

The Relative Specific Ionization of Fast Mesons*

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An experiment has been performed with cosmic-ray mesons to study the specific ionization in the relativistic region. A cloud chamber was used to measure the rate of droplet production for two groups of mesons, one with momentum p between 70 and 250 Mev/ c and the other with $p > 1500$ Mev/ c . These two groups of momenta were selected by the appropriate arrangement of Geiger counters. The cloud chamber used to measure the ionization was also used to measure the magnetic rigidity of the particles in the lower group of momenta. The particles in the other group produced tracks which were essentially straight in the magnetic field of 4800 gauss. The group of mesons with lower momenta ionized at or near the minimum rate which was determined to be 14.7 ± 0.35 droplets/mm on the photographic film. The other group of mesons gave an experimental average of 17.9 ± 0.25 droplets/mm, in good agreement with the value of 18.5 droplets/mm predicted on the basis of the Bethe-Bloch ionization theory and the accepted momentum spectrum for sea-level cosmic-ray mesons. The good agreement obtained between the experimental and theoretical distributions shows that the present ionization theory with its rise in the relativistic region adequately describes the observed rate of energy loss for energetic mesons with values of p/μ up to approximately 100. Too few mesons were available with momenta large enough to check the effect of polarization in helium.

INTRODUCTION

THE theoretical curve for the rate of energy loss by particles passing through matter due to collisions with electrons was first developed by Bohr.¹ It has been modified by quantum-mechanical calculations, and relativistic corrections have been made. A convenient summary of these equations giving complete references appears in an article by Rossi and Greisen.² For collision losses caused by small transfers of energy a separate calculation must be made to include the binding energy of the electron. This has been done quantum-mechanically by Bethe.³ The energy loss per unit distance dE/dx is shown to depend only on the charge and velocity of the moving particle. Thus the curve should be universally applicable to all particles of the same charge, independent of their mass. This curve shows that dE/dx decreases rapidly as the velocity βc increases toward the velocity of light c , reaching a minimum at $\beta = 0.97$. Beyond this minimum, the rate of energy loss rises logarithmically with increasing velocity. The dependence of the energy loss on the momentum p and rest mass μ of the incident particle can be obtained by noting that $p/\mu = \beta/(1 - \beta^2)^{1/2}$. Swann,⁴ Fermi,⁵ Wick,⁶ and Halpern and Hall⁷ have developed the theory of an additional effect due to polarization which reduces the rate of ionization in the relativistic region. This effect is very important for condensed media but is considerably less important in dispersed media like gases. (See in particular the calculations of Halpern and Hall.⁷)

The theoretical curve has been well substantiated by experiments on electrons and mesons for values of momenta up to $p/\mu = 3$.⁸⁻¹⁰ The rise due to relativistic effects has also been shown to agree well with experiments on electrons, but a similar check on this rise for mesons has been difficult because of the high momenta involved. Not only are such momenta difficult to measure, but mesons with such momenta are found as yet only in cosmic radiation where the intensity decreases very rapidly with increasing momentum. The rise in energy loss is due directly to the Lorentz contraction of the Coulomb field of the incident particle. This contraction causes significant transfers of energy at greater distances from the path of the particle than would otherwise be the case. It has been generally expected, therefore, that energetic mesons should show the same rise as that observed for electrons.

Various attempts have been made to detect for mesons the predicted rise in the rate of energy loss in the relativistic region. The early investigations were performed by determining the specific ionization of fast mesons by counting droplets in a cloud chamber.⁸⁻¹¹ These all indicated little or no rise, but the experiments were subject to appreciable difficulty entailed by the measurement of the high momenta. Also, these early experiments were performed before the composition of high energy cosmic rays was well known. With the advent more recently of solid ionization detectors such as silver chloride conduction counters and scintillation crystals, attempts were subsequently made to investigate the energy loss in thin layers of these materials by studying the pulse sizes produced by ener-

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¹ N. Bohr, *Phil. Mag.* **25**, 10 (1913).

² B. Rossi and K. Greisen, *Revs. Modern Phys.* **13**, 240 (1941).

³ H. Bethe, *Ann. Physik* **5**, 325 (1930); *Z. Physik* **76**, 293 (1932).

⁴ W. F. G. Swann, *J. Franklin Inst.* **226**, 598 (1938).

⁵ E. Fermi, *Phys. Rev.* **57**, 485 (1940).

⁶ G. C. Wick, *Nuovo cimento* **21**, 7 (1943).

⁷ O. Halpern and H. Hall, *Phys. Rev.* **73**, 477 (1948).

⁸ Sen Gupta, *Nature* **146**, 65 (1940).

⁹ D. Corson and R. Brode, *Phys. Rev.* **53**, 773 (1938).

¹⁰ W. Hazen, *Phys. Rev.* **65**, 259 (1944); *Phys. Rev.* **67**, 269 (1945).

¹¹ E. Hayward, *Phys. Rev.* **72**, 937 (1948).

getic mesons.^{12,13} These experiments revealed little or no rise in the rate of ionization for mesons of relativistic energy, but these results were to be expected in view of the magnitude of the effect of polarization, even for p/μ as low as 10. On the other hand, this effect for gases at standard pressure is not important for p/μ less than about 100–500.⁷ The first experiment in which the momenta of the fast mesons were measured directly was published by Goodman, Nicholson, and Rathgeber.¹⁴ They used a momentum spectrometer capable of measuring momenta up to 8×10^{10} ev/c. A gas proportional counter was used to measure the ionization of the fast cosmic-ray mesons. According to the theory, the increase of ionization for the fastest mesons whose momenta they could measure should be 50 percent above minimum. However, they detected only a negligible rise for mesons, although the measurements of other workers on electrons agree well with the theoretical curve both below and above minimum. Because of the fundamental nature of this rise mentioned previously, it is very hard to explain this discrepancy between the behavior of electrons and mesons on a theoretical basis. Thus it was of considerable interest to check the results of Goodman *et al.*, using a different experimental technique. After the experiment to be reported here was completed and this paper was being written, the excellent results of Ghosh, Jones, and Wilson¹⁵ were published. Their results showed a rise in the rate of energy loss in the relativistic region for mesons, in agreement with theory and in agreement with the results reported in this paper.

Before becoming interested in this problem, we had spent considerable effort developing a technique for the reliable operation of a Wilson cloud chamber. Such a technique allowed us to maintain a constant efficiency for the formation of droplets on a track during a period of several weeks. A chamber operated in the above manner with provision also for measuring momentum seemed quite well suited to an investigation of the energy loss for energetic mesons.

APPARATUS

The apparatus consisted of a counter-controlled cloud chamber in a magnetic field of 4800 gauss. The chamber was 9 inches in diameter and 3 inches deep. Only the center $1\frac{1}{4}$ inches were used. This insured good focus and a minimum amount of distortion. The experimental arrangement is shown in Fig. 1. The counter telescope *A B C* selected cosmic-ray particles. The 4 inches of lead above the chamber greatly reduced the probability of observing single electrons. Thus only single mesons and an occasional chance shower

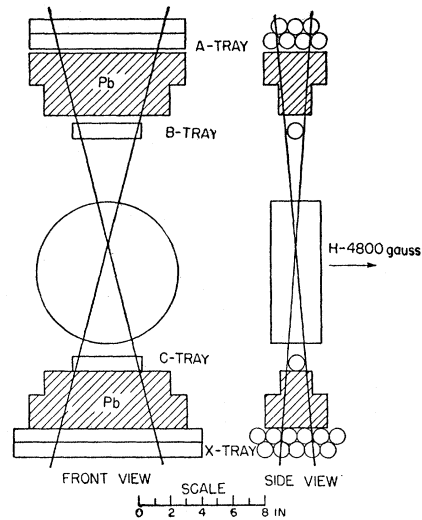


FIG. 1. Experimental arrangement.

were seen in the chamber. By using the X-tray of counters in anticoincidence with the telescope one could require that the mesons stop in the $4\frac{1}{4}$ inches of lead below the chamber. Thus one could either select mesons with momentum between approximately 70 and 250 Mev/c or at random from the cosmic-ray spectrum, if the X-tray were not used.

The technique used in the operation of the cloud chamber was designed to maintain a constant efficiency for the formation of droplets on ions. It is well known that when tracks are split, a reliable ion count may be made on the denser column provided droplets form on about one-fourth or more of the lighter column. However, when one measures the momentum of a particle by measuring its curvature in a magnetic cloud chamber, the accuracy of such measurements is appreciably reduced by splitting the track. We preferred not to split the track, since we wanted to measure in the same cloud chamber the magnetic rigidity as well as the ionization. To insure reproducible droplet counts without splitting the track, we took special precautions to maintain the same condensate mixture of 70 parts by volume of ethyl alcohol to 30 parts water. Another necessary requirement was *never* to change the expansion ratio of the chamber, which was 1.076. A paper describing the details of the operating technique outlined here is being prepared.

In order to obtain a track with few enough ions so that they could be counted accurately, helium was chosen as the operating gas. This allows the droplets to diffuse sufficiently for easy counting without using an intentional time delay in the expansion. The inherent delay in the expansion mechanism in our apparatus is such that the tracks diffuse to 1.9-mm width. A greater diffusion would introduce difficulties into the measurement of the curvature of the tracks.

¹² W. Whittemore and J. C. Street, Phys. Rev. **76**, 1786 (1949).

¹³ T. Bowen and F. Roser, Phys. Rev. **85**, 992 (1952).

¹⁴ Goodman, Nicholson, and Rathgeber, Proc. Phys. Soc. (London) **A64**, 96 (1951).

¹⁵ Ghosh, Jones, and Wilson, Proc. Phys. Soc. (London) **A65**, 68 (1952).

DISCUSSION AND RESULTS

Over a period of a month, pictures of 200 particles were obtained. Out of these, 129 were selected as being in sufficiently good focus for reliable droplet count. About one-third of our pictures were obtained using the X-tray in anticoincidence with the telescope. This assured us of at least 40 particles whose momenta could be accurately determined. These particles fall on the curve in the region which has been checked in previous experiments. Thus they can be used to calibrate the droplet count in our chamber. These particles were interspersed throughout our data, so approximately every third meson had momentum in the range from 70 to 250 Mev/c. To assure a good calibration of our data, only runs including at least one meson in this range were included in our analysis.

The maximum momentum a particle could have and still be reliably distinguished from a straight track was 1500 Mev/c. This momentum in our magnetic field of approximately 5000 gauss corresponds to a radius of curvature of 10 meters. Our accuracy was determined by measuring the curvature of no field tracks taken throughout each day's run. No systematic curvature could be detected. About 80 percent of these tracks had a radius of curvature greater than 12 meters. Therefore we chose a radius of curvature of 10 meters as our upper limit on measurable tracks. Particles whose paths had greater radii of curvature were classified as having momenta greater than 1500 Mev/c. Of the 129 particles selected, 52 were in this high momentum range, while the remaining 77 had measureable momenta.

The specific ionization of each particle was measured by counting the droplets. Because the density of droplets which were counted was small, the correction for overlapping droplets was negligible. All clusters containing more than 20 droplets were excluded from our count. This exclusion both decreased the statistical fluctuation and eliminated the error introduced by estimating the number of droplets in these larger clusters. If we assume approximately 30 ev/ion pair, we are essentially excluding all energy transfers greater than 300 ev. Thus, in comparing our result with theory, we must use the formula developed by Bethe^{3,4} for

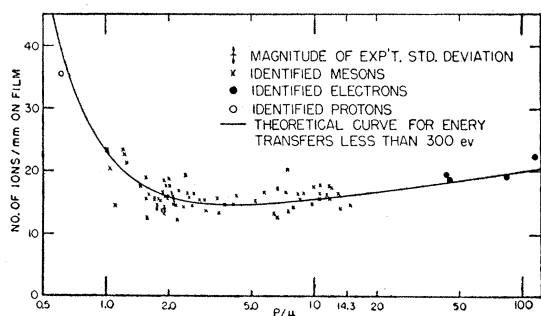


FIG. 2. Theoretical and experimental specific ionization as a function of p/μ .

small transfers of energy, where the maximum transfer of energy in any one collision is η . This formula for k_η is

$$k_\eta = \frac{2C\mu_e}{\beta^2} \left[\ln \frac{2\mu_e\beta^2\eta}{(1-\beta^2)I^2(Z)} - \beta^2 \right]$$

where k_η = energy loss through collisions in ev/g-cm⁻², η = maximum energy transferred in any one collision, $C = \pi NZ/Ar_0^2 = 0.150(Z/A)$, μ_e = rest mass of electron, and $I(Z)$ = average ionization potential of an atom of atomic number Z .

Since $p/\mu = \beta/(1-\beta^2)^{1/2}$, this curve may be plotted as a function of p/μ . For comparison with our data we calculated the curve for helium using $\eta = 300$ ev. The shape of the curve, however, is only slightly dependent on η , since η enters only in the argument of the logarithm. Thus it differs but slightly from that used by other investigators. This curve, normalized to our data, is plotted as a function of p/μ in Fig. 2.

We obtain our droplet count as the number of droplets per mm measured on the film. The demagnification of our lens was approximately 8. So we must divide the number of droplets/mm on the film by 8 and multiply by 10 to obtain the number of droplets/cm along the actual path in the cloud chamber. We obtained a count of approximately 15 droplets/mm on the film for minimum particles. This corresponds to approximately 19 droplets/cm in the cloud chamber. Although we are only concerned here with the relative specific ionization in order to check the shape of the curve, it is of some interest to compare our specific ionization with the theoretical value. In order to do this, one must consider the number of electrons in the ethyl alcohol and water mixture which has approximately 3.5 cm of Hg vapor pressure. There are twelve times as many electrons in the alcohol molecule as in the helium. Consequently it forms about 12 times as many ions per cm of pressure. The pressure of helium in the cloud chamber is approximately 98 cm of Hg. Thus the total helium equivalent pressure is approximately $98 + 12 \times 3.5 = 140$ cm. Taking this into account and reducing our measurements to atmospheric pressure, we get $I_{N.T.P.} = 5.1$ ion pairs/cm. If we assume that each ion pair represents a loss of 27 ev, we obtain from the theoretical value for k_η , $I_{N.T.P.} = 8.0$ ion pairs/cm. Thus one sees that droplets form on slightly over half of the ions produced. This is only a rough calculation including an approximate correction for the presence of the alcohol. It checks quite well, however, with our experimental estimate of 75 percent efficiency of droplet formation based on a few split tracks. This efficiency could be increased by increasing the expansion ratio of the chamber, but this would cause too much background.

The absolute theoretical values of k_η have already been checked by experiment for low momentum particles so we are interested only in checking the shape

of the curve to see if it rises above minimum for high momentum particles. Thus we shall use the specific ionization measured in droplets/mm on the film and normalize the theoretical curve to these units. The density of mesons in the region from 70 to 250 Mev/ c is greater than for any other value of momentum due to our preferred selection in this range. Thus we chose our normalization point at $p/\mu=2.0$. The average droplets/mm for the range of p/μ from 1.70 to 2.50 is 15.75 ± 0.35 droplets/mm based on thirty particles. k_η evaluated at $p/\mu=2.0$ and compared to this average introduces a normalization constant equal to 12.33. The curve

$$k_\eta(\text{droplets/mm}) = 12.33(\text{Mev/g-cm}^{-2})$$

is plotted in Fig. 2 and compared to the 77 points representing the particles which had measurable momentum. It should be noted that we would have obtained essentially the same normalization constant if we had normalized k_η at minimum. There were 15 particles which had values of p/μ between 2.0 and 7.0. These limits correspond to a 5 percent rise above minimum on either side of minimum. Thus, one would expect the average ionization of the particles to be about 2 percent above minimum. Using the normalized curve, one obtains $k_\eta=14.7\pm 0.35$ droplets/mm at minimum. This gives $k_\eta=15.0$ when evaluated at 2 percent above minimum. The actual average of the 15 particles obtained was 15.0 ± 0.5 . Thus the normalization constant obtained from these figures would be the same as that obtained at $p/\mu=2.0$, but with a somewhat larger error.

Figure 2 shows that our data agrees quite well with the theoretical curve for those particles whose momenta we can measure. In particular, our data exhibits the rise at low values of p/μ which is known to exist. The root mean square deviation for our data calculated from the deviation of each of the 77 particles from the theoretical curve is 1.85 drops/mm. This is the basis for the errors indicated in the preceding paragraph. The theoretical standard deviation due to fluctuations in energy loss for transfers of energy smaller than 300 ev per collision has been calculated using the Rutherford differential energy loss expression. For the energy lost by such transfers in an average path length for a typical meson, one finds the fluctuation to be 12 percent. Using 16.0 droplets/mm as a typical ion count this gives 1.92 droplets/mm as the theoretical fluctuation. Thus we see that our over-all fluctuation does not indicate any instrumental error. Furthermore, the fluctuation in droplets/mm from day to day was not larger than was to be expected on the basis of statistical fluctuation. This is shown by the average deviation for each day of the 14 days on which data were taken. Eight of these days each had an algebraic average deviation smaller than the standard deviation for the number of particles obtained that day. Five had a deviation greater than 1 standard deviation, and only

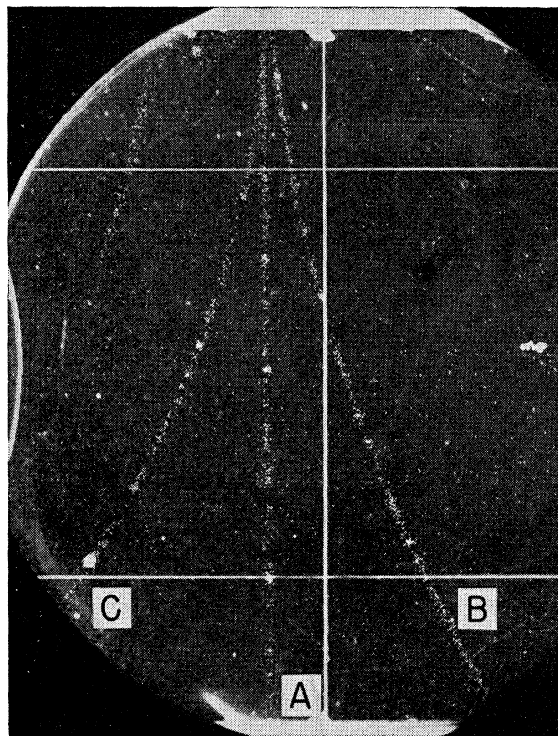


Fig. 3. Reproduction of a cloud-chamber picture showing a relativistic meson (A) accompanied by a high energy electron-pair (B), (C).

1 had a deviation greater than 2 standard deviations. Thus we feel that our operating procedure achieved its purpose of giving reproducible operation. This enabled us to group all our data together with no normalization from day to day.

In Fig. 2, it should be noted that there are four electrons which exhibit the relativistic rise. These were distinguished as electrons by their low momentum (30 to 45 Mev/ c) and by the fact that none of them appeared singly. Two appeared as tracks diagonally across the chamber, not through the telescope, and not accompanied by counter selected mesons. The other two are of particular interest. They appeared in a single picture accompanied by a meson with $p/\mu=11.8$. Thus complete data could be taken on all three tracks. This picture is reproduced in Fig. 3. We were very fortunate to have all three tracks lie in the plane perpendicular to the axis of the camera. This put all three tracks completely in the lighted region of the chamber and in good focus. The event was probably caused by the radiation of a γ -ray by the meson A in the lead above the chamber. Then the γ -ray must have materialized in the wall of the cloud chamber to form the pair B and C. (The stereoscopic views show that B and C originate from one point approximately coincident with the cloud-chamber wall.) The data for this picture are given in Table I. These data show the extent to which the chamber is capable of detecting

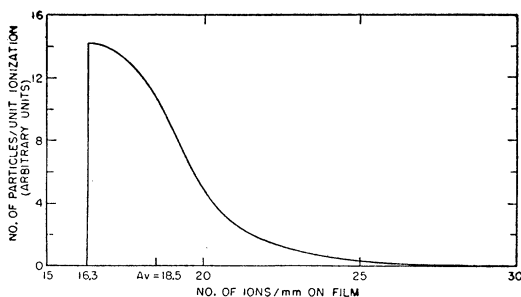


FIG. 4. Theoretical differential ionization distribution of mesons, uncorrected for statistical fluctuation in energy loss.

the small change in ionization due to the relativistic rise. The picture is also a good example of the quality of track that we had to count. The background is negligibly low and the droplets quite distinct.

Most of the 52 particles whose momenta are greater than 1500 Mev/c must be mesons. Single electrons are eliminated by the 4 inches of lead above the chamber. A very few protons may be included in this number, but it is known that there are very few protons compared to the number of mesons in this region. Therefore we may treat the 52 particles as mesons. The inclusion of 3 or 4 protons would serve only to decrease the average ionization slightly, but certainly could not introduce a false rise. Assuming all 52 particles to be mesons, they all have a $p/\mu > 14.3$. Figure 2 shows that they should all have ionization greater than 16.3 according to theory. The average of the experimental data on these 52 particles is 17.9 ± 0.25 droplets/mm. The error indicated is the standard deviation to be expected based on an individual experimental root-mean-square deviation of 1.85 droplets/mm. The minimum is 14.7 droplets/mm. If there were no rise at all, this average would be 12 standard deviations removed from the value at minimum which one should expect. The chance that this is actually the case is only 1 in 450 million. Thus this separation alone shows the existence of a rise.

The experimental data can best be compared with theory, however, by considering the distribution of the

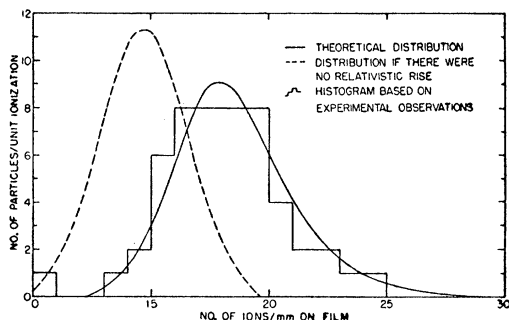


FIG. 5. Experimental differential ionization distribution for mesons compared with the theoretical distribution corrected for the statistical fluctuation in energy loss.

particles in the various ranges of ionization. Figure 4 shows the theoretical distribution for mesons with $p/\mu > 14.3$, with no account taken of the fluctuation in ionization. The ordinate is the number of particles (arbitrary units) per unit range of ionization, plotted against ionization. The curve is based on the momentum distribution of mesons as given by Rossi,¹⁶ and upon the theoretical ionization expected for each of several momentum ranges. The theoretical curve cuts off sharply at 16.3 droplets/mm since this corresponds to the ionization of a meson with $p/\mu = 14.3$. All mesons with lower p/μ are eliminated from the 52 by direct measurement of their momenta. The average ionization for mesons in this region is obtained from Fig. 4 and is 18.5. This agrees fairly well with the experimental average of 17.9 when, in addition to the statistical fluctuation of ion count, one includes the possibility of a variation in the momentum spectrum of the 52 particles from the true distribution.

A direct comparison is given in Fig. 5 between the experimental histogram representing the distribution of the 52 particles having $p > 1500$ Mev/c and the theoretical curve representing the expected distribution. The theoretical curve is that of Fig. 4 modified

TABLE I. Comparison of theoretical and experimental relative ionization for a meson *A* and two electrons *B*, *C*.

Track	p/μ	Droplets/mm	I/I_{\min}	
			Exp.	Theory
<i>A</i>	11.8	15.6	1.06 ± 0.13	1.07
<i>B</i>	116	22.1	1.50 ± 0.18	1.33
<i>C</i>	84.1	19.0	1.29 ± 0.15	1.29

to take account of the actual fluctuation in ionization of a particle with a finite path length. The fluctuation is approximately Gaussian and the experimental root-mean-square deviation of 1.85 droplets/mm is used in the calculation. The theoretical curve is normalized to have the same area as the histogram, thus representing the expected distribution of 52 particles. It should be pointed out that this curve is based purely on ionization theory, accepted momentum distribution of sea-level cosmic-ray mesons, and the experimentally determined droplet count for minimum ionization. Therefore, its good agreement with the experimental histogram is a true measure of the correctness of the predicted rise of the rate of ionization in the relativistic region. Figure 5 also includes a dotted curve which represents the distribution of the 52 particles to be expected if there were no rise at all. The histogram agrees well with the curve representing the existing theory and is far removed from the dotted curve. It should be pointed out that the spectrum of mesons at sea level contains such a small proportion of mesons with $p/\mu > 100$ that any decrease in the rate of ioniza-

¹⁶ B. Rossi, *Revs. Modern Phys.* **20**, 537 (1948).

tion due to the effect of polarization would not be detected in the present experiment.

CONCLUSION

The present paper presents experimental evidence for the existence of the theoretically predicted relativistic rise in the rate of ionization up to values of $p/\mu=100$. Our results are in disagreement with the results found by Goodman, Nicholson, and Rathgeber¹⁴

using a proportional counter, but they are in good agreement with the results of Ghosh, Jones, and Wilson¹⁵ who used a cloud chamber for their ionization measurement.

We wish to thank the staff and personnel of the Harvard Physics Department who made this work possible, and in particular we want to thank Professor J. C. Street and Dr. G. M. Nonnemaker for their interest and many valuable suggestions.

The Stopping Cross Section of D₂O Ice*

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The energy loss of protons and deuterons in D₂O ice has been measured over the energy range $E_p=18-541$ kev using the double focusing magnetic spectrometer to measure the energy of the particles after they have traversed a known thickness of the ice target. One method of measurement is used to determine relative values of the stopping cross section as a function of energy, another method measures the absolute values. The results are in very good agreement with the values calculated from Bethe's semi-empirical formula. Possible sources of error are considered and the accuracy of these measurements is estimated to be ± 4 percent.

I. INTRODUCTION

KNOWLEDGE of the stopping cross section of the target material is essential in the measurements of nuclear cross sections if the target cannot be weighed to determine its thickness or if a thick target is used. In the course of making measurements of the $D(d,p)T$ cross section it seemed advisable to remeasure the stopping cross section for the D₂O ice targets, since the poor accuracy claimed for earlier measurements¹ would limit the accuracy with which the $D(d,p)T$ cross section could be measured. Further interest lay in the striking disagreement of the earlier measurements of the stopping cross section with the theoretical predictions. The double focusing magnetic spectrometer for charged particles is admirably suited to measurements of stopping cross sections, and this application of the instrument is described in this paper.

II. ABSOLUTE VALUE MEASUREMENTS

When a beam of protons is scattered from a thick target, the momentum spectrum of the scattered protons, as observed in the magnetic spectrometer, is made up of a succession of steps, one for each isotope in the target. Such a step spectrum, for protons scattered from O¹⁶ in D₂O, is shown by the solid curve in Fig. 1. The height of the step N_{\max} , is proportional to the scattering cross section ($d\sigma/d\omega$) and inversely proportional to the effective stopping cross section ϵ_{eff} of the target material. N_{\max} can be measured easily since the

top of the step is nearly flat, and if ($d\sigma/d\omega$) is known ϵ_{eff} can be determined from the following expression²

$$\begin{aligned} \epsilon_{\text{eff}} &\equiv \epsilon(E_1)(\partial E_2/\partial E_1) + \epsilon(E_2)(\cos\theta_1/\cos\theta_2) \\ &= (d\sigma/d\omega)_{\theta_c}(E_2Q/N_{\max})(\Omega_c/R_c)4\pi \times 10^{-15} \text{ ev-cm}^2, \end{aligned}$$

where ($d\sigma/d\omega$) _{θ_c} is the scattering cross section in millibarns per steradian at the angle θ_c in the center-of-mass system; Ω_c is the solid angle subtended by the aperture of the magnetic analyzer at the target (c.m. system); R_c is the momentum resolution, $P/\Delta P$, a known function of the geometry of the spectrometer. N_{\max}/Q is the number of scattered particles of energy E_2 (measured in ev) counted in the spectrometer per Q microcoulombs of protons incident on the target, and N_{\max} is to be evaluated for particles scattered from the front surface of the target. The molecular stopping cross section for protons of energy E is given by $\epsilon(E) = (1/N)dE/dX$, where N is the number of molecules per cubic centimeter and dE/dX is the energy loss per cm of path for protons of energy E . The subscripts 1 and 2 refer to incident and scattered particle, respectively. θ_c is the angle of scattering in the center-of-mass system; θ_{1ab} and θ_1 and θ_2 are defined in the diagram of the target geometry in Fig. 2.

The ϵ_{eff} for protons in D₂O ice was determined by measuring the N_{\max}/Q for protons scattered from the oxygen in a thick D₂O ice target. The ice was condensed on a copper target cooled with liquid nitrogen by letting a jet of water vapor containing 99.8 percent

* Assisted by the joint program of the ONR and AEC.

¹ Previous measurements of D₂O are summarized by A. P. French and F. G. P. Seidl, *Phil. Mag.* **42**, 537 (1951).

² Snyder, Rubin, Fowler, and Lauritsen, *Rev. Sci. Instr.* **21**, 852 (1950).

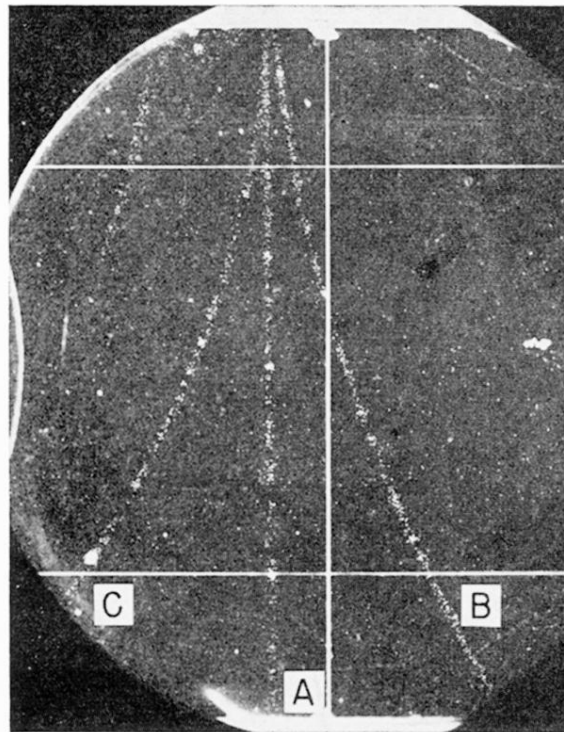


FIG. 3. Reproduction of a cloud-chamber picture showing a relativistic meson (*A*) accompanied by a high energy electron-pair (*B*), (*C*).