The Polyneutron Theory of the Origin of the Elements

R. E. PEIERLS, K. S. SINGWI,* AND D. WROE†
Department of Mathematical Physics, The University, Birmingham, England
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Assuming as an early stage in the expansion of the universe a homogeneous fluid of nuclear density and low temperature, it is shown that, for reasonable values of the constants, this will, on expansion, leave the matter in the form of droplets of the same properties as those found in the Mayer-Teller "polyneutron" theory. However, this model leads necessarily to an abundance curve in which the amount of heavy elements is at least comparable to that of the light elements, contrary to experience.

INTRODUCTION

IN 1948 Mayer and Teller[1] discussed a new mechanism for the origin of heavy elements. In their theory the starting point is a condensed body of matter of density of the order of nuclear density, which they call "polyneutron." Its mass was assumed to be less than that of an ordinary star to avoid gravitational considerations. Briefly the Mayer-Teller mechanism is the following: Inside a polyneutron a small proportion (about 10^{-5}) of the neutrons will transform into protons, until the gain of energy due to the reduction of the kinetic energy of the neutrons and the increased binding energy are balanced by the kinetic energy of the β-electrons. In equilibrium the maximum kinetic energy of the electrons is about 9 Mev. Near the surface, the electrons will extend beyond the surface of the nuclear matter until the resulting electrostatic field prevents further extension. This extension of the electrons reduces the energy near the surface and results in a negative contribution to the surface tension, which may exceed the positive surface tension of the nuclear matter by itself and thus will lead to instability. Due to this surface instability large droplets of nuclear matter break off, which will increase their charge later by further β-decay and then undergo fission, giving rise to smaller secondary droplets. These evaporate neutrons and undergo further β-decay, until finally they stabilize into heavy elements.

This theory has had some measure of success in predicting the observed isotopic abundances of heavy elements. It provides an alternative to the theory of Gamow, Alpher, and Herman.[2] The theory does not deal with the light elements.

In order to derive these results Mayer and Teller had to make certain assumptions about the binding energy of neutron-rich nuclear matter and about the magnitude of its surface tension. They proposed no mechanism for the formation of the original polyneutron. In view of its instability, it is clearly not easy to imagine a mechanism for its formation.

In this paper we shall discuss a model in which it is assumed that the expansion of the universe can be traced back to a stage when the whole universe was filled uniformly with matter of nuclear density. As such a system expands, it will at one stage have the matter in the form of polyneutrons, as postulated by Mayer and Teller, and their arguments can be taken over directly for the subsequent history. We shall see, however, that our model cannot account for the fact that the heavy elements constitute only a small portion of the matter in the present universe.

I. DISCUSSION OF THE MODEL

We consider a universe initially at very high density. At that stage its expansion is likely to be very rapid. We cannot write any precise equations since the large value of the energy-momentum density may somewhat alter the usual equations of the expanding universe and also because at such extreme conditions new factors may come in. It is likely, however, that near the time when the density is equal to the equilibrium density of nuclear matter the rate of expansion is too rapid to allow β-decay to maintain the proportion of protons to neutrons in equilibrium.

The actual ratio of protons to neutrons must therefore depend on the previous history, but two typical cases are of interest: (a) that the matter originally consisted only of neutrons and that no appreciable number of protons were formed prior to the stage we consider; (b) that in the earlier stages when the higher densities would have made faster β-decay possible and favored a finite proton-neutron ratio, some protons were actually formed. In that case, we would reach equilibrium density with more than the equilibrium number of protons.

One might suspect that extended matter of such high density would be unstable in the sense of tending to collapse under its own gravity if its homogeneous distribution is disturbed. This point has been investigated by Wroe,[3] and it was found that there was no tendency towards such a catastrophic collapse.

As the expansion proceeds beyond the equilibrium

* On leave of absence from the University of Delhi, Delhi, India. Present address: University of Illinois, Urbana, Illinois.

† Now at the Wool Industries Research Association, Torridon, Headingley, Leeds 6, England.

[1] Maria G. Mayer and E. Teller, Phys. Rev. **76**, 1226 (1949); also Proceedings of the 1948 Solvay Conference, Brussels.

[2] Alpher, Herman, and Gamow, Phys. Rev. **75**, 332 (1949); R. A. Alpher and R. C. Herman, Revs. Modern Phys. **22**, 153 (1950).

[3] D. Wroe, Ph.D. thesis, University of Birmingham, 1951.

density for that particular mixture of neutrons and protons present at that time, the energy per particle will increase with increasing volume, thus giving rise to a tension.

Nuclear matter in a state of tension must be unstable, since the energy can be reduced by forming cavities. However, the formation of a small cavity requires energy and hence the process will involve the passage through a potential barrier. In Sec. II we shall discuss the properties of this barrier and show that the number of cavities formed will depend on the rate of expansion, but that within a wide range of conditions their mean distance will be sufficiently small to expect non-uniformities on a cosmic scale. The further development is then different for cases (a) and (b) defined above. In (a) when there are only neutrons present, the surface tension is positive, and the cavities will grow in a regular way, leaving the nuclear fluid ultimately in the form of large lumps which will separate, but will be stable until β-decay has raised the proton (and electron) concentration inside them to a value which makes the surface tension negative. From then on, the development follows the mechanism discussed by Mayer and Teller. In case (b) the presence of protons may make the surface tension negative from the start, and then the cavities will immediately form in complicated shapes and the whole matter will break up into a kind of foam, the ultimate outcome being droplets of the same kind and size as those resulting from the break-up of a polyneutron.

It is, however, essential for this picture that all the matter in the universe has passed through the high density stage, from which the heavy elements can be obtained by the process already discussed, whereas the light elements can be formed only from the evaporation neutrons and the protons into which they will decay. This process will be discussed in Sec. III, and it will be seen that the amount of evaporation is small so that the heavy elements would constitute most of the matter, whereas in fact they constitute something like one part in a million.

It follows, therefore, that the polyneutron model is tenable only if one assumes that only a small part of the matter in the universe has condensed into polyneutrons, but no model has so far been proposed which would explain their origin.

II. FORMATION OF CAVITIES

Assume nuclear matter at a specific volume exceeding the equilibrium value, so that there is a tension T. In order to form a spherical cavity of radius a we have to create a new surface of area $4\pi a^2$ and the increase in nuclear surface energy will be

$$4\pi a^2\sigma, \tag{1}$$

where σ is the nuclear surface tension. On the other hand, we gain, by releasing the tension, the energy

$$-(4\pi/3)a^3T. \tag{2}$$

In case (a), where we deal only with neutrons, there is no further term. In case (b), we have to allow for the electronic contribution to the surface tension, which is negative. For a small cavity, the reduction of energy is

$$-(4/3)\pi a^3 P,$$

where P is the pressure of a relativistic degenerate electron gas. Inserting for P, the contribution to the energy becomes

$$-(a/c\hbar)^3(E_0^4/9\pi), \tag{3}$$

where E_0 is the Fermi energy which for the equilibrium mixture is 9 Mev, according to Mayer and Teller. The net effect is to increase the tension by 1.2×10^{28} dynes/cm^2. We shall be interested in much larger tensions, and the electronic effect is therefore negligible for the formation of cavities; hence we need not distinguish the two cases at this stage.

The theory of the formation of a small cavity is given in the appendix, and the probability per unit volume and unit time is

$$w = b^{-3}(2\sigma/\rho l^3)^{\frac{1}{2}}\exp(-kT^{-7/2}), \tag{4}$$

[see Eqs. (A20) and (A21)] where k is given by

$$k = 336\sigma^4\rho^{\frac{1}{2}}/\hbar. \tag{5}$$

l is the distance between nucleons, and b is a length somewhat larger than l [see Eq. (A21)]. ρ is the density. The probability of a cavity being formed in a unit volume up to time t_1 is then

$$W(t_1) = \int_{-\infty}^{t_1} w(t)dt = \frac{2}{7}b^{-3}\left(\frac{2\sigma}{\rho l^3}\right)^{\frac{1}{2}}\left(\frac{T^{9/2}}{k\dot{T}}\right)\exp(-kT^{-7/2}), \tag{6}$$

provided the exponent is large. In (6) T and $\dot{T}$ refer to time t_1. The mean distance between cavities at time t_1 will then be

$$L \sim [W(t_1)]^{-\frac{1}{3}}. \tag{7}$$

The formation of cavities will continue until enough have formed to relieve the tension everywhere. It is important to make sure that this will not happen too early, since a large L would mean that after further expansion the matter would be in large pieces, with large gaps in between. If the scale were large enough to result in an appreciable curvature of space, the further development might lead to an universe quite unlike the one we know. However, the release of tension will spread only with a velocity which is practically equal to the velocity C of sound in the nuclear fluid, so an upper limit to L is obtained from the condition

$$C(t_1 - t_0) \sim L, \tag{8}$$

where t_1 is the time defined by (7) and t_0 the time when the tension first appears.

Let us assume for orientation that the equations of general relativity held good at the time under discussion. Since in our model the temperature is low, the expansion would be governed by the material pressure only. In that case the density of matter is related to time by[4]

$$\rho t^2 = 8\times 10^5 \text{ g sec}^2/\text{cm}^3. \tag{9}$$

Taking the equilibrium density ρ_0 as 10^{14} g/cm³ this gives

$$t_0 \sim 10^{-4} \text{ sec}. \tag{10}$$

If the tension is not too large, we may use the relation

$$T = C^2(\rho_0 - \rho), \tag{11}$$

and hence from (9)

$$\dot{T} = -C^2\dot{\rho} = 2C^2\rho_0/t. \tag{12}$$

Using the values $\sigma = 4\times 10^{17}$ dynes/cm², $\rho_0 = 10^{14}$ g/cm³, $C = 3\times 10^8$ cm/sec, $b \sim l = 3\times 10^{-13}$ cm, Eqs. (7) and (8) become

$$kT^{-7/2}\exp(kT^{-7/2}) = 2\times 10^{49}L^4, \tag{13}$$

$$k = 8\times 10^{106}, \tag{14}$$

$$L = 1.7\times 10^{-27}T. \tag{15}$$

The solution of these equations is

$$T \sim 10^{30} \text{ dynes/cm}^2, \quad L \sim 2\times 10^3 \text{ cm}. \tag{16}$$

This shows that the cavities are certainly copious enough to prevent any inhomogeneity on a cosmological scale. With the above value of T, we see from (11) that $\rho_0 - \rho \sim \frac{1}{10}\rho_0$, which justifies the use of (11), which is valid only for small expansions. The exponent of (4) becomes 136, which is large enough to justify our asymptotic formula (6). The radius, for which the energy of the cavity is just zero, is

$$3\sigma/T \sim 1.2\times 10^{-12} \text{ cm}, \tag{17}$$

or about four times larger than l. Hence the uncertainty implied in (A21) is not important. On the other hand, the incipient cavities are so small compared to their mean distance that the compression wave starting from each will be very weak before it has covered distance L. This justifies our approximation assuming that these expansion waves run with sound velocity. In fact, it is evident that these compression waves will be far too weak to relieve the tension completely, so that the formation of new cavities will continue much further than the point determined by our estimate. In the same direction goes the assumption, implied in (8), that the time available for the compression waves is fhe whole time for which a tension existed, whereas in tact most of the cavities will have formed just before the instant we consider. We conclude, therefore, that the hypothesis of a uniform breaking up of the nuclear fluid is reasonable, and that it would be compatible even with a considerably slower rate of expansion than that corresponding to (9).

[4] See the second part of reference 2.

III. LIGHT ELEMENTS

Starting from a cold neutron fluid we have seen how polyneutrons could be formed. We shall now attempt to make an estimate of the actual physical state of matter, i.e., the average temperature and density in the model at the time of the formation of heavy elements, and whether the model can give the right abundance distribution of elements.

We assume that the lifetime of a neutron against β-decay inside the polyneutron is roughly the same as that of a free neutron, i.e., 10^3 seconds. In fact, the lifetime in the beginning is much shorter than this but increases later on as the Fermi energy of the electrons increases with the increase of the electron concentration. The equilibrium concentration corresponding to the proton neutron ratio of 10^{-5} would be attained in a time roughly of the order of 10^3 seconds. Assuming that the zero of the time scale concides with the time of the starting of the expansion, it would take approximately 10^3 seconds before the break-up of the polyneutron starts. It would not be unreasonable to assume that the equations of general relativity are qualitatively correct at this stage of the expansion and later. We then find for the density of matter the formula

$$\rho_{\text{matter}} = 8\times 10^5/t^2,$$

or

$$\rho_{\text{matter}} = 0.8 \text{ g/cm}^3 \quad \text{for} \quad t = 10^3 \text{ seconds}.$$

Hence the average density of matter in our model at the time of the breakup of the polyneutron is roughly equal to that of water.

We shall now make a rough estimate, in our model, of the temperature which would develop as a result of the nonequilibrium processes. The surface instability of the polyneutron leads to the evaporation of droplets, the energy of formation of which has been calculated by Mayer and Teller as a function of the size of the droplets. According to these authors droplets whose total charge is less than 37 are not formed since their energy of formation comes out to be negative. As a particular case we shall consider a droplet of charge $Z = 100$. The energy of formation, from the formula of Mayer and Teller, of such a droplet, is roughly 400 Mev. The temperature T corresponding to an excitation U of a nucleus of mass number A is given by

$$U = KAT^2, \tag{18}$$

where both U and T are in units of Mev and K is about 0.1. Assuming that half the energy of formation appears as the excitation energy of the droplet and substituting the numerical values of U and $A(\sim 10^7)$ for the droplet under consideration, we have

$$T \sim 0.01 \text{ Mev}.$$

The excitation energy would chiefly lead to neutron evaporation. The number N of neutrons evaporated is roughly given by

$$N = AT^2/E_B, \tag{19}$$

where E_B is the average binding energy of a neutron inside the droplet. In deducing formula (19) the cooling of the droplet, as a result of neutron evaporation, is neglected. Substituting for AT^2 from (18) and for E_B the value 1 Mev in (19), we have

$$N = 10^3,$$

which is a negligible fraction of the total neutron content of the droplet. Each evaporated neutron has, approximately, a kinetic energy of 10^{-2} Mev.

The droplet is still very neutron rich. The lifetime of a neutron in such a nucleus is roughly of the order of 0.1 sec. The β-decay of the droplet would proceed until enough charge has accumulated to exceed the limit for fission. This limit is given by the well-known formula

$$(Z^2/A)_{\lim} = 45, \tag{20}$$

or numerically $Z \sim 10^4$ for $A = 10^7$. In other words the droplet undergoes fission when nearly 0.1 percent of the neutrons have changed into protons.

One can make an estimate of the energy released in the fission process. This energy would roughly be equal to the Coulomb energy of repulsion of two daughter droplets when in contact. The fission energy is therefore

$$E_f = \left(\frac{Ze}{2}\right)^2 \frac{1}{2R} = \frac{45}{8} \frac{e^2 A^{\frac{4}{3}}}{1.5 \times 10^{-13}} \sim 2 \times 10^5 \text{ Mev}, \tag{21}$$

using (20). Even if we assume that the whole of the fission energy goes to excite the fragments, the excitation energy would evaporate roughly 10^5 neutrons, i.e., 2 percent of the total neutron content. The average kinetic energy of the evaporated neutrons is roughly 10^{-2} Mev, which also gives the value of the temperature in our assembly. At the time of the formation of heavy elements our model gives the following physical conditions:

$$T \sim 10^{-2} \text{ Mev}, \quad \rho \sim 1 \text{ g/cm}^3.$$

The fission fragments are still neutron rich and would undergo β-decay until the limit for fission is reached again. In this way a few fissions would take place before the resulting fragments stabilize into known heavy nuclei. It is clear, however, from the above picture that the total mass of matter which remains in the form of heavy nuclei is much greater than the total mass of the available free neutrons which may later combine to form light elements. Thus our model gives a large abundance of heavy nuclei over the light ones. It therefore fails to account for the abundance of the light elements. We have been unable to find any simple modification of the scheme (e.g., by assuming a high initial temperature) which would lead to the right abundance of the light elements while still preserving the conditions for the Mayer-Teller mechanism of the formation of the heavy ones.

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APPENDIX

In order to calculate the probability of formation of a cavity, we need to know, for our purpose, an approximate wave function for the radial motion of the cavity.[5] We make use of the B.W.K. method to solve the Schrödinger equation.

From classical hydrodynamics the kinetic energy of a hole expanding in an incompressible liquid of density ρ is given by

$$E_{\text{kin}} = 2\pi\rho R^3 \dot{R}^2, \tag{A1}$$

where $\dot{R}$ is the radial velocity and R the radius of the hole. The potential energy is

$$V = 4\pi R^2 \sigma - (4/3)\pi R^3 T. \tag{A2}$$

The Hamiltonian of the system is

$$H = (P^2/8\pi\rho R^3) + V, \tag{A3a}$$

where P, the radial momentum, is given by

$$P = dE_{\text{kin}}/d\dot{R} = 4\pi\rho R^3 \dot{R}.$$

Symmetrizing (A3) we have

$$H = \frac{1}{16\pi\rho}\left(\frac{1}{R^3}P^2 + P^2\frac{1}{R^3}\right) + V. \tag{A3b}$$

The Schrödinger equation is

$$\frac{\hbar^2}{16\pi\rho}\frac{1}{R^3}\frac{d^2\psi(R)}{dR^2} + \frac{\hbar^2}{16\pi\rho}\frac{d^2}{dR^2}\left(\frac{1}{R^3}\psi(R)\right) + (E - V)\psi(R) = 0. \tag{A4}$$

We solve Eq. (A4) by using the B.W.K. method. Let the solution in the first approximation be

$$\psi = e^{\phi}. \tag{A5}$$

Substituting (A5) in (A4) and retaining terms of the appropriate order, we have

$$\phi' = \pm[R^3(V - E)/2A]^{\frac{1}{2}}, \tag{A6}$$

where the prime denotes the first derivative with respect to R, and

$$A = \hbar^2/16\pi\rho. \tag{A7}$$

[5] The problem of the eigenvalues of a hole has been considered by Auluck and Kothari (Proc. Cambridge Phil. Soc. **41**, 180, 1945) when the tension term is absent.

In the second approximation let

$$\psi = e^{\phi+\chi}. \tag{A8}$$

Substituting (A8) in (A4) and again retaining terms of the appropriate order, we have

$$\chi' = -\tfrac{1}{2}\phi''/\phi' + 3/2R,$$

which gives on integration

$$\chi = -\tfrac{1}{2}\log\phi' + \tfrac{3}{2}\log R.$$

Hence, in the region where $V > E$ the solution of Eq. (4) is

$$\psi = BR^{\frac{3}{2}}(\phi')^{-\frac{1}{2}}e^{-\phi}, \tag{A9}$$

where B is a constant and ϕ is given by

$$\phi = \int \left\{ \frac{R^3}{2A}\left(4\pi R^2\sigma - \frac{4\pi}{3}R^{\frac{3}{2}}T\right)\right\}^{\frac{1}{2}} dR$$

$$= \int R^3\left(\frac{\alpha}{R} - \beta\right)^{\frac{1}{2}} dR, \tag{A10}$$

where

$$\alpha = 4\pi\sigma/2A, \quad \beta = 4\pi T/6A. \tag{A11}$$

Integrating (A10), we have

$$\phi_{R_1} - \phi_0 = \frac{\beta^{\frac{1}{2}}}{24}\cdot\left(\frac{\alpha}{\beta}\right)^4 \left[\frac{15}{8}\tan^{-1}\left(\frac{\alpha}{R\beta} - 1\right)^{\frac{1}{2}} + \left(1 - \frac{R\beta}{\alpha}\right)^{\frac{1}{2}}\left\{\frac{15}{8} + \frac{5}{4}\left(\frac{R\beta}{\alpha}\right)^{\frac{1}{2}} + \left(\frac{R\beta}{\alpha}\right) - 6\left(\frac{R\beta}{\alpha}\right)^{\frac{3}{2}}\right\}\right]_0^{R_1},$$

where R_1 is the value of R at which the integrand is zero, i.e., $R_1 = 3\sigma/T$, or

$$\phi_{R_1} - \phi_0 = \frac{\beta^{\frac{1}{2}}}{24}\left(\frac{\alpha}{\beta}\right)^4\left(\frac{15\pi}{16} + \frac{5}{8}\right). \tag{A12}$$

Substituting the values of α, β from (A11) in (A12) and using (A7), we have

$$\phi_{R_1} - \phi_0 = \frac{5\pi}{8\hbar}\left(\frac{T\rho}{\sigma}\right)^{\frac{1}{2}}\left(\frac{3\sigma}{T}\right)^4\left(1 + \frac{\pi}{2}\right) = 168\frac{\sigma^4\rho^{\frac{1}{2}}}{\hbar T^{7/2}}. \tag{A13}$$

Since for our purpose we are not interested in finding the exact value of the constant B, which one can do by the appropriate fitting of the solutions in different regions, we make use of a dimensional argument to get an approximate value. Near $R=0$

$$\psi = BR^{\frac{3}{2}}(\phi')^{-\frac{1}{2}} \sim \frac{BR^{\frac{3}{2}}}{(\alpha^{1/2}R^{5/2})^{\frac{1}{2}}}e^{\phi},$$

since $\phi_{R\to 0}' \sim \alpha^{\frac{1}{2}}R^{5/2}$. Therefore

$$|\psi|^2 \sim B^2R^{\frac{1}{2}}/\alpha^{\frac{1}{2}}. \tag{A14}$$

The macroscopic approximation which we have used will fail for radii comparable with l, the mean nucleon distance. If we had solved the exact equation for the lowest state of an incipient cavity, the solution would have extended over distances of the order l. Hence we may take as normalization condition

$$\psi \sim l^{-\frac{3}{2}}, \quad R \sim l. \tag{A15}$$

From (A14) and (A15) we get

$$B^2 \sim (\alpha/l^3)^{\frac{1}{2}}. \tag{A16}$$

From Eq. (A4) the expression for the current is given by

$$S = (\hbar/8\pi\rho R^3)(\psi^*\partial\psi/\partial R - \psi\partial\psi^*/\partial R) \tag{A17}$$

or

$$S = (\hbar/8\pi\rho)B^2e^{-2\phi(R_1)}, \tag{A18}$$

which, on substituting the value of B^2 from (A16) and using (A11) and (A7), becomes

$$S = (2\sigma/\rho l^3)^{\frac{1}{2}}e^{-2\phi(R_1)}. \tag{A19}$$

Hence the probability of formation of a particular cavity per unit time is

$$w = (2\sigma/\rho l^3)^{\frac{1}{2}}e^{-2\phi(R_1)}, \tag{A20}$$

where $\phi(R_1)$ is given by $168(\sigma^4/\hbar)(\rho^{\frac{1}{2}}/T^{7/2})$.

We are interested in the rate of formation of cavities per unit volume; for this we have to multiply (A20) by a weight factor which is of the dimension of an inverse volume. Its inverse evidently cannot be less than the volume for a nucleon nor greater than the volume of a cavity when it has become energetically stable. We therefore multiply (A20) by b^{-3}, where

$$l < b < 3\sigma/T, \tag{A21}$$

and thus obtain Eq. (4) of the text.