100.3-kev level if their order of decay is reversed) because of the uncertain state of knowledge concerning the K/L internal conversion intensity ratio for the 100.3-kev transition or the conversion coefficients of either transition.

The fact that the 60.3-kev transition belongs to Nd^{144}, as proposed by Keller and Cork,[4] is verified by our results. The observed intensity, however, appears insufficient for it to follow a large fraction of the Pr^{144} betas. It is, however, possible for it to represent a transition to the ground state of Nd^{144} from a level 60 kev above the ground state level without changing the shape of our spectra. A calculation of the F-K plots resulting from the addition of two spectra whose end-point energies are 2.97 and 2.91 Mev, respectively, shows no distinct change from a straight line except in the region in the immediate vicinity of the end-point energy. This is the result regardless of the relative abundance of the two groups. Our investigations of the beta-spectra, therefore, are not inconsistent with such a scheme.

The present investigations indicate that the difference in ft values for ground-state to ground-state transitions is real. For the disintegration scheme proposed here the values are such that $\log ft=7.43$ for the Ce^{144} ground-state transition (assuming 70 percent abundance), $\log ft \geqslant 7.03$ for Ce^{144} decay to the 134.2-kev excited state of Pr^{144} (assuming $\leqslant 30$ percent abundance), and $\log ft=6.53$ for the Pr^{144} decay (assuming 98 percent ground-state to ground-state transition). The 60-kev transition does not appear to have sufficient total intensity to alter the latter value appreciably.

The authors wish to thank Mr. O. H. Zinke and Mr. J. A. Whalen for making the scintillation spectrometer measurements.

PHYSICAL REVIEW VOLUME 87, NUMBER 3 AUGUST 1, 1952

The Electrical Conductivity in the Solar Atmosphere

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(Received April 14, 1952)

The electrical conductivity in the solar atmosphere is computed from the physical properties of the solar material. It is shown that as a consequence of the high abundance of hydrogen the mean free paths of the free electrons are determined by collisions with neutral hydrogen atoms and not by collisions with the much less abundant ions. Furthermore, for magnetic fields less than about 750 gauss the curvature of the trajectories of the free electrons is small enough for the ordinary Ohm's law to be a good approximation.

THE conditions under which convection can begin in a thermally unstable fluid that is a good electrical conductor have recently been studied by Chandrasekhar.[1] He finds that the presence of a magnetic field has a large effect in inhibiting the onset of convection and that the higher the electrical conductivity of the fluid is, the larger this effect is. One of the proposed applications of this theory is to the sun and to sunspots, and in this connection values of the electrical conductivity of the solar atmosphere are needed. The present paper contains calculations of these values.

It is well known that the electrons in a gas consisting of a mixture of electrons, ions, and neutral atoms, are responsible, on account of their small masses and consequently large velocities, for carrying practically all the current. Thus in finding the electrical conductivity of such a gas one is essentially finding the mean free paths of the electrons. Values of the conductivity of the solar atmosphere have been computed in the past by using the formulas derived for the case of an ionized gas, in which the mean free path of the electrons is set by the Coulomb scattering cross section of the positive ions. Now in the solar atmosphere the number of positive ions is relatively small, but the number of neutral hydrogen atoms is very large, so that in spite of the very large Coulomb cross sections of the ions, the effect of scattering by neutral hydrogen atoms may be expected to be of some importance. The elastic scattering cross section of a hydrogen atom is rather large at the low velocities of importance in the solar atmosphere and in fact the calculations summarized below show that the neutral hydrogen atoms reduce the mean free path of the electrons and hence the electrical conductivity by a factor of at least ten from the case in which only Coulomb scattering is taken into account.

To compute the electrical conductivity for electrons scattered by neutral hydrogen atoms, we shall use the formalism developed by Chapman and Cowling.[2] Collecting the formulas from their book (the notation is somewhat simplified here), we can reduce the calculation of the conductivity σ in the first approximation to the

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[1] S. Chandresekhar, Phil. Mag. (to be published).
[2] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases* (Cambridge University Press, Cambridge, 1939). The expression for the conductivity is given in Eq. 18.11, 5 of this book and all the other equations may be found by tracing the definitions of the various symbols in this equation.

following equations:

$$\sigma = 3n_e e^2/2na, \tag{1}$$

$$a = 8(\pi)^{\frac{1}{2}} m \int_0^\infty \exp(-g^2) g^4 \phi dg, \tag{2}$$

$$\phi = \int_0^\pi (1-\cos\chi)\alpha(v,\chi)\sin\chi d\chi. \tag{3}$$

Here n_e is the number of electrons per unit volume; n, the number of neutral atoms; e and m, the charge and mass of the electron, respectively; $g=(m/2kT)^{\frac{1}{2}}v$, where v is the velocity of an electron being scattered; and χ is the polar angle measured from the original direction of motion of the electron. The function $\alpha(v,\chi)$ is a weighting factor over the angle of scattering and has the form

$$\alpha(v,\chi) = (v/4j^2)\left|\sum_0^\infty (2l+1)(e^{2i\delta_l}-1)P_l(\cos\chi)\right|^2, \tag{4}$$

where $j=2\pi mv/h$ and δ_l is the phase shift of the lth partial spherical wave. In these equations and throughout this paper, the motion of the atoms is neglected in comparison with the motion of the electrons; that is, the approximation $1+m/M\approx 1$ is made, where $M=$ mass of a proton or hydrogen atom.

Now the kinetic energies of the electrons which are appropriate for conditions in the solar atmosphere are very low in comparison with the energy of the bound electron, and so we are interested only in the low energy limit of the cross section. The mean speeds of the electrons at temperatures of 4000°K and 7000°K, respectively, are 3.94×10^7 cm/sec and 5.43×10^7 cm/sec, which correspond to values of ja_0 of 0.18 and 0.25 ($a_0=$ Bohr radius of the hydrogen atom). Less than 2 percent of the electrons have speeds greater than twice the mean speed, so the only values of ja_0 of interest are those smaller than 0.5.

At these low energies only the s wave scattering ($l=0$) is of importance and in this case

$$\alpha(v,\chi) = (v/j^2)\sin\delta_0 = (v/4\pi)Q_0, \tag{5}$$

where Q_0 is the elastic scattering cross section, due only to s wave scattering at low energies. Substituting this value in the integral (3), we obtain

$$\phi = (v/2\pi)Q_0; \tag{6}$$

and hence, from Eq. (2),

$$a = 4(2\pi mkT)^{\frac{1}{2}} \int_0^\infty \exp(-g^2) g^5 Q_0 dg. \tag{7}$$

Now it must be remembered that Q_0, the elastic scattering cross section of a neutral hydrogen atom for electrons, is a function of v, the relative velocity of the electron with respect to the atom. Naturally this cross section has not been measured experimentally, but theoretically calculated values have been published by Massey and Moiseiwitsch[3] and the values used in the present paper are those recommended by these authors for the low energy range, namely those given in Table III, column (4) of their paper. They are the cross sections as calculated from the exchange-polarization approximation and averaged properly over the two possible orientations of electron spin.

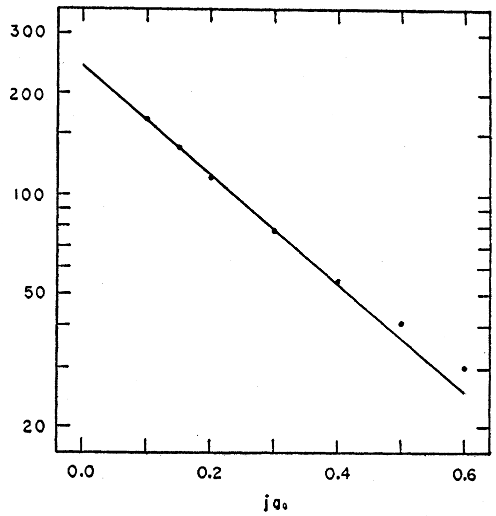

FIG. 1. The scattering cross section of a neutral hydrogen atom for electrons as a function of electron momentum, both expressed in atomic units. The circles are values computed by Massey and Moiseiwitsch; the straight line is the interpolation formula used in the present paper.

These values are plotted on a semilogarithmic scale in Fig. 1 and it can be seen that for the low energy range in which we are interested the cross sections may be represented very well by the empirical equation

$$Q_0 = 244 a_0^2 e^{-3.76 j a_0}. \tag{8}$$

If we change the independent variable from j to g, this becomes

$$Q_0 = Ae^{-\beta g}, \tag{9}$$

with

$$A = 244 a_0^2, \quad \beta = [(3.76)(137)/c](2kT/m)^{\frac{1}{2}}. \tag{10}$$

Therefore we have

$$a = 4(2\pi mkT)^{\frac{1}{2}} A \int_0^\infty \exp(-g^2-\beta g) g^5 dg$$
$$= 4(2\pi mkT)^{\frac{1}{2}} A \exp(\beta^2/4) \int_0^\infty \exp[-(g+\beta/2)^2] g^5 dg. \tag{11}$$

This integral has been evaluated from the tables of $\mathrm{Hh}_5(x)$, tabulated in the British Association tables.[4]

[3] H. S. W. Massey and B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A205**, 483 (1951).

[4] *Mathematical Tables*, Volume I (London: Office of the British Association, 1931).

TABLE I. Electrical conductivity.

Temperature	Scattering by H alone	Scattering by ions alone
4000°	3.78×10^{11} sec^{-1}	3.94×10^{12} sec^{-1}
5000°	3.73×10^{11} sec^{-1}	5.92×10^{12} sec^{-1}
6000°	3.68×10^{11} sec^{-1}	8.36×10^{12} sec^{-1}
7000°	3.74×10^{11} sec^{-1}	1.13×10^{13} sec^{-1}

The conductivities for the solar atmosphere can immediately be computed once the values of a are known. In this paper the ratio of number of electrons to number of neutral hydrogen atoms has been taken for purposes of computation as $n_e/n=10^{-3.7}=2\times10^{-4}$, which is a representative value for the solar atmosphere[5] around the mean optical depth $\tau=0.6$. Computed values are given in Table I for $T=4000°$ to $T=8000°$ and for this value of n_e/n; the conductivity is of course simply proportional to n_l/n.

The conductivity resulting from the scattering of electrons by ions may easily be computed numerically from the formulas given by Cohen, Spitzer, and Routly.[6] This conductivity depends only weakly on the number of electrons per unit volume, logarithmically through the cut-off parameter but not directly on n_e/n. The values of p_e given in Münch's model solar atmosphere[5] with $\log A=3.8$ have been used to find n_e and hence the logarithmic factor for each temperature separately. The computed values of the conductivity are collected in Table I.

In the actual solar atmosphere both positive ions and hydrogen atoms scatter the electrons and the contributions of both effects must be taken into account in computing the electrical conductivity. The net conductivity σ is given by the relation[7]

$$1/\sigma=1/\sigma_1+1/\sigma_2, \tag{12}$$

where σ_1 and σ_2 are the conductivities as computed for the cases of scattering by H alone and by the ions alone, respectively. In the region in which only the metals are ionized the scattering by hydrogen atoms determines the conductivity entirely. It is only when the regions are reached in which hydrogen is 1 percent or more ionized, so that $n_e/n>10^{-2}$, that the scattering by ions begins to predominate. Of course, in the region in the sum where convection is strong, the hydrogen convection zone, hydrogen is appreciably ionized and the scattering by ions is the main effect, but in the outer solar atmosphere and in particular in the visible portions of sunspots, the neutral hydrogen atoms are responsible for limiting the conductivity.

According to the analysis by Cowling[8] of the conductivity of a gas in a magnetic field, the electric current density is not given by the ordinary Ohm's law

$$\mathbf{j}=\sigma\mathbf{E}, \tag{13}$$

but rather by an expression of the form

$$\mathbf{j}=\sigma^{\mathrm{I}}\mathbf{E}+\sigma^{\mathrm{II}}(\mathbf{H}\times\mathbf{E})/|\mathbf{H}|, \tag{14}$$

if a magnetic $\mathbf{H}$ is imposed. The direct and transverse conductivities are given in the case in which only the electrons contribute appreciably to carrying the current by the expression

$$\sigma^{\mathrm{I}}+i\sigma^{\mathrm{II}}=\sigma/(1-i\omega\tau). \tag{15}$$

Here $i=(-1)^{\frac{1}{2}}$, $\omega=eH/mc=$Larmor precession frequency of an electron, where e is the absolute value of the electronic charge and τ is the mean time between collisions of an electron, defined by the relation

$$\tau=(m/n_e e^2)\sigma. \tag{16}$$

For the case of the solar atmosphere adopting $n=10^{17}$ cm^{-3} as a representative value of the number of hydrogen atoms, we find $\tau\approx7.5\times10^{-11}$ sec over the whole range in which the conductivity is fixed by hydrogen. Accordingly, the effects of the magnetic field on the conductivity are small if $H\lesssim760$ gauss, since for fields this small $\omega\tau\lesssim1$. However, for the larger fields that occur in strong sunspots, this effect must be taken into account.

The electrical conductivity of interstellar matter is also of interest in connection with problems of interstellar magnetic fields. In H II regions in which hydrogen is ionized, the electrical conductivity is of course given by the standard Coulomb formula. One might expect, however, that in H I regions the scattering by neutral hydrogen atoms might reduce the electrical conductivity below the value given by this formula, for only the less abundant elements with ionization potential below that of hydrogen are ionized. Numerical calculation shows, however, that even in the most favorable case in which all the hydrogen is in the atomic form and the temperature has its maximum value[9] of 200°, the effect of the ions far outweighs that of the atoms. The reason of course is that the ions have very large scattering cross sections at these low temperatures.

I wish to thank Professor S. Chandrasekhar for his constant encouragement and many stimulating discussions during the course of preparation of this paper.

[5] The model solar atmosphere used is that given by G. Münch, Astrophys. J. **106**, 217 (1947).
[6] Cohen, Spitzer, and Routly, Phys. Rev. **80**, 230 (1950).
[7] See, for example, reference 8, page 465 in the limit $H=0$.
[8] T. G. Cowling, Proc. Roy. Soc. (London) **A183**, 453 (1945). The equation given as (8) below is on page 465 of this paper, expressed there in electromagnetic units.
[9] L. Spitzer, Jr., and M. Savedoff, Astrophys. J. **111**, 593 (1950).