(k), \tag{9}$$

provided the integral is neither zero nor infinite; for singular potentials with $\lim_{r\to 0}r|V(r)|=\beta$,

$$\lambda_T\beta=\pi k/\log k+O(k/\log k). \tag{10}$$

Expansion of $S_l\equiv e^{2i\eta_l}$:—Calling the radius of convergence λ_S, we find

$$k=0:\quad \lambda_S\int_0^\infty r|V(r)|dr=\lambda_T\int_0^\infty r|V(r)|dr\geqslant 2l+1; \tag{3'}$$

$$\text{All } k:\quad \lambda_S\int_0^\infty r|V(r)|dr\geqslant s_l;$$

$$\begin{array}{c||c|c|c|c|c|c|} l & 0 & 1 & 2 & 3 & & \text{large} \\ \hline s_l & 1 & 2.047 & 2.783 & 3.416 & & \approx 0.86(2l+1)^{\frac{2}{3}} \end{array}\ . \tag{4'}$$

Again the numbers are optimal. Equation (5) holds also for λ_S, but no inequality corresponding to (6) was found. For

$$l\geqslant 1:\quad \left.\frac{\partial\lambda_S}{\partial(k^2)}\right|_{k=0}=\left.\frac{\partial\lambda_T}{\partial(k^2)}\right|_{k=0}, \tag{11}$$

so that (7) and (8) hold also for λ_S.

At high energies

$$\lambda_S/k\to\infty, \tag{9'}$$

provided $|\int_0^\infty V(r)dr|<\infty$.

A number of properties of Born expansions have been derived, some of which have been previously observed.[4]