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Decay of a Neutral Scalar Heavy Meson

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The decay of a neutral scalar heavy meson into two pseudoscalar π-mesons is renormalized by the introduction of a contact interaction $\chi\Phi_s\varphi_{ps}^*\varphi_{ps}(x)$, where the finite part of χ is an empirical constant chosen to fit the observed lifetime for this decay. This interaction contributes to the decay into two γ-rays in order $G_sg_{ps}^2e$, and all contributions to this order have been calculated. The total fifth-order contribution is negative relative to the third-order matrix element. Consequently, for a particular relationship between the heavy and light meson coupling constants, the 2γ-decay lifetime will be e^4/g^4 slower than the 2π-decay lifetime. Hence, heavy scalar mesons strongly enough coupled to nucleons to be produced copiously at high energies could be sufficiently stable to be observed experimentally; in particular the V_2^0 particle may be of this type.

INTRODUCTION

THIS paper is an investigation into the stability of neutral, scalar, heavy mesons against decay into two pseudoscalar π-mesons or two γ-rays, undertaken in order to discover if it is possible to explain any of the recently discovered unstable particles in such terms. Experimentally, the Manchester group finds two groups of neutral V-particles which they interpret according to the decay schemes $V_1^0\rightarrow p+\pi^-$ and $V_2^0\rightarrow\pi^++\pi^-$,[1] although not all workers in this field agree with their analysis.[2] Accepting their analysis for the moment the V_1^0 must have half-integral spin and will not be discussed here. Further, since decay into two scalar particles is strictly forbidden by parity for a pseudoscalar particle (and decay into three pseudoscalar particles for a scalar particle),[3] pseudoscalar heavy mesons such as the rarely observed $\tau^\pm\rightarrow\pi^\pm+\pi^++\pi^-$ [4] will only be briefly considered at this time. Although numerical results will be quoted only for the V_2^0 mass and lifetime, the results can readily be extended to other unstable particles with the same decay scheme such as the ζ^0 meson[5] which are not yet easily studied by present techniques.

The difficulty in explaining any heavy particle energetically capable of decaying into π-mesons, which is coupled strongly enough to nucleons to be produced in large numbers, is that according to present field theories it would be expected to undergo such possible decays through virtual nucleon states in an unobservably short time. This problem has been previously investigated by Van Wyk using perturbation theory and by several Japanese authors using the Tomonaga-Schwinger formalism.[6] Most of the heavy boson decay schemes turn out to be divergent, making the interpretation of these results uncertain. The Japanese obtained finite answers by the use of Pauli regulators but found that in many cases the regulated results could not be made unambiguous. This is in accord with Matthews' result[7] that in fact no unambiguous prescription exists for theories that cannot be renormalized to all orders. However, the work of Salam[8] has shown that scalar and pseudoscalar meson theories without derivative couplings can be renormalized by the introduction of direct interaction terms into the Lagrangian. This means that in these cases finite and unambiguous results should be obtainable, and because of the possible importance of heavy mesons in the nuclear force problem, it seemed worthwhile to reinvestigate these cases in detail.

The lowest order process by which a scalar meson can decay into two π-mesons is logarithmically divergent but can be renormalized by the introduction of an (infinite) contact interaction between the heavy and light meson fields of the form $\chi\Phi_s\varphi_{ps}^*\varphi_{ps}(x)$ (Φ_s = heavy meson field; φ_{ps} = π-meson field). Since this interaction can contain a finite part, we obtain the uninformative result that any desired lifetime will be given by a correct choice of this constant. However, this interaction has further observable consequences that must be in agreement with experiment. A necessary but not sufficient condition is that the heavy neutral particle must also be sufficiently stable against decay into two γ-rays. This process is convergent to lowest order (Ge^2) and the only divergence (outside of the usual charge and mass divergences) that occurs in next order (Gg^2e^2) is precisely that divergence which is removed by the infinite part of the contact interaction term introduced to renormalize the 2π-decay. It might

[1] Armenteros, Barker, Butler, and Cachon, Phil. Mag. **42**, 1113 (1951).
[2] Proceedings of the Rochester Conference on Meson Physics (1951).
[3] In the center-of-mass system, decay into three or fewer spin zero particles depends at most on two vectors. Since no invariant pseudoscalar can be constructed from less than three vectors, the matrix element must be a scalar, and there can be no parity change.
[4] Fowler, Menon, Powell, and Rochat, Phil. Mag. **42**, 1040 (1951), see also for references.
[5] Danysz, Lock, and Yekutieli, Nature **169**, 364 (1952).
[6] C. B. Van Wyk, Proc. Phys. Soc. (London) **A62**, 697 (1949); S. Ozaki, Prog. Theoret. Phys. **5**, 373 (1950); Fukuda, Hayakawa, and Miyamoto, Prog. Theoret. Phys. **5**, 352 (1950); Sasaki, Oneda, and Ozaki, Prog. Theoret. Phys. **5**, 25 (1950).
[7] P. T. Matthews, Phys. Rev. **81**, 936 (1951).
[8] A. Salam, Phys. Rev. **82**, 217 (1951); see also J. C. Ward, Phys. Rev. **79**, 406 (1950) and F. Rohrlich, Phys. Rev. **80**, 666 (1950).

be argued that since it is necessary to introduce a direct interaction term in the case of the 2π-decay, it is unreasonable not to introduce a second direct interaction term in this case, even though it is not needed for the purposes of renormalization. However, the analogous interaction term in this case, $\chi'\Phi A_\mu A_\nu(x)$, is not gauge invariant, while the gauge invariant interaction $\chi''\Phi F_{\mu\nu}F_{\mu\nu}(x)$, because it involves derivatives, would be likely to introduce nonrenormalizable infinities in higher orders. This situation exists in spinor electrodynamics, where it is also possible to introduce direct interaction terms; here the empirical result is that the correct value for the Lamb shift, anomalous magnetic moment, etc., are obtained without such terms. Further, they would again be likely to cause divergences not removable by present techniques. Granted then that the interaction with the electromagnetic field should not contain direct interaction terms, the constant that occurs in the 2π decay is also the only unknown in the two process (to order Gg^2e^2), and it is no longer trivial to ask whether it is possible to pick this interaction in such a way as to make both decay lifetimes of the right order of magnitude.

The result of the calculation is that for the value of the interaction constant which renders the decay lifetime into two π-mesons long enough to fit the experimental facts, the particle can also be made sufficiently stable against decay into two γ-rays by postulating (in principle) a precisely determinable relationship between the coupling constants of the heavy and light mesons to nucleons. This demonstration has in fact been carried out to order Gg^2e^2 in the usual S matrix expansion in the coupling constants. However, provided only that this expansion converges, it can be argued that there will be a relationship between the coupling constants for which this stability exists even if all orders in the expansion are included. Thus, contrary to expectation, the instability of neutral, scalar, heavy mesons due to transitions through virtual nucleon states can be reduced sufficiently to make their lifetime as long as that of the observed $V_2{}^0$ particles.

DECAY INTO TWO PSEUDOSCALAR MESONS

The Feynman diagrams for the decay of a scalar, neutral meson into two pseudoscalar charged π-mesons are given in Fig. 1. After taking the spur the matrix element for this process is

$$\int d^3q d^3k_1 d^3k_2 (2\pi)^4\delta(q+k_1-k_2) \times\Phi(q)\varphi^*(k_1)\varphi(k_2)\frac{i(G_p+G_n)g^2}{(2\pi)^4}\int d^4t \times\frac{4iM(k_1k_2-t^2-M^2)}{[(t-k_1)^2+M^2][t^2+M^2][(t+k_2)^2+M^2]}, \quad (1)$$

where k_1 and k_2 are the four-momenta of the final π-

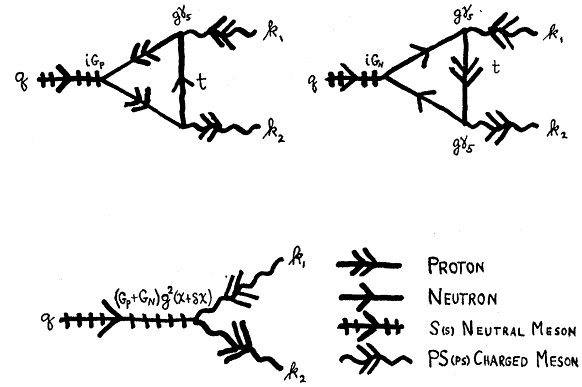

FIG. 1. Decay of a neutral scalar meson into two charged pseudoscalar mesons.

mesons, k_1-k_2 the four-momentum of the heavy meson, and hence $(k_1-k_2)^2=-K^2=k_1{}^2+k_2{}^2-2k_1k_2$; $2k_1k_2=K^2-2\mu^2$ (K=heavy meson mass, $\mu=\pi$-meson mass $=276m_e$, M=nucleon mass; for general notation see Appendix I). The t^2 term gives a logarithmic divergence, so that it is necessary to assume that the Lagrangian also contains a contact interaction of the form:

$$-(G_p+G_n)g^2(\chi+\delta\chi)\int d^4x\Phi\varphi^*\varphi(x), \quad (2)$$

where

$$\delta\chi=4iM(2\pi)^{-4}\int d^4t/(t^2+M^2)^2. \quad (3)$$

Adding the lowest order diagram due to this term and using the identity[9]

$$\int\frac{d^4t}{(2\pi)^4}\left\{\frac{1}{[t^2+M^2]^2}-\frac{1}{[t^2+M^2+C]^2}\right\} =\frac{\pi^2 i}{(2\pi)^4}\log\left(1+\frac{C}{M^2}\right), \quad (4)$$

the renormalized matrix element after integration over t becomes

$$iA\left\{\frac{(2\pi)^2M}{K^2}\chi-\frac{M^2}{K^2}\int_0^1 dx\log\left[1-\frac{K^2x(1-x)}{M^2}\right] -\left(\tfrac{1}{2}-\frac{\mu^2}{K^2}\right)\int_0^1 xdx\int_0^1 dy\frac{1}{D}\right\}, \quad (5)$$

where

$$A=\frac{(G_p+G_n)g^2K^2}{(2\pi)^2M}\int d^3qd^3k_1d^3k_2\Phi(q)\varphi^*(k_1) \times\varphi(k_2)(2\pi)^4\delta(q+k_1-k_2), \quad (6)$$

and

$$D=1-x^2y(1-y)K^2/M^2-x(1-x)\mu^2/M^2.$$

If the finite interaction constant χ were zero, this would

[9] R. Karplus and N. M. Kroll, Phys. Rev. **77**, 536 (1950).

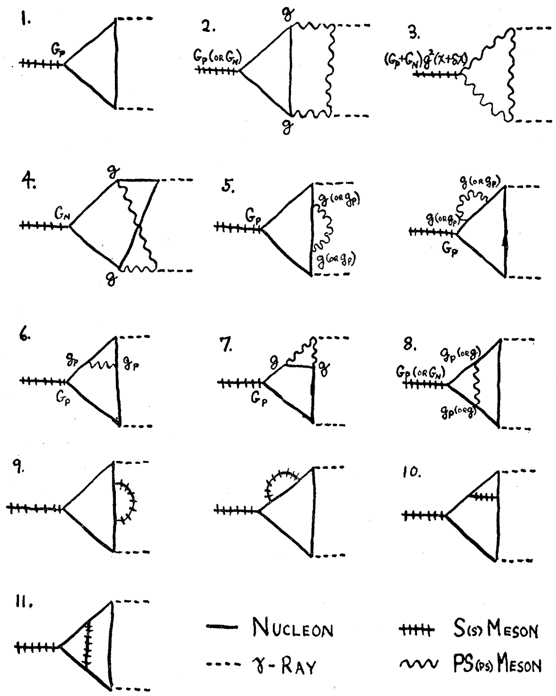

FIG. 2. Decay of a neutral scalar meson into two γ-rays to order Gg^2e^2 and G^3e^2.

give a decay probability of

$$\tau^{-1}\approx\frac{(G_p+G_n)^2g^4K}{(4\pi)^3}\left(1-\frac{4\mu^2}{K^2}\right)^{\frac{1}{2}}\left[\frac{(\frac{1}{6}-\mu^2/K^2)K}{8\pi M}\right]^2 \quad (7)$$

to the lowest order in an expansion in powers of K^2/M^2. It is interesting to note that this vanishes for $K=\mu\sqrt{6}=678m_e$, which number is only changed by about one electron mass if higher terms in the K^2/M^2 expansion are included. So that this particular renormalization prescription, if it could be justified on other grounds, would predict a unique stable mass for the heavy scalar meson. For the observed $V_2{}^0$ mass of about $800m_e$ the lifetime would be approximately $[(4\pi)^3/(G_p+G_n)^2g^4]\times 3.2\times 10^{-15}$ second, so that in order to obtain the observed lifetime of about 10^{-10} second, the finite part of the interaction constant must be chosen to be

$$\frac{\chi}{K}=\frac{K}{(2\pi)^2M}\left\{\frac{M^2}{K^2}\int_0^1 dx\log\left(1-\frac{x(1-x)K^2}{M^2}\right)+\left(\frac{1}{2}-\frac{\mu^2}{K^2}\right)\int_0^1 xdx\int_0^1 dy\frac{1}{D}\right\}\pm 1.6\times 10^{-4}. \quad (8)$$

This choice of χ affects the 2γ-decay, as will be discussed in the next section.

DECAY INTO TWO γ-RAYS

This process has already been investigated by Steinberger and by Fukada and Miyamoto,[10] who, after dropping nongauge-invariant terms, obtain for the lowest order matrix element

$$-iB\int_0^1 xdx\int_0^1 dy\frac{1-4x^2y(1-y)}{1-x^2y(1-y)K^2/M^2}, \quad (9)$$

where

$$B=G_pe^2K^2/(2\pi)^2M\int d^3qd^3k_1d^3k_2(2\pi)^4\delta(q+k_1+k_2)\times\Phi(q)A_\mu(k_1)A_\nu(k_2)(\delta_{\mu\nu}+2k_{1\nu}k_{2\mu}/K^2). \quad (10)$$

This term gives a decay probability of about

$$\tau^{-1}\approx G_p{}^2e^4K/(4\pi)^3(K/6\pi M)^2\approx 2.8\times 10^{16}(G_p{}^2/4\pi)\text{sec}^{-1}\quad\text{for}\quad K=800m_e,$$

which would be much too great for the scalar meson to exist long enough to be observed as a $V_2{}^0$ particle. (Schwinger also obtains this result by a different method.[11]) However, the interaction constant χ, which it was found necessary to introduce in order to give the observed lifetime for the decay into two π-mesons, contributes to this process in order Gg^2e^2 and because $g^2/(4\pi)$ is not small can significantly alter this conclusion. The Feynman diagrams to this order are given in Fig. 2. Number 1 is the lowest order diagram already discussed. Number 2 contains a logarithmic divergence that is canceled by the $\delta\chi$ term from number 3. Both of these matrix elements are complex due to the occurrence of two real π-mesons as an intermediate state in addition to the virtual states that do not conserve energy (Dyson's "displaced poles").[12] But χ has been picked to cancel the contribution from this part of the diagram when the two π-mesons are real. Consequently, it is to be expected that for this choice of χ the imaginary part contributed to the matrix element by diagrams 2 and 3 will cancel identically. Unfortunately, it has not proved feasible to evaluate exactly the fivefold integrals that are left after combining denominators by Feynman's famous formula and carrying out the four-dimensional integrations over intermediate momenta. However, the first term in an expansion of these integrals in powers of K^2/M^2 has been obtained, and to this order the expected cancellation of the imaginary parts does occur. The exact integrals for all the diagrams in Fig. 2 are given in Appendix II, and the approximations introduced to obtain the values given below are discussed there. It should be noted that although K^2/M^2 is about 0.2, the expansions actually involve $x(1-x)K^2/M^2$ with $0\leq x\leq 1$, so that the error made by dropping these terms is probably less than 5 percent, which is more than adequate for the purposes of this paper. Neglecting terms of order K^2/M^2, the total contribution from diagrams 2 and 3 is $[-iB(1+G_n/G_p)g^2/(4\pi)^2](-2/9)$. Diagram 4 is irreducible and

[10] J. Steinberger, Phys. Rev. **76**, 1180 (1949); H. Fukada and Y. Miyamoto, Prog. Theoret. Phys. 4, 347, 392 (1949).

[11] J. Schwinger, Phys. Rev. **82**, 664 (1951).

[12] F. J. Dyson, Phys. Rev. **82**, 428, 438 (1951), and lectures at Birmingham.

finite and gives a contribution $[-iBG_n g^2/G_p(4\pi)^2]\times(7/36)$. Diagrams 5 are obtained by self-energy insertions in 1. They therefore contain linear divergences which are canceled in the usual way by mass renormalization; the finite contribution is $[-iB(g^2+g_p^2)/(4\pi)^2](-0.0209)$. Diagrams 6, 7, and 8 are vertex insertions containing logarithmic divergences which are removed by using Dyson's prescription of replacing γ_μ by $\bar{\Gamma}_\mu$ and 1 by $\bar{1}$ in the original integral for diagram 1. They give rise, respectively, to the finite contributions

$$[-iBg_p^2/(4\pi)^2](-0.360);\ [-iBg^2/(4\pi)^2](0.242);$$
$$\text{and } [-iB(g_p^2+G_nG_p^{-1}g^2)/(4\pi)^2](-0.452).$$

Diagrams 9, 10, and 11 are the corresponding self-energy and vertex insertions due to neutral, scalar mesons in virtual states; they are functions of the heavy meson mass, even using the above-mentioned expansion, and have been evaluated for $K=800\ m_e$ giving a total finite contribution of $[-iBG_p^2/(4\pi)^2](-3.290)$.

Charged heavy mesons have not been included as their introduction would require an additional contact interaction $(\chi'+\delta\chi')\Phi_0\Phi^*\Phi(x)$, where χ' is a new empirical constant that would require the calculation of a third distinct process for evaluation. The diagrams contributed by charged heavy mesons to the 2γ-decay of the neutral heavy meson are given in Fig. 3. The neglect of these diagrams does not, however, alter the argument below, as they will contribute simply an additional term proportional to $-iBG^2/(4\pi)^2$ which merely increases the complexity of the relationship between coupling constants of heavy and light mesons for which cancellation occurs.

Collecting all the above contributions, the real part of the matrix element for decay into two γ-rays is

$$-iB\left\{\tfrac{1}{3}-\frac{g^2}{(4\pi)^2}\left[0.833\frac{g_p^2}{g^2}+0.672\frac{G_n}{G_p}-0.193\right]-3.290\frac{G_p^2}{(4\pi)^2}\right\} \quad (11)$$

and clearly can be zero for certain values of the coupling constants. In particular, for $g^2=g_p^2=g_n^2$ and $G_p=G_n$, this relationship is $g^2/4\pi+2.5G_p^2/4\pi=3.08$, which is clearly compatible with the usually assumed values for these constants. There is in addition a small imaginary term, corresponding to the interaction that allows a lifetime of about 10^{-10} second for the 2π-decay, but this clearly gives a lifetime for 2γ-decay approximately $g^4/e^4\cong 2\times 10^4$ longer. Therefore, for that value of χ which gives a reasonable lifetime for the 2π-decay, the probability of decay into two γ-rays is sufficiently small to insure that only the former process will be observed.

The extension of this calculation to higher order terms in the S matrix cannot be expected to alter this conclusion. As χ is evaluated as a power series in the coupling constants, the coefficients of the higher order terms can be chosen to keep the 2π-decay lifetime of the same order of magnitude. But since the identical diagrams will occur as the only real intermediate states in the 2γ-decay (neglecting higher terms in e), this choice will also insure that there will be negligible higher order contributions to the imaginary part of the 2γ-decay matrix element. Therefore, provided only the series converges sufficiently not to change the sign of the higher order terms relative to the third-order matrix element, there will still exist some (in principle) precisely determinable relationship between the heavy and light meson coupling constants for which the heavy neutral scalar meson will be more stable against decay into two γ-rays than into two pseudoscalar π-mesons.

FURTHER CONSIDERATIONS

It has already been pointed out that the inclusion of charged heavy scalar mesons will not alter this conclusion, although a new empirical constant enters the theory. Further, if empirical evidence should be obtained that the charged V particle decays into a charged and a neutral π-meson, the above calculation can be readily extended to this case, most of the integrals being the same as have already been evaluated.

The problem of whether the neutral scalar heavy meson is also sufficiently stable against other modes of decay is more serious. Decay into two γ-rays and a π-meson was found by Van Wyk and S. Power[13] to be of the order of $10^{-11}/G^2g^2$ seconds, which is compatible with the above relationship between the coupling constants, but might require rather too small a G to be reconciled with the copious production of V_2^0 particles. *A priori*, the decay into two π-mesons and a γ-ray would be expected to have a still shorter lifetime, and this process is now be inginvestigated.* Pais[14] has

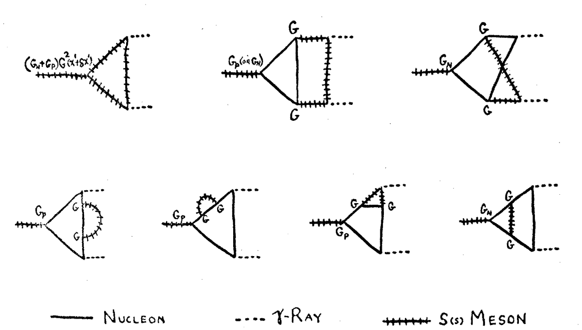

FIG. 3. Contributions of charged scalar mesons to the decay of a neutral scalar meson into two γ-rays in order G^3e^2.

[13] C. B. Van Wyk, Proc. Phys. Soc. (London) **A63**, 350 (1950); S. Power, Phys. Rev. **76**, 865 (1949).

* *Note added in proof:*—For $K=850m_e$ R. S. Grover finds $\tau^{-1}=(G_p+G_n)^2g^4(2\pi)^{-3}\times 4.6\times 10^{17}$ sec^{-1} for decay into two charged mesons and a γ-ray and $G_p^2g^4\,(2\pi)^{-3}\times 1.8\times 10^{18}$ sec^{-1} for decay into two π^0 mesons and a γ-ray. The former process can be forbidden by taking $G_p\approx -G_n$, but the latter makes the neutral scalar particle too unstable to be observed unless a cancellation similar to that discussed in this paper occurs when Gg^6e terms are included.

[14] A. Pais, Phys. Rev. **86**, 663 (1952); see also reference 2.

shown that this lifetime for the decay of a pseudoscalar heavy meson is of the order of $10^{-17}/G^2g^4$, but the contact interaction introduced to renormalize the 2π-decay contributes in the scalar case and could conceivably alter this situation in the right direction.

The extension of this approach to pseudoscalar heavy mesons does not look hopeful, unless one follows Pais in postulating a strong interaction allowing the heavy particles to be produced in pairs and a weak interaction governing their subsequent decay. In the first place the neutral particle would presumably decay into two γ-rays in a time shorter than the lifetime of the neutral π-meson or into a γ-ray and two π-mesons, as already indicated, but this may be actually the case, since there is at present no evidence for the existence of such neutral particles with long lifetimes. However, the relatively stable charged τ-meson that decays into three π-mesons has been seen. This decay in itself offers no problem, as it is renormalizable and hence can have the observed lifetime; the finite part of the renormalization interaction contributes to the $(\pi+2\gamma)$ decay in much the same way as the scalar renormalization contributes to the 2γ-decay, so this situation could well reproduce the features already discussed. But this interaction does not contribute to the $(2\pi+\gamma)$ decay in sufficiently low order to make calculation feasible, and since Pais' result for the lowest order matrix element gives such a short lifetime, it may well prove impossible to achieve sufficient stability against this decay. A detailed investigation of this case would be required in order to determine if any new features enter which could alter this conclusion.

CONCLUSION

The decay of a neutral scalar particle strongly coupled to nucleons into two pseudoscalar π-mesons can be renormalized by the introduction of a direct interaction between the heavy and light meson fields, the finite part of which can be chosen to given any desired lifetime for this process. If the interaction is chosen to fit the observed lifetime of the V_2^0 particle, the neutral scalar particle can also be made sufficiently stable against decay into two γ-rays to insure that only the decay into two π-mesons will occur appreciably. This stability can, however, only be achieved if one is willing to accept (in principle) a precisely determinable relationship between the coupling constants of heavy and light mesons to nucleons. The present estimates of the values for these coupling constants are compatible with this relationship. Therefore, the possibility that there can exist scalar mesons strongly enough coupled to nucleons to affect the nuclear force problem cannot be ignored. The approach used in this paper is likely to prove inadequate to explain the τ-meson.

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APPENDIX I. NOTATION

$$tx=t_\mu x_\mu=\mathbf{t}\cdot\mathbf{x}-t_0x_0;\ \boldsymbol{\gamma}=-i\beta\boldsymbol{\alpha};\ \gamma_4=\beta;\ \gamma_5=\gamma_1\gamma_1\gamma_3\gamma_4;$$
$$t=t_\mu\gamma_\mu;\ \bar{\psi}=i\psi^*\gamma_4.$$

$$\text{Lagrangian density}=-\tfrac{1}{2}[(\partial\Phi/\partial x_\mu)^2+K^2\Phi^2]$$
$$-\tfrac{1}{2}[(\partial\varphi_0/\partial x_\mu)^2+\mu^2\varphi_0^2]-\tfrac{1}{2}[(\partial\varphi^*/\partial x_\mu-ieA_\mu\varphi^*)$$
$$\times(\partial\varphi/\partial x_\mu+ieA_\mu\varphi)+\mu^2\varphi^*\varphi]+i\bar{\psi}_p[\gamma_\mu(\partial/\partial x_\mu$$
$$+ieA_\mu)+M]\psi_p+i\bar{\psi}_n[\gamma_\mu\partial/\partial x_\mu+M]\psi_n-iG_p\bar{\psi}_p\psi_p\Phi$$
$$-iG_n\bar{\psi}_n\psi_n\Phi-g_p\bar{\psi}_p\gamma_5\psi_p\varphi_0-g_n\bar{\psi}_n\gamma_5\psi_n\varphi_0$$
$$-g[\bar{\psi}_p\gamma_5\psi_n\varphi+\bar{\psi}_n\gamma_5\psi_p\varphi^*],$$

where $\psi_{p(n)}$=proton (neutron) wave function, mass M; Φ=scalar neutral meson wave function, mass K; φ=charged pseudoscalar meson wave function, mass μ; φ_0=neutral pseudoscalar meson wave function, mass μ; and A_μ=electromagnetic potential function.

Propagation of a fermion

$$\psi^\cdot(x)\bar{\psi}^\cdot(y)=-i\int\frac{d^4t}{(2\pi)^4}\frac{e^{it(x-y)}}{t-iM}.$$

Propagation of a boson

$$\varphi^\cdot(x)\varphi^{*\cdot}(y)=i\int\frac{d^4t}{(2\pi)^4}\frac{e^{it(x-y)}}{t^2+\mu^2}.$$

Transformation to momentum space

$$\Phi(x)=\int d^3qe^{iqx}\Phi(q).$$

APPENDIX II. CALCULATION OF MATRIX ELEMENTS

The gauge invariant contribution from diagram 2 is

$$S_2=-iB\frac{(1+G_n/G_p)g^2}{(4\pi)^2}\bigg\{\bar{I}_2+\frac{2i(1+J\mu^2/K^2)M^2}{\pi^2K^2}\times\int d^4t\frac{1}{[(t-k_1)^2+M^2][(t+k_2)^2+M^2]}\bigg\},\quad(12)$$

where

$$\bar{I}_2=2\int_0^1 xdx\int_0^1 dy\int_0^1 du\int_0^1 dv\int_0^1 w(1-w)dw \times\left\{\frac{a+b-8ab}{D}-\frac{K^2}{M^2}abx(1-x)\frac{1-a-b+2ab}{D^2}\right\}$$

$$\begin{aligned}&a=wvy+(1-w)u;\ b=wv(1-y)+(1-w)(1-u);\\&D=wv+(1-wv)x(1-x)\mu^2/M^2\\&\qquad-wvxy(1-y)K^2/M^2-abx(1-x)K^2/M^2;\end{aligned} \quad (13)$$

$$J=2\int_0^{\frac{1}{2}} dx\frac{\log|K^2(x^2-x_0^2)/\mu^2|}{\frac{1}{4}-x^2}-2\pi i\log\frac{1+2x_0}{1-2x_0};$$

$$x_0^2=\tfrac{1}{4}-\frac{\mu^2}{K^2}.$$

To order K^2/M^2, D may be written as

$$D_0=wv+x(1-x)\left[1-u(1-u)(1-w)^2\frac{K^2}{\mu^2}\right]\frac{\mu^2}{M^2}, \quad (14)$$

where the second term must be retained because of the pole that gives the imaginary contribution to the matrix element; in this expansion higher powers of K^2/M^2 will then have integrals of the same form as coefficients. By using care to drop all higher order terms during the integration and some algebraic manipulation $\bar{I}_2$ can be reduced to the form

$$\bar{I}_2=-2/9+(\tfrac{1}{2}-\mu^2/K^2)(1+J\mu^2/K^2). \quad (15)$$

The contribution from diagram 3 is

$$S_3=\frac{iB(1+G_n/G_p)g^2}{(4\pi)^2}(\chi+\delta\chi)\frac{(4\pi)^2M}{2K^2}\left(1+J\frac{\mu^2}{K^2}\right). \quad (16)$$

Using the value of $\delta\chi$ already assumed and the identity (4) gives for the total contribution due to both diagrams

$$S_2+S_3=\frac{iB(1+G_n/G_p)g^2}{(4\pi)^2}\left\{-\bar{I}_2+\frac{(4\pi)^2M}{2K^2}\left[\chi-\frac{4M}{(4\pi)^2}\times\int_0^1 dx\log\left(1-\frac{x(1-x)K^2}{M^2}\right)\right]\left(1+J\frac{\mu^2}{K^2}\right)\right\}. \quad (17)$$

But to order K^2/M^2,

$$\chi-4M\int_0^1 dx[\log(1-x(1-x)K^2/M^2)]/(4\pi)^2$$

is by Eq. (8), simply $2K^2(\frac{1}{2}-\mu^2/K^2)/(4\pi)^2M$, so that not only the divergent terms, but also the imaginary parts of these two diagrams cancel identically.

Diagram 4 is irreducible, and after dropping non-gauge-invariant terms, leads to the integral

$$S_4=\frac{-2iB(G_n/G_p)g^2}{(4\pi)^2}\int_0^1 du\int_0^1 dv\int_0^1 w(1-w)dw \times\int_0^1 x^2dx\int_0^1 dy\left\{\frac{1-16abw+14aw-12awv}{w(1-w)D_1}-\frac{K^2}{M^2}ab\frac{1-b+2aw+(8b-4)awv}{D_1^2}+\frac{2xy(v-1)b}{D_2}+\frac{2xyv(b-1)}{D_3}\right\}, \quad (18)$$

where $a=xy(v-u)$; $b=x(1-vy)$ and

$$D_i=xy-wv(1-v)K^2/M^2+(1-xy)w(1-w)\mu^2/M^2-bw(1-w)[a\delta_{1i}+xy(v-1)\delta_{2i}+xyv\delta_{3i}]K^2/M^2,$$

$i=1, 2, 3$.

Diagrams 5 are obtained by replacing $1/(t-iM)$ by

$$\frac{(g^2+g_p^2)}{(4\pi)^2}\int_0^1 dx\int_0^1 dy\left\{\frac{x(1-x)[-iMtx+(1-x)t^2+M^2]}{\Lambda_0^2+yx(1-x)[t^2+M^2]}-\frac{2M^2x^2(1-x)}{\Lambda_0^2}\right\}\frac{1}{t-iM}, \quad (19)$$

where

$$\Lambda_0^2=x^2M^2+(1-x)\mu^2=M^2\lambda_0^2,$$

in diagram 1 [plus corresponding insertions for $1/(t-k_1-iM)$ and $1/(t+k_2-iM)$] giving

$$\begin{aligned}S_5=\frac{iB(g^2+g_p^2)}{(4\pi)^2}&\int_0^1 x(1-x)dx\int_0^1 dy\int_0^1 udu\int_0^1 dv\\&\times\left\{-\frac{6x}{\lambda_0^2}\frac{[1-4u^2v(1-v)]}{[1-u^2v(1-v)K^2/M^2]}+x\left[\frac{2u-1}{D_1}-\frac{1-3u+2u^2+2uv(1-u)}{D_2}-\frac{1-u-2uv(1-u)}{D_3}\right]\right.\\&+(1-x)\left[\frac{1-4u^2v(1-v)}{D_1}+\frac{1-4u(1-u)(1-v)}{D_2}+\frac{1-4u(1-u)v}{D_3}\right]+2yx(1-x)(1-u)\\&\left.\times\int_0^1 dw\left[(1-4u^2v(1-v))\left(\frac{1}{D_4^2}+\frac{1}{D_5^2}+\frac{1}{D_6^2}\right)-4u(1-u)w\left(\frac{1-v}{D_5^2}+\frac{v}{D_6^2}\right)\right]\right\},\end{aligned}$$

where

$$D_i=yx(1-x)+(1-u)\lambda_0^2-yx(1-x)u[uv(1-v)\delta_{1i}+(1-u)(1-v)\delta_{2i}+(1-u)v\delta_{3i}]K^2/M^2;\ i=1,2,3$$

$$D_i=yx(1-x)+w(1-u)\lambda_0^2-yx(1-x)u[uv(1-v)\delta_{4i}+(1-v)(uv+w-uw)\delta_{5i}+v(u-uv+w-uw)\delta_{6i}]K^2/M^2;\ i=4,5,6.$$

To order K^2/M^2 this leads to the integral

$$S_5=-\frac{iB(g^2+g_p^2)}{(4\pi)^2}\int_0^1 dxF_0(x)=-\frac{iB(g^2+g_p^2)}{(4\pi)^2}(-0.0209), \quad (20)$$

where

$$F_0=x(f_1+f_2-2f_3)+(1-x)(7f_0/3-5f_1+6f_2-10f_3/3)-4\alpha^2x(g_0-2g_1+g_2)-5\alpha x/3,$$

$$\alpha=\frac{x(1-x)}{\lambda_0^2};\quad f_n=\int_0^1 dww^n\log\frac{w+\alpha}{w};$$

$$g_n=\int_0^1 dww^n\log\frac{w+\alpha}{\alpha}.$$

This and the following integrals were evaluated numerically.

Similarly, the replacement of γ_μ by

$$\frac{g_p^2}{(4\pi)^2}\int_0^1 xdx\int_0^1 dy\times\left\{\frac{[(1-x)t-(1-xy)k_1-iM]\gamma_\mu[(x-1)t-xyk_1+iM]}{\Lambda_0^2+x(1-x)[(t-yk_1)^2+M^2]}-\frac{x^2M^2}{\Lambda_0^2}\gamma_\mu-\gamma_\mu\int_0^1 dz\frac{x(1-x)[(t-yk_1)^2+M^2]}{\Lambda_0^2+zx(1-x)[(t-yk_1)^2+M^2]}\right\} \quad (21)$$

or

$$\frac{-2g^2}{(4\pi)^2}\int_0^1 xdx\int_0^1 dy\left\{\frac{-(1-x)t_\mu(xt-xyk_1-iM)}{\Omega_0^2+x(1-x)[(t-yk_1)^2+M^2]}+\frac{(1-x)^2M^2}{\Omega_0^2}\gamma_\mu-\gamma_\mu\int_0^1 dz\times\frac{x(1-x)[(t-yk_1)^2+M^2]}{\Omega_0^2+zx(1-x)[(t-yk_1)^2+M^2]}\right. \quad (22)$$

(where $\Omega_0^2=(1-x)^2M^2+x\mu^2$) leads (including a similar substitution for γ_ν) to the contributions

$$\frac{2iBg_p^2}{(4\pi)^2}\int_0^1 xdx\left(F_1-F_2-\frac{x^2}{3\lambda_0^2}\right)=-\frac{iBg_p^2}{(4\pi)^2}(-0.360) \quad (23)$$

and

$$\frac{-2iBg^2}{(4\pi)^2}\int_0^1 dx\left(\alpha F_3-(1-x)F_2+\frac{2\alpha x}{3}\right)=\frac{-iBg^2}{(4\pi)^2}(0.242), \quad (24)$$

where

$$F_1=\frac{x^2}{\lambda_0^2}\left[\left(\frac{4}{3}\alpha+\frac{1}{2}\alpha^2-\alpha^3-\frac{2}{3}\alpha^4\right)\log\frac{1+\alpha}{\alpha}-\frac{11}{12}-\frac{7}{9}\alpha+\frac{2}{3}\alpha^2+\frac{2}{3}\alpha^3\right]+\frac{x^2}{2\lambda_0^2}\log\frac{1+\alpha}{\alpha}-\frac{1}{2}\alpha(1+\alpha-\alpha^2-\alpha^3)\times\log\frac{1+\alpha}{\alpha}-\alpha\left(-\frac{7}{12}+\frac{1}{4}\alpha+\frac{1}{2}\alpha^2\right);$$

$$F_2=\left(\frac{1}{3}+\frac{2}{3}\alpha+\frac{1}{2}\alpha^2+\frac{1}{3}\alpha^3+\frac{1}{6}\alpha^4\right)\log\frac{1+\alpha}{\alpha}+\frac{1}{3}\log\alpha-\frac{7}{18}\alpha-\frac{1}{4}\alpha^2-\frac{1}{6}\alpha^3;$$

$$F_3=\frac{5}{9}+\frac{19}{36}x+\left(\frac{37}{24}-\frac{35}{72}x\right)\alpha+\left(\frac{11}{12}-\frac{9}{4}x\right)\alpha^2-\frac{4}{3}x\alpha^3-\left[\frac{1}{6}+\frac{1}{3}x+\left(\frac{5}{4}+\frac{5}{12}x\right)\alpha+\left(2-\frac{3}{2}x\right)\alpha^2+\left(\frac{11}{12}-\frac{35}{12}x\right)\alpha^3-\frac{4}{3}x\alpha^4\right]\log\frac{1+\alpha}{\alpha}.$$

The replacement of 1 (i.e., the absorption of a scalar meson) by

$$\frac{(g_p^2+G_nG_p^{-1}g^2)}{(4\pi)^2}\int_0^1 xdx\int_0^1 dy\left\{\frac{(1-x)M^2}{\Lambda_0^2}-2\int_0^1 dz\frac{x(1-x)[(t-yk_1+(1-y)k_2)^2+M^2]-K^2xy(1-y)}{\Lambda_0^2-K^2xy(1-y)+zx(1-x)[(t-yk_1+(1-y)k_2)^2+M^2]}+\frac{(x-1)(tk_1-k_2t+k_1k_2)+iM[2(x-1)t+(1-2xy)k_1-(1-2x+2xy)k_2]+(1-x)t^2+(x-1)M^2+(1-x)\mu^2}{\Lambda_0^2-K^2xy(1-y)+x(1-x)[(t-yk_1+(1-y)k_2)^2+M^2]}\right\} \quad (25)$$

gives ultimately

$$\frac{-2iB(g_p^2+G_nG_p^{-1}g^2)}{(4\pi)^2}\int_0^1 dx\left(\frac{x^2F_4}{\lambda_0^2}-\frac{1}{6}\frac{\alpha\mu^2}{M^2}-xF_5\right)=\frac{-iB(g_p^2+G_nG_p^{-1}g^2)}{(4\pi)^2}(-0.452), \quad (26)$$

where

$$F_4=\frac{1}{6}\left(\frac{1}{6}-\frac{1}{2}\alpha+\alpha^2+\alpha^2(1+\alpha)\log\frac{1+\alpha}{\alpha}\right);$$

$$F_5=\alpha\left(1+\frac{7}{6}\alpha+\frac{2}{3}\alpha^2+\frac{1}{6}\alpha^3\right)\log\frac{1+\alpha}{\alpha}$$

$$+\frac{1}{3}\log(1+\alpha)-\alpha\left(\frac{10}{9}+\frac{7}{12}\alpha+\frac{1}{6}\alpha^2\right).$$

All of the above results were checked by an independent substitution of auxiliary Feynman variables, leading not only to the same numerical results, but also (after considerable algebra) to the same algebraic form for F_0 to F_5. Hence, the fact that the total fifth-order contribution from virtual π-mesons is negative relative to the third-order term is quite certain. The entirely analogous results obtained for the heavy meson insertions were not checked as carefully, since they only affect the numerical constants in the relationship between the coupling constants and not the fact that cancellation can occur.

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The Ratio of Soft and Hard Particles in Air Showers*

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While the density spectrum of air showers is well established for energies above 10^{13} ev, doubt has recently been cast upon the customary procedure of identifying the energy spectrum of the shower-initiating particles calculated from this density distribution with the primary cosmic-ray spectrum. These two distributions would differ if the mechanism of energy transfer to the electron component itself varied with energy. As a result, one would then expect to find a variation with energy also in the amount and composition of the penetrating component of extensive showers. Previous investigations of penetrating particles in air showers do not extend below 10^{13} ev because of the bias imposed, in the case of low density showers, by the addition of a shielded tray. An experiment was therefore carried out at Mt. Evans, Colorado, altitude 4300 m, and at Echo Lake, Colorado, altitude 3260 m, in order to compare the penetrating components of showers in the 10^{13}-ev region and in the 10^{14}-ev region at a similar stage of their cascade development. The relative abundance of penetrating particles was $(2.06\pm0.11)\times10^{-2}$ at Mt. Evans, and $(1.20\pm0.11)\times10^{-2}$ at Echo Lake, and the fraction of N-particles among the penetrating component was (62 ± 7) percent for the lower, and (38 ± 6) percent for the higher shower energies. The ratio of μ-mesons to electrons was the same for both energy ranges. The results are therefore consistent with the assumption that the development of the large mixed cascades in air is based on the same processes for all energies above 10^{13} ev, demanding only a slight increase in the multiplicity of meson production with increasing primary energy.

I. INTRODUCTION

THE study of extensive air showers is certainly the most obvious, and hitherto probably the most successful, method of research in the high energy region of cosmic radiation. Unfortunately, though, the results of such a study yield only very indirect evidence on the nature of the primary interactions, since in these experiments a great many steps intervene between the initial acts and the final recording of the fully-grown shower. Thus, for instance, one can hardly expect from an observation on the intensity of the N-component in these showers, to obtain a unique answer as to the multiplicity of the primary nuclear collision. Or, to choose a simpler example, the old idea of deriving the energy spectrum of the primary cosmic radiation from the density spectrum of air showers (e.g., Heisenberg,[1] Hilberry[2]) is correct only if over the entire range of energies considered, the fraction of the primary energy which goes into the electronic component remains constant.

This latter assumption is certainly subject to serious doubt. Recent advances have shown the processes involved to be very complex, and to include the production of a considerable variety of different fundamental particles. It would seem quite likely that the composition of the shower secondaries produced in the initial collision varies with the energy of the primary. As the contribution to the electron cascade differs for the various components originated in the first interaction, a variation with energy of their relative abundance would, or could, also lead to an energy dependence of the abundance of electrons. The power law for the shower energies derived from the density spectrum of the air shower electrons would then no longer be applicable to the primary cosmic radiation.

The density spectrum of air showers has been thoroughly investigated. In particular, in the experi-

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[1] W. Heisenberg, *Cosmic Radiation* (Dover Publications, New York, 1946), Chap. I.
[2] N. Hilberry, Phys. Rev. **60**, 1 (1941).