The experiments were put aside at that time owing to the press of other work and approximately a year later they were again investigated with the help of a scintillation counter. In the present work two sets of experiments were performed.

In the first set of experiments, in order to get a good calibration point for this low energy radiation, the low energy radiations from Tl^{204} were compared with those of Os^{185}, a known K-capturer.[2,3] By using the line produced by Os^{185} (K_α of Re) as a calibration point, the radiation from Tl^{204} had an energy of 68 kev.

In a second set of experiments, the photon radiations from Tl^{204} were compared directly with those from Tl^{200}, which is known to decay to Hg^{200} by electron capture.[4] The results of these experiments, taken with a scintillation counter are shown in Fig. 1. It will be seen that the peak of the curve, owing to the K_α x-rays of Hg from Tl^{200}, falls at exactly the same place as that of the photons from Tl^{204}.

From the above experiments, it is to be concluded that the photons from Tl^{204} are the K_α x-rays of Hg and that Tl^{204} decays to Hg^{204} by K-electron capture. While these experiments were in progress, a similar conclusion was obtained by Lidofsky, Macklin, and Wu.[5]

† Supported by the joint program of the ONR and AEC.
1 Mitchell, Canada, and Cuffey (unpublished).
2 Bunker, Canada, and Mitchell, Phys. Rev. **79**, 610 (1950).
3 M. M. Miller and R. G. Wilkinson, Phys. Rev. **83**, 1050 (1951).
4 H. I. Israel and R. G. Wilkinson, Phys. Rev. **83**, 1051 (1951).
5 Lidofsky, Macklin, and Wu, Phys. Rev. **87**, 204 (1952).

A Proposed Re-Interpretation of Quantum Mechanics

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IN two recently published extensive papers,[1] Bohm has given a full discussion of an alternative interpretation of nonrelativistic quantum mechanics proposed by him. It is Bohm's opinion that there exists the possibility of looking on nonrelativistic quantum mechanics as a form of classical mechanics involving special quantum forces. While the bulk of the papers is devoted to various applications of this view, the fundamental point is contained in a few remarks concerning the solution of Schrödinger's equation, at the beginning of Bohm's first note.

Writing, as customarily,

$$\psi = R(x, y, z, t)\exp(iS/\hbar) \quad (R, S \text{ real}), \tag{1}$$

the Schrödinger equation is split into two equations:

$$(\partial R/\partial t) + (1/2m)(R\Delta S + 2\,\mathrm{grad}R\cdot\mathrm{grad}S), \tag{2a}$$

$$(\partial S/\partial t) + (1/2m)(\mathrm{grad}S)^2 + V(x, y, z, t) - \frac{\hbar^2}{2m}\frac{(\mathrm{grad}R)^2}{R}. \tag{2b}$$

Equation (2b) is looked upon as the Hamilton-Jacobi equation of the quantum-mechanical problem, in which the terms containing R and its derivatives are a sort of quantum potential energy. Since it is necessary to satisfy also (2a), Bohm simply suggests the use of an arbitrary solution ψ of the Schrödinger equation. One is then certain that both (2a) and (2b) will be satisfied.

We do not want to enter here into a discussion of the individual cases treated by Bohm. It is the thesis of the present letter that so far no possibility of a mechanical interpretation has been demonstrated.

To obtain the solution of a mechanical problem, it is necessary to have a solution S which depends on the coordinates of the system, the physical parameters entering into the expression for kinetic and potential energy and f nonadditive integration constants. Bohm has failed to show that such a function can be found.[2]

It is, of course, true that, thanks to the infinite number of eigenstates, as many constants as one wishes can be introduced into S; but these are not integration constants in the sense of the Hamilton-Jacobi theory, since they will in general also be present in the "quantum potential" described by R and its derivatives. There does not seem to exist a general formulation by which (2a) and (2b) can be satisfied and S still be made to contain f (nonadditive) integration constants that are not present in R. Until a way to find these integration constants has been devised, the similarity with the Hamilton-Jacobi theory is purely extraneous, and one cannot talk about a mechanical interpretation of the wave equation.

It may perhaps be kept in mind that the whole procedure suggested by Bohm is, even in its present incomplete form, strictly limited to nonrelativistic quantum mechanics and there is no indication given as to how it could possibly be extended to include relativistic (spin) phenomena; nor has an attempt been made in any of the examples to analyze the fundamental relativistic questions of observables and their measurability.[3]

Keeping in mind these factors together with the difficulties in the purely mechanical part of the interpretation, one may feel justified in concluding that no alternative interpretation of quantum mechanics has been offered.

1 D. Bohm, Phys. Rev. **85**, 166; **85**, 180 (1952).
2 To put the momentum vector equal to the gradient of S (as defined by Bohm) disposes of f integration constants in a nonmechanical manner which may be compared to the quantization of the phase-integrals in the old quantum theory.
3 O. Halpern and M. H. Johnson, Phys. Rev. **59**, 896 (1941).

Reply to a Criticism of a Causal Re-Interpretation of the Quantum Theory

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IN a letter[1] criticizing a causal re-interpretation of the quantum theory proposed by the author,[2] Halpern comes to the conclusion that "no alternative interpretation of the quantum theory has been offered." In the present letter, the author wishes to discuss in some detail the arguments by which Halpern is led to this conclusion.

Halpern's first objection is that because the function, $S(\mathbf{x})$, appearing in paper I, Eq. (6), does not depend on f nonadditive integration constants, one has not yet obtained a solution of the mechanical problem. It was stated in paper I, Sec. 4, however, that the Hamilton-Jacobi theory was being used only for the purpose of indicating in a simple way how one might arrive at a causal interpretation of the quantum theory; while the theory itself was to be based directly on the equations of motion [paper I, Eq. (8a)]:

$$md^2\mathbf{x}/dt^2 = -\nabla\{U(\mathbf{x}) + V(\mathbf{x})\},$$

where $V(\mathbf{x})$ is the classical potential, and $U(\mathbf{x})$ is the "quantum-potential." Now, the solution of these equations by the Hamilton-Jacobi technique would certainly require the f nonadditive integration constants referred to by Halpern, but the author intended to employ another method of solution in his papers; namely, that of guessing a function and verifying it by direct substitution in the differential equations. Since the treatment given in the papers is perhaps not completely explicit, it may be useful to amplify it here. Guided by the Hamilton-Jacobi theory, we guess tentatively that if the momentum were equal to $\mathbf{p} = \nabla S(\mathbf{x})$, this function would satisfy the equations of motion. Now,

$$d\mathbf{p}/dt = \partial\mathbf{p}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{p} = \partial(\nabla S)/\partial t + \nabla(\nabla S)^2/2m.$$

But since S is defined as the phase of the wave function, we have (paper I, Eq. (6))

$$\partial S/\partial t = -(\nabla S)^2/2m - V - U, \quad d\mathbf{p}/dt = -\nabla\{V + U\}.$$

Thus, we verify that a particle traveling with velocity $\mathbf{v}=\nabla S/m$ will satisfy our equations of motion.[3] This means that the condition $\mathbf{p}=\nabla S(\mathbf{x})$ is a consistent subsidiary condition. It was then pointed out in paper I, Secs. 4 and 9 that if we assumed this subsidiary condition to be satisfied initially, we would be able to obtain a mechanical explanation of the results of the quantum theory; if not, we obtained a wider theory. Since it was shown to be possible to choose this wider theory in such a way that $\mathbf{p}$ approached $\nabla S(\mathbf{x})$ at the atomic level, but differed significantly from it at the level of 10^{-13} cm, it was concluded that the wider theory might be needed in the domain of very small distances, where present theories seem to be inadequate.

Halpern then objected that the suggested reinterpretation was not relativistic and did not deal with spin phenomena. The author has, however, recently developed an extension of the causal interpretation which applies to the Dirac relativistic wave equation and therefore also takes care of spin phenomena. This interpretation will soon be submitted for publication.

As for Halpern's statement that no attempt has yet been made to analyze the question of the measurability of observables from a relativistic point of view, the answer is, of course, that this will now be done with the aid of the causal interpretation of Dirac's relativistic wave equation.

It is therefore the author's feeling that his previous articles[2] do in fact present a causal reinterpretation of quantum mechanics.

[1] O. Halpern, preceding letter, Phys. Rev. **87**, 389 (1952).
[2] D. Bohm, Phys. Rev. **85**, 166 (1952) (Paper I); **85**, 180 (1952) (Paper II).
[3] To solve for $\mathbf{x}$, one must, of course, integrate the differential equations $d\mathbf{x}/dt=\nabla S(\mathbf{x})/m$ with the appropriate initial position of the particles.

The Decay Scheme of the V^0-Particle*

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Observations of the V^0-particle are based on the detection of two charged decay fragments which in many cases are probably a proton and a negative pion.[1–3] Accordingly, the tentative decay scheme

$$V^0 \rightarrow p+\pi+Q^* \tag{1}$$

has been assigned with a Q^*-value given by various groups as ranging from 10 to 250 Mev, the majority of the Q^*-values falling between 30 and 50 Mev. A neutral decay product cannot be directly observed in the cloud chamber,[4] but would be expected to produce detectable dynamic effects on the observed decay fragments and their orientation with respect to the apparent origin of the V^0-particle.

If the V^0-particle decays according to the scheme

$$V^0 \rightarrow p+n_0+\pi+Q, \tag{2}$$

where n_0 is a neutral decay product, then the energy carried off by the charged particles and accordingly the apparent Q^*-value calculated for the assumed 2-particle decay scheme of Eq. (1) will depend on the energy taken up by the neutral particle. The relation between the true energy release Q and the apparent energy release Q^* is

$$Q^*=\frac{2(M+\mu)+Q}{2(M+\mu)+Q^*}\left(1-\frac{T_0}{T_0^{\max}}\right)Q, \tag{3}$$

where M and μ are the masses of the proton and pion, respectively, T_0 is the kinetic energy of the neutral particle in the true c.m. system, and $T_0^{\max}$ is the maximum possible value of T_0. In case $Q \ll 2(M+\mu)$, this exact expression can be approximated by the relation

$$Q^*=[1-(T_0/T_0^{\max})]Q. \tag{4}$$

In this approximation, since the proton is of large mass compared to the other decay fragments, it takes up very little kinetic energy and $Q^*=T$, the kinetic energy of the pion in the true c.m. system. Thus the probability distribution for Q^* is identical to that for T, which can be easily obtained if it is assumed that the partition of energy among the decay products is determined by the statistical factor alone, i.e., that the matrix elements which characterize the decay are constant. Thus, under the approximation that the proton is heavy, the result for the relative probability distribution is

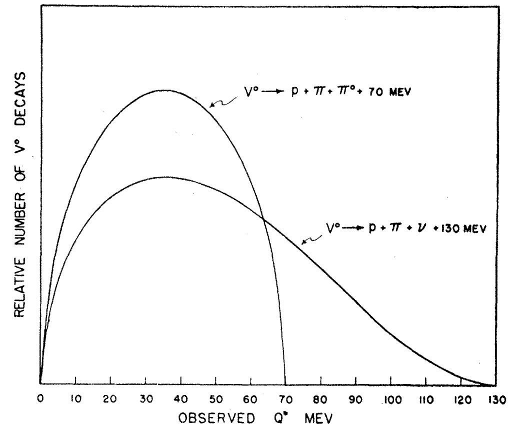

Fig. 1. Probability distribution of observed Q^*-values.

$$P(T)=EqE_0q_0, \tag{5}$$

where E, q, E_0, q_0 are the total energies and momenta of the pion and neutral particle respectively considered to be functions of the pion kinetic energy T with the restriction that $E+E_0=Q+\mu+\mu_0$. The distribution in Q^* is given in Fig. 1 for the cases of zero and mesonic[5] neutral particle mass. The true energy release Q has been adjusted to give a most probable value for Q^* of 35 Mev, to correspond approximately to experiment.

The problem of coplanarity can be treated similarly. If $\mathbf{p}'$ and $\mathbf{q}'$ are the momenta of the proton and pion in the laboratory system, then the angle δ between the plane of the charged particles and the true direction $\boldsymbol{\beta}/\beta$ of the V^0-particle is given by the expression

$$\sin\delta=(\mathbf{q}'\times\mathbf{p}')\cdot\boldsymbol{\beta}/(|\mathbf{q}'\times\mathbf{p}'|\beta). \tag{6}$$

This result can be simplified if, in the numerator, we substitute $\mathbf{p}'=\mathbf{P}_0'-\mathbf{q}'-\mathbf{q}_0'$, where $\mathbf{P}_0'$ is the momentum of the V^0-particle. In the denominator, we make use of the fact that for the decay of a rapidly moving V^0-particle, the velocity and direction of the decay proton are essentially the same as for the V^0-particle, so that $|\mathbf{q}'\times\mathbf{p}'|\cong|\mathbf{q}'\times\boldsymbol{\beta}|\gamma M$, where $\gamma=(1-\beta^2)^{-\frac{1}{2}}$. We then find, for small angles

$$\delta=\frac{1}{\gamma\beta M}\frac{\mathbf{q}'\times\boldsymbol{\beta}}{|\mathbf{q}'\times\boldsymbol{\beta}|}\cdot\mathbf{q}_0'. \tag{7}$$

Aside from the numerical factor $p'=\gamma\beta M$ in the denominator, the right hand side of this expression represents the component $q_{0\perp}'$ of $\mathbf{q}_0'$ perpendicular to the plane defined by the vectors $\mathbf{q}'$ and $\boldsymbol{\beta}$ and may be evaluated in the c.m. system. Thus

$$\delta \cong q_{0\perp}/(\gamma\beta M), \tag{8}$$

and the distribution of δ is identical to that for $q_{0\perp}$ except for the scale factor $1/\gamma\beta M$. In the present approximation that the proton is heavy, the distribution of $\mathbf{q}_0$ with the plane of $\mathbf{q}'$ and $\boldsymbol{\beta}$ considered fixed is spherically symmetric, so that the distribution of $q_{0\perp}$, for a particular magnitude of q_0 is given by

$$\begin{aligned} N(q_{0\perp})&=1/q_0, \quad q_{0\perp}<q_0; \\ &=0, \quad\quad\ q_{0\perp}>q_0. \end{aligned} \tag{9}$$

The actual distribution in $q_{0\perp}$ is obtained by integration over the