An Optical Model for Nucleon-Nuclei Scattering*

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An optical model for the scattering of nucleons by nuclei has been investigated. The model is one in which the interaction is described by a complex square well potential $V(r)=-V_0-iW_0$, $r<R_0$. Comparison with experiment indicates that for the scattering of 18-Mev protons by Al, reasonable agreement is obtained using well parameters which are consistent with the requirement of the transparency model and of low energy neutron scattering. The boundary condition model of Feshbach and Weisskopf has also been examined and a region of quasi-equivalence indicated for the two models.

I. DESCRIPTION OF THE MODEL

THERE is an energy range for which the problem of nucleon-nuclei interaction is somewhat analogous to the scattering of light by a conducting glass sphere, e.g., a colloidal suspension of gold particles in a glass bead. Such a medium is conventionally described by introducing a complex index of refraction,

$$n^2=\epsilon+i(4\pi\sigma/\omega).$$

The dielectric constant ϵ and the conductivity σ are clearly macroscopic parameters which describe the average effects of a multitude of microscopic events. Although a rigorous treatment would require a detailed examination of the individual scattering processes taking place at each metallic particle, the use of macroscopic parameters is quite sufficient if only such features as the distribution of the scattered intensity and the total energy absorbed are desired. Similarly, it might be expected that the elastic scattering of nucleons by nuclei and the total absorption cross section could be described approximately by assigning macroscopic parameters to nuclear matter—at least at energies sufficiently great that many levels contribute. Consequently, we have investigated an optical model for nucleon-nuclei scattering in which a complex square well potential is used, this being equivalent to a complex index of refraction.[1] A similar model was used by Pasternack and Snyder[2] to discuss neutron scattering at 90 Mev.

The introduction of a non-Hermitian potential into the Hamiltonian means that the probability density is no longer conserved. If the potential is

$$V(r)=-V_0-iW_0, \quad r<R_0,$$

then the continuity equation reads

$$\partial\rho/\partial t+\mathrm{div}\mathbf{j}=-2\rho W_0/\hbar,$$

where $\rho=\psi^*\psi$ and

$$\mathbf{j}=(\hbar/M)\ \mathrm{Im}(\psi^*\nabla\psi).$$

Thus, per unit of its volume, the nucleus appears to absorb incident nucleons at the rate $2\rho W_0/\hbar$. This absorption corresponds, of course, to the over-all effect of any inelastic processes that can occur, and it is just the point of the model that these processes are taken into account through the macroscopic parameter W_0.

The actual calculation of cross sections proceeds as usual, using a partial wave analysis and the tables of Coulomb wave functions published recently by Breit and co-workers at Yale University[3] and the tables issued by the National Bureau of Standards. Before discussing the results, however, it is of some interest to compare the features of this model with a boundary condition model, proposed by Feshbach and Weisskopf,[4] in which the complicated phenomena occurring inside the nucleus are described in terms of surface values of the wave function. Specifically, the logarithmic derivative of the wave function at the surface of the nucleus is required to satisfy the boundary condition

$$R_0\left[\frac{\partial(r\psi)}{\partial r}\Big/r\psi\right]_{r=R_0}=-iKR_0,$$

where

$$K=k(1+V_0/E)^{\frac{1}{2}}, \quad k=(2mE/\hbar^2)^{\frac{1}{2}},$$

and R_0 is the radius of the nucleus. Since the logarithmic derivative is taken to be independent of angle this model cannot be regarded as exactly equivalent to the optical approximation. Indeed, for a complex well, the logarithmic derivative for the Lth partial wave, which

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[1] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).

[2] S. Pasternack and H. S. Snyder, Phys. Rev. **80**, 921 (1950).

[3] Bloch, Hull, Broyles, Bouricius, Freeman and Breit, Phys. Rev. **80**, 553 (1950).

[4] H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949).

we denote by Γ_L, is given by

$$\Gamma_L = 1 + z j_L'(z)/j_L(z),$$

where

$$z = u + iv = kR_0[1 + (V_0 + iW_0)/E]^{\frac{1}{2}},$$

and j_L is the regular spherical Bessel function of order L. However, in the limit $|z| \gg L$ we note, on using asymptotic forms for the Bessel functions, that

$$\Gamma_L \simeq v - iu; \quad u, v \gg L.$$

Since Γ_L is essentially independent of L, we see that in this limit the optical model leads to a value for the logarithmic derivative of the entire wave function which is at least approximately independent of angle, as was assumed in the boundary condition model, and accordingly the cross sections can turn out to be quite similar for the two cases at appropriate energies.

II. COMPARISON FOR ALUMINUM

Experimental results for the elastic scattering of protons by Al, Ni, Pd, and W at 18.6 Mev have recently been published.[5] Relative angular distributions were obtained from 26° to 106° at 15° intervals. An attempt has been made to fit these data for Al[6] using both the optical model and the boundary condition model. In making this fit, the parameters of the well have not been permitted unrestricted variation since some information exists which serves to limit them. The radius R_0 was not regarded as adjustable at all, but chosen according to the accepted relation

$$R_0 = 1.42 \times 10^{-13} A^{\frac{1}{3}} \text{ cm}.$$

With respect to the well depth V_0, only two values were considered in any detail. One was the Fermi gas value of 30 Mev which has been found to be consistent with the requirements of the transparency model for high energy scattering.[7] The second was the value of 45 Mev obtained by Bohm and Ford in their study of slow neutron resonances.[8] With respect to W_0, its range of values was determined as follows. The transparency model gives a value of absorption coefficient $K = 2.4 \times 10^{12}$ cm^{-1} at 90 Mev. Assuming a gross $1/E$ dependence in the effective collision cross sections, this would indicate an absorption coefficient of about 5 or 6×10^{12} cm^{-1} at 18-Mev depending on the assumed well depth. Hence, only values of W_0 corresponding to absorption coefficients of this magnitude were used. It was then found a better fit could be made with the 45-Mev Bohm well than with a Fermi well. The best absorption depth was found to be $W_0 = 20$ Mev corresponding to an absorption coefficient $K = 6.2 \times 10^{12}$ cm^{-1} in reasonable agreement with the value extrapolated from the high energy transparency data. It was similarly found that the boundary condition model fit the data better with a Bohm rather than a Fermi well. In Fig. 1 the differential cross sections which give the best fit are shown. The experimental points have been normalized to give agreement with the optical model at 26°.

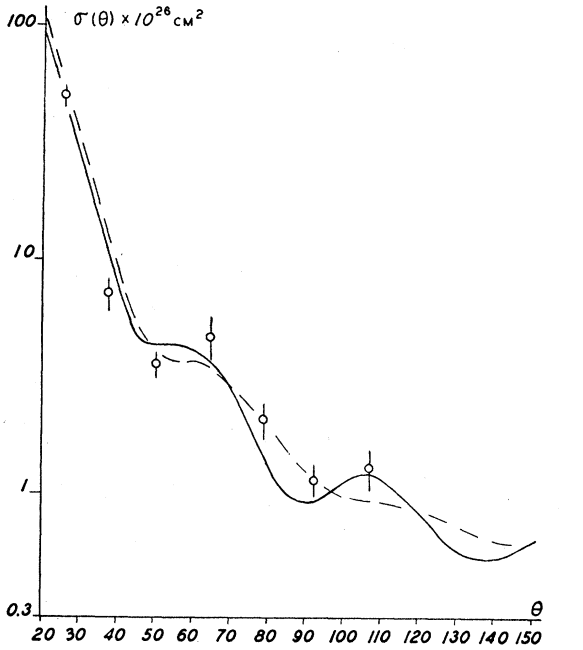

FIG. 1. Differential cross section *vs* angle of scattering. The solid curve is the optical model result, the dashed curve the boundary condition model result. The experimental points are from reference 5.

The reaction cross section predicted by each of the models is 0.60 barn ($\pi R_0^2 = 0.57$ barn) but unfortunately no experimental results are available as yet.

An examination of the curve seems to indicate that the optical model is in slightly better agreement with the experimental results than is the boundary condition model. This is rather pleasing, since the former seems to offer a more fundamental description than the latter and since the well parameters are reasonably consistent with those fitting other results at other energies as previously mentioned.

It is hoped that additional tables of Coulomb wave functions will become available in the near future so that comparisons can be made with experiment for the other elements. However, until experimental values for the absolute differential scattering cross sections and the reaction cross sections are determined, the success of this model is difficult to estimate.

[5] J. W. Burkig and B. T. Wright, Phys. Rev. **82**, 451 (1951).
[6] Because of limitations in the tables of Coulomb wave functions available, only phase shifts up to $L = 5$ can be calculated. For aluminum, $kR = 4.00$. Phase shifts in this case for $L = 6$ and 7 were estimated by means of a Born approximation. For tungsten, $kR = 7.60$ and phase shifts up to $L = 9$ or 10 are important.
[7] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[8] D. Bohm and K. W. Ford, Phys. Rev. **79**, 745 (1950).