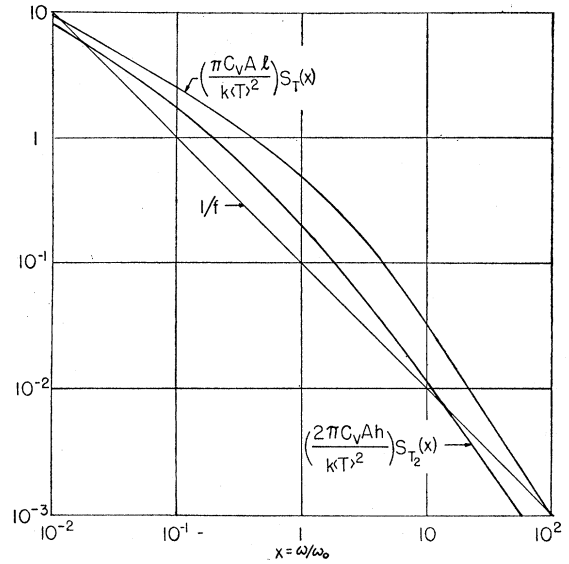

FIG. 1. Power spectra of temperature fluctuations.

The theory of contact noise that we are proposing is based on the idea that temperature fluctuations in the "neighborhood" of the contact are the source of the contact noise.

In order to illustrate this idea we consider a $P-N$ junction in, say, germanium and assume that the "neighborhood" whose temperature controls the current extends a distance l each side of the center of the junction. Later we use a weighting factor to decrease the influence of regions further away from the junction in a continuous manner. The assumption of a finite uniform "neighborhood" is considerably simpler to treat mathematically and illustrates the features of the theory.

Considering a fluctuation ΔT in the temperature of the "neighborhood," the current is:

$$i=B(\langle T\rangle)\exp\{-E/[k\langle T\rangle(1+\Delta T/\langle T\rangle)]\}; \tag{2}$$

[we neglect in this note temperature fluctuation effects in $B(\langle T\rangle)=R[1-\exp(-eV/k\langle T\rangle)]$—they can be shown to be small compared to the effects in $\exp(-E/kT)$].

Expanding the exponential we have:

$$\Delta i=i-\langle i\rangle=\langle i\rangle E\Delta T/(k\langle T\rangle^2). \tag{3}$$

We can write down the total fluctuations in the current, using the general result that the temperature fluctuations of a body of thermal capacity C in contact with a heat reservoir are given by the expression:[1]

$$\langle\Delta T^2\rangle=k\langle T\rangle^2/C. \tag{4}$$

For our problem $C=C_VAl$, where C_V is the specific heat of the material, and A is the cross-sectional area of the junction. Then:

$$\langle\Delta i^2\rangle=\langle i\rangle^2E^2/(k\langle T\rangle^2C_VAl). \tag{5}$$

This result is in qualitative agreement with certain characteristics of contact noise: (1) The noise power is proportional to the square of the average current, (2) the noise is not strongly temperature dependent, (3) point contact rectifiers and transistors are noiser than broad area devices.

To investigate the power spectrum of the noise we first find the correlation function of the current and then the power spectrum:

$$\langle\Delta i(0)\Delta i(t)\rangle=[\langle i\rangle^2E^2/(k^2\langle T\rangle^4)]\langle\Delta T(0)\Delta T(t)\rangle,$$
$$G_i(f)=[\langle i\rangle^2E^2/(k^2\langle T\rangle^4)]S_T(f),$$
$$S_T(f)=4\int_0^\infty(\cos 2\pi ft)\langle\Delta T(0)\Delta T(t)\rangle dt. \tag{6}$$

The power spectrum of T is that of a diffusion mechanism since heat flow is basically a diffusion process. A one-dimensional heat flow seems appropriate since heat flow to and from the "neighborhood" will occur primarily within the sample.

The power spectrum of the one-dimensional diffusion mechanism has been obtained by McFarlane[2] and Miller[3] in regard to a theory of contact noise based on density fluctuations of ions near the contact. The corresponding expression for the power spectrum of the temperature fluctuations is:

$$S_T(x)=\frac{k\langle T\rangle^2}{C_VAlx^{\frac{1}{2}}}\{1-\exp(-x^{\frac{1}{2}})[\cos x^{\frac{1}{2}}+\sin x^{\frac{1}{2}}]\}, \tag{7}$$

where $x=\omega/\omega_0$, $\omega_0=2D/l^2$, $D=K/C_V$, and K is the coefficient of heat flow. The spectrum is plotted in Fig. 1.

An important characteristic of this theory is that the turnover frequency ω_0 is not strongly temperature dependent. The parameter l is involved in ω_0 and also in the magnitude of the noise [Eq. (7)] making an internal check of the theory possible.

Weighing the effect of a local temperature fluctuation $\Delta T(y)$ at a distance y from the center of the junction by an exponential factor $\exp(-y/h)$ the power spectrum is:

$$ST_2(x)=\left[\frac{k\langle T\rangle^2}{C_VAh2\pi}\right]\frac{1}{x^{\frac{1}{2}}[1+2x+2x^{\frac{1}{2}}]}, \tag{8}$$

where $x=\omega/\omega_0$, $\omega_0=2D/h^2$, and we have used mathematical results of Miller.[3] $ST_2(x)$ and the characteristic $1/f$ spectrum of contact noise are plotted on Fig. 1 and examination shows that ST_2 approximates a $1/f$ law within ± 3 db over nearly five decades.

A more detailed account of this theory is under preparation for publication.

Helpful discussions with Prof. Uhlenbeck and with Dr. W. W. Scanlon are gratefully acknowledged.

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The Mass of a Photon

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THE method, developed by Riesz, of solving the hyperbolic equation by the analytical continuation of an integral which is an analytic function of an arbitrary parameter α has been applied to different problems in electrodynamics by various authors.[1-4] Fremberg[1] uses a definition for the Riesz potential which is a generalization of the Maxwell potential in the α-plane. The equivalence of this definition and the λ-limiting process has been established by Ma.[5] Auluck and Kothari[4] use a modified definition for the Riesz potential and have shown that it is a generalization of the Wentzel potential in the α-plane. The purpose of the present note is to show that in any problem the use of the modified definition of Auluck and Kothari is equivalent to assuming a finite rest mass for the photon and finally letting it tend to zero. This method of assuming a finite rest mass for the photon has been widely used in present day electrodynamics to avoid certain infrared divergences. There the rest mass is introduced arbitrarily. This unphysical assumption can, it seems, be justified by the fact that one would get the same results using consistently the analytical continuation method. However, in any problem it is simpler to introduce the mass directly.

We define the metric tensor $g_{\mu\nu}$ as $g_{00}=1$, $g_{11}=g_{22}=g_{33}=-1$; $g_{\mu\nu}=0(\mu\neq\nu)$. The velocity of light is taken as unity. The scalar product of two four-vectors A_μ and B_μ is denoted by $[AB]$. The (positive) length of the space part of A_μ is written as $|A|$.

The Riesz potential at any point x due to a point electron moving with a velocity ϑ_μ is defined as [Eqs. (5) and (11) of I]

$$A_\mu{}^\alpha(\kappa)=H(\alpha)e/2\pi^2\int_D\int_{-\infty}^{z}k^{\alpha-2}\sin[k,x-z']\vartheta_\mu' d^4k d\tau', \tag{1}$$

where

$$H(\alpha)=\frac{2}{\Gamma(\alpha/2)\Gamma[1-(\alpha/2)]},$$

τ is the proper time of the electron, and the domain of integration, D, is $[kk]\geqslant 0$, $k_0>0$. It can be shown that $A_\mu{}^\alpha(x)$ satisfies the following relations:

$$\Box A_\mu{}^\alpha(x)=A_\mu{}^{\alpha+2}(x), \tag{2}$$

$$\partial A_\mu{}^\alpha(x)/\partial x_\mu=-e\mathfrak{D}^\alpha(x-z), \tag{3}$$

where

$$\mathfrak{D}^\alpha(x)=H(\alpha)\int_D k^{\alpha-2}\sin[k,x]d^4k. \tag{4}$$

Now consider the function

$$\mathfrak{D}^\alpha(x)=H(\alpha)\int_D k^{\alpha-2}\sin[kx]d^4k.$$

Changing the variables of integration form k_0, k_1, k_2, k_3 to k, k_1, k_2, k_3, where $k=(k_0^2-|k|^2)^{\frac{1}{2}}$ and integrating over k_1, k_2, k_3 we have[6]

$$\mathfrak{D}^\alpha(x)=H(\alpha)\int_0^\infty k^{\alpha-1}\left\{\Delta(x)+\frac{1}{|x|}\frac{\partial U}{\partial|x|}\right\}dk, \tag{5}$$

where

$$\Delta(x)=(1/|x|)\{\delta(x_0-|x|)-\delta(x_0+|x|)\}, \tag{5a}$$

and

$$U=\begin{cases} J_0\{k(x_0^2-|x|^2)^{\frac{1}{2}}\} & \text{for } x_0>|x|,\\ 0 & \text{for } |x|>x_0>-|x|,\\ -J_0\{k(x_0^2-|x|^2)^{\frac{1}{2}}\} & \text{for } x_0<-|x|.\end{cases} \tag{5b}$$

Thus from (5) we see that so far as the variation of $\mathfrak{D}^\alpha(x)$ with respect to x is concerned, it behaves like a D-function for a particle of mass k. On analytically continuing $\mathfrak{D}^\alpha(x)$ to $\alpha=0$, it reduces to

$$\mathfrak{D}^0(x)=\Delta(x),$$

i.e., the effect of analytically continuing $\mathfrak{D}^\alpha(x)$ to $\alpha=0$ is to let k tend to zero.

This shows that the Riesz potential $A_\mu{}^\alpha(x)$ defined by (1) refers to an infinite number of particles of different masses k. Only $k=0$ remains when $A_\mu{}^\alpha(x)$ is analytically continued to $\alpha=0$.

I would like to express my thanks to Mr. J. Hamilton for his interest in the work. I am also thankful to the Atomic Energy Commission of the Government of India for the award of a scholarship.

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Line Shape of Monochromatic γ-Rays in the Scintillation Spectrometer

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IN order to facilitate quantitative analysis of complex[1] or continuous[2] γ-ray spectra by means of the NaI scintillation technique, we have calculated the detailed line shape for several monochromatic γ-radiations. This shape will always lie between the following extreme cases:

(1) *Primary effect:* Out of n_0 quanta of energy γ (expressed in mc^2 units) falling on a crystal of thickness L, a number np_1 given by

$$(np_1/n_0)=(\mu_p/\mu)(1-e^{-\mu L}) \tag{1a}$$

will undergo photoelectric absorption. μ_p and μ denote photoelectric and total absorption coefficients, both of which may be taken from Victoreen's tables.[3] On the other hand the number of primary Compton processes is calculated using the Klein-Nishina formula which in its original form[4] gives the scattering cross section as a function of scattering angle ϑ. After some transformations we get the number of recoil electrons as a function of their energy ϵ (in mc^2 units):

$$\frac{\delta n_c}{n_0}=\frac{\pi r_0^2NZ}{\mu}(1-e^{-\mu L})\left[2+\epsilon\frac{\epsilon(\gamma+1)^2-\gamma(\epsilon^2+2\gamma)}{\gamma^2(\gamma-\epsilon)^2}\right]\frac{\delta\epsilon}{\gamma^2}, \tag{1b}$$

where $\pi r_0^2NZ=0.232$ cm^{-1} for NaI(Tl). Since we shall confine our considerations to γ-ray energies between 0.1 and 1 Mev, the total primary effect consists of np_1 pulses corresponding in amplitude to the full quantum energy and the n_c-distribution with amplitudes between zero and $\epsilon_{\max}=2\gamma^2/(2\gamma+1)$, according to Eqs. (1a) and (1b). This simple procedure approximates the true distribution for geometric conditions where contributions from the absorption of secondary quanta may be neglected (i.e., for small crystals).

(2) *Complete secondary absorption:* Instead of a Compton distribution, a single "photo peak" of

$$(np_2/n_0)=1-e^{-\mu L} \tag{2}$$

pulses would result from an overestimation of secondary and higher order effects (approximation for very large crystals).

In order to calculate the true line shape we note that the number of secondary quanta, scattered into angles $\vartheta\cdots\vartheta+\delta\vartheta$ and escaping from the crystal, is given by an expression of the form

$$\frac{\delta n_e}{n_0}=\frac{\pi r_0^2NZ}{\mu}\left(\frac{\gamma'}{\gamma}\right)^2\left(\frac{\gamma'}{\gamma}+\frac{\gamma}{\gamma'}-\sin^2\vartheta\right)G(\vartheta,\gamma)\sin\vartheta\delta\vartheta, \tag{3}$$

where $\gamma'=\gamma/[1+\gamma(1-\cos\vartheta)]$=energy of scattered quanta. Similarly, the number of secondary quanta which are absorbed within the crystal is

$$\frac{\delta n_a}{n_0}=\frac{\pi r_0^2NZ}{\mu}\left(\frac{\gamma'}{\gamma}\right)^2\left(\frac{\gamma'}{\gamma}+\frac{\gamma}{\gamma'}-\sin^2\vartheta\right)H(\vartheta,\gamma)\sin\vartheta\delta\vartheta, \tag{4}$$

where G and H are geometric factors related by

$$G+H=1-e^{-\mu L}.$$

For the case of a cylindrical crystal (radius R, length L), irradiated on its axis by a well-collimated beam of γ-rays, we have expressed G and H explicitly by ϑ, γ and $\mu'=\mu'(\vartheta,\gamma)$=total absorption coefficient for secondary γ-rays.

If we apply the above-mentioned extreme assumptions (1) and (2) to the treatment of secondary quanta, we obtain two limits for the secondary contributions. Thus we get from (2) the following modified line shape:

$$np_2'=np_1+\int_0^\pi(\delta n_a/\delta\vartheta)d\vartheta \tag{5}$$

$$\delta nc_2'=\delta\epsilon\delta n_e/\gamma'^2\sin\vartheta\delta\vartheta; \tag{6}$$

and from (1a), (1b):

$$np_1'=np_1+\int_0^\pi(\mu_p'/\mu')(\delta n_a/\delta\vartheta)d\vartheta \tag{7}$$

$$\delta nc_1'=\delta nc_2'+\delta\epsilon\int_{\vartheta_1}^{\vartheta_2}\frac{\mu_c'}{\mu'}\frac{2\gamma'+1}{2\gamma'^2}\frac{\delta n_a}{\delta\vartheta}d\vartheta, \tag{8}$$

where $np_{1,2}'$=number of pulses with full energy, $\epsilon=\gamma$, and $\delta nc_{1,2}'$=number of pulses with energies $\epsilon\cdots\epsilon+\delta\epsilon$; for (6),

$$\epsilon=\gamma^2(1-\cos\vartheta)/[1+\gamma(1-\cos\vartheta)]\leqslant 2\gamma^2/(2\gamma+1);$$

for the second term in (8), ϵ arbitrary within $0\leqslant\epsilon\leqslant 4\gamma^2/(4\gamma+1)$. The limits of the integral in (8) are determined by

$$\left.\begin{array}{lll}\cos\vartheta_1=1 & \text{or} & 3-\epsilon/\gamma(\gamma-\epsilon)\\ \cos\vartheta_2=1-\epsilon/\gamma(\gamma-\epsilon) & \text{or} & -1\end{array}\right\}\begin{array}{l}\text{depending on}\\ \epsilon\leqslant 2\gamma^2/(2\gamma+1),\end{array}$$

for (5) and (7), respectively; (8) and (6) represent upper and lower limits for the modified photopeaks and Compton distributions, respectively. After computing the limits (5)+(6) and (7)+(8) for all γ-ray energies we may use them—instead of the crude assumptions (1) and (2)—in the treatment of secondary absorption. This procedure leads to higher order approximations which converge towards the true line shape. As an example, the "true" curves for $\gamma=1$ and for several crystal sizes are shown in Fig. 1 where also the effect of a light pipe, consisting of Lucite and extending from behind the crystal to infinity, is indicated (dotted lines). The curve