

The lowest eigenvalue is certainly smaller than the minimum of (7) taken with respect to l_0 . We thus obtain

$$l_0 = (\hbar^2/2m^2g)^{\frac{1}{2}} \quad (8a)$$

$$(E_1/mg) = \frac{3}{4}(\hbar^2/2m^2g)^{\frac{1}{2}}. \quad (8b)$$

The agreement with (5) is obviously very satisfactory. Since both kinetic and potential energy are positive, it is clear that the density must be vanishingly small for values of z sizeably in excess of $l_0/2$. This leads for He to a film thickness of approximately $4 \cdot 10^{-4}$ cm.

We would like to maintain the view expressed in our first letter that the conditions analyzed by Lamb and Nordsieck and by ourselves clearly indicate that the behavior of an ideal gas is not even approximately indicative of that of He II; the difference between He⁴ and He³, if not mainly a mass effect, may still be caused by the influence of the $E-B$ statistics on the behavior of a liquid. The fact that ordinary He⁴ condenses with a density about one thousand times smaller than that of an ideal condensed gas is, in our opinion, convincing evidence that the inter-atomic forces are the determining factor in the distribution law.

¹ O. Halpern, Phys. Rev. **86**, 126 (1952).

² W. Lamb and A. Nordsieck, Phys. Rev. **59**, 677 (1941).

³ R. Becker, Z. Physik **128**, 120 (1950); G. Leibfried, Z. Physik **128**, 133 (1950).

The Influence of Charge Independence of Nuclear Forces on Electromagnetic Transitions

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IN a recent paper Trainor¹ has considered the effect of symmetry on the nuclear dipole radiation. The result of his calculations is that the symmetry properties of nuclear states give rise to certain selection rules for electric dipole radiation. We want to draw attention to the fact that this result is much more general and applies not only to electric dipole radiation but also to radiation of any multipolarity. Secondly, we will show that the selection rule obtained is not restricted to the case (considered by Trainor) when the nuclear forces are pure Wigner- and Majorana-type forces, but holds provided only that the forces are charge-independent. This seems a reasonable approximation and is consistent, for example, with the most recent spectroscopic evidence on the excited nuclear states.²

The interaction between the electromagnetic field and the nucleus can be represented, in a nonrelativistic approximation by the Hamiltonian

$$H = (e/mc) \sum_i \frac{1}{2} p_i \cdot A(x_i) (1 - \tau_z^{(i)}) + \sum_i \left\{ \frac{1}{2} \mu_n (1 + \tau_z^{(i)}) + \frac{1}{2} \mu_p (1 - \tau_z^{(i)}) \right\} \sigma^{(i)} \cdot \text{curl} A(x_i), \quad (1)$$

where p_i is the momentum of the i th nucleon, $A(x_i)$ the vector potential of the electromagnetic field at the position x_i of the nucleon, μ_n and μ_p the neutron and proton magnetic moments, $\sigma^{(i)}$ the Pauli spin matrix, and $\tau_z^{(i)}$ the z -component of the isotopic spin matrix having eigenvalues $+1$ for neutrons and -1 for protons.

If the forces are charge-independent, the total isotopic spin

$$T = \sum_i \tau^{(i)}$$

is a constant of the motion, so that each nuclear stationary state can be assigned a definite eigenvalue $T(T+1)$ of T^2 .

Consider now the matrix element

$$\langle \alpha' J' T' | H | \alpha J T \rangle \quad (2)$$

between two states of angular momentum J and J' , isotopic spin T , T' (α, α' indicate any other quantum number necessary to specify the state). It is easy to see that the matrix element (2) vanishes unless

$$T - T' = 0, \pm 1. \quad (3)$$

This follows immediately if we note that H can be written as

$$H = H_0 + K_z = H_0 + \sum_i f_i \tau_z^{(i)}, \quad (4)$$

where H_0 represents the part of H independent of the isotopic spin, and f_i is the factor multiplying $\tau_z^{(i)}$. In isotopic spin space H_0 transforms like a scalar and K_z like the third component of a vector. The matrix elements of H_0 vanish unless $T = T'$. As for K_z , using the same considerations developed by Condon and Shortley³ for a general vector operator one deduces immediately Eq. (3).

Equation (3) represents a generalization of the result obtained by Trainor¹ for the case of dipole radiation and for nuclei containing an equal number of protons and neutrons. If we restrict ourselves to a consideration of electric dipole radiation, it is well known that the scalar term H_0 [Eq. (4)] gives no contribution to the matrix element since it represents the contribution of a system of particles each with a charge $e/2$. For the second term of Eq. (4) another restriction is immediately obtained considering the dependence of the matrix element (2) on the z -component of the isotopic spin: With the notations of Condon and Shortley one has⁴

$$\langle \alpha J T T_z | K_z | \alpha' J' T T_z \rangle = \langle \alpha T || K_z || \alpha' T \rangle T_z.$$

If $T = 0$ the right-hand side vanishes, so we can conclude that an electric dipole transition cannot take place between two states with $T = 0$. Of course, the transition can still occur as magnetic dipole or as an electric transition of higher multipolarity, since in such a case the matrix element of H_0 does not vanish.

The validity of the selection rule (3) is not restricted to the validity of Wigner's model of nuclear structure; in particular it is still true also when a strong spin-orbit interaction is present, in which case the symmetry of the space part of the wave function is no longer a constant of the motion.

On the other hand, since the Coulomb energy

$$H_0 = (e^2/8) \sum_{i < j} (1 - \tau_z^{(i)})(1 - \tau_z^{(j)})/r_{ij}$$

does not commute with T , the isotopic spin is no longer a constant of the motion, and Eq. (3) will not be strictly satisfied even if the specific nuclear forces are strictly charge-independent. The extent to which Coulomb forces destroy the validity of the selection rule (3) is under investigation and will be reported later.

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¹ L. E. H. Trainor, Phys. Rev. **85**, 962 (1952).

² S. Baskin and H. T. Richards, Phys. Rev. **84**, 1124 (1951); W. A. Fowler and T. Lauritsen, Phys. Rev. **82**, 197 (1951); J. D. Seagrave, Phys. Rev. **85**, 197 (1952).

³ E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1951) p. 59.

⁴ Reference 3, p. 63.

Are Direct Nucleon-Lepton Interactions Charge-Independent?

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VOTRUBA¹ has recently suggested that direct nucleon-lepton interactions might be charge-independent. If this were so, the coefficients of $H_{PN\nu\nu}$, $H_{NP\nu e}$, $\frac{1}{2}H_{PPee}$, $-\frac{1}{2}H_{NNee}$, $-\frac{1}{2}H_{PP\nu\nu}$, and $\frac{1}{2}H_{NN\nu\nu}$ in the Hamiltonian should be equal, if H_{ijkl} means the space integral of any of the invariants $\sum_n \bar{\psi}_i \omega_n \psi_j \bar{\psi}_k \omega_n \psi_l$, where we assume the neutrino ν to be different from the antineutrino.²

In this connection it is interesting to remark that the phenomenologic direct-interaction terms with $\omega_n = \gamma_\lambda$ (vector coupling; $H_{ijkl} = -\psi_i \alpha_\lambda \psi_j \psi_k \alpha^\lambda \psi_l$) do not satisfy this condition. In fact, the repulsive interaction $\frac{1}{2}H_{PPee}$ describing phenomenologically the Lamb shift³ has a coefficient $(4.91 \pm 0.01) \times 10^{-41}$ cm³ erg, while $(-\frac{1}{2}H_{NNee})$ describing phenomenologically the attractive electron-neutron interaction⁴⁻⁶ has a coefficient (2.8