hindered although in lesser degree than the three highest energy groups.

There is at present no quantitative explanation to account for the degrees of hindrance of the various alpha-groups. It has been pointed out,[3] and Preston[19,25] has demonstrated, that no explanation to include such high degree of hindrance as for several of the Am^{241} groups is likely to come from spin changes in the alpha-transitions. An hypothesis which we shall consider further is that the delay is involved in assembling the components of the alpha-particle and that the quantum states of the affected nucleons are involved.

This work was performed under the auspices of the United States Atomic Energy Commision. We wish to acknowledge the assistance of Mr. James Vanderveen in counting the alpha-tracks and to thank Drs. B. B. Cunningham, S. G. Thompson, and W. W. T. Crane for the preparations of Am^{241} and Cm^{242} used in these studies.

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Positron-Electron Scattering in Helium*

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Positron-electron scattering in a cloud chamber filled with helium at a pressure of 105 cm of Hg has been studied. The positron source was Na^{22}, and the primary positron energies ranged from about 20 to 600 kev. On 2420 meters of track length 1129 scattering events have been found with a fractional energy exchange between positron and electron greater than 10 percent. The frequency of scattering is in good agreement with the theory of Bhabha, but not enough events are available to discrimate between the theory with the effect of exchange taken into account and the theory without exchange (i.e., "ordinary"scattering).

I. INTRODUCTION

THIS research deals with the single scattering of positrons by electrons. The first work on this problem[1] indicated a discrepancy between theory and experiment. As this sort of discrepancy was observed in the early work on electron-electron scattering and was resolved by later more precise work, it is hoped that such may prove to be the case for positron-electron scattering.

The early results for electron-electron scattering gave cross sections greater than expected according to the relativistic theory of Möller[2] which is now accepted. The more recent work of Groetzinger *et al.*,[3] and of Page[4] is in good agreement with the theory.

Electron-electron scattering is different from positron-electron scattering in several essential ways. In the case of an electron-electron scattering process it is impossible to determine, after the collision, which was the primary electron. One cannot, therefore, separate the cases of strong energy exchange from those of weak energy exchange. It is the convention to take the electron with the lower energy as the secondary. In positron-electron scattering, on the other hand, it is easy to determine from the curvature which track in the cloud chamber is due to the positron. A second difference is that the contribution of exchange is different since the positron and electron can be created and annihilated in pairs.

In the first direct work on positron-electron scattering Ho Zah-Wei used a cloud chamber to study the positrons from Mn^{52} and F^{18}. The chamber was filled with air, a magnetic field was used, and stereoscopic photographs were made. On 395 meters of path length, 328 events were observed with an energy exchange $\epsilon = E^-/E_0^+ \geqslant 10$ percent, where E_0^+ and E^- are the kinetic energies of the initial positron and the secondary electron, respectively.

The result was that the frequency of single scatterings of positrons by electrons, in those cases in which the energy exchange was large, was greater than the expected frequency on the basis of the theory of Bhabha.[5] For very large energy exchanges ($\geqslant 70$ percent) the experimental frequencies were two or three times the theoretical values.

The most recent work, that of Von O. Ritter *et al.*,[6] was carried out completely independently of the present work but is similar in many ways. In their work the cloud chamber was filled with methane; the magnetic field was 300 gauss; the positron source, Cu^{64}, was outside the chamber; and on 5000 stereoscopic

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[1] Ho Zah-Wei, Phys. Rev. **70**, 224 (1946); Compt. rend. **226**, 1083 (1948).

[2] C. Möller, Ann. Physik **14**, 531 (1932); Z. Physik **70**, 786 (1931).

[3] Groetzinger, Leder, Ribe, and Berger, Phys. Rev. **79**, 454 (1950).

[4] L. A. Page, Phys. Rev. **81**, 1062 (1951).

[5] H. J. Bhabha, Proc. Roy. Soc. (London) **A154**, 195 (1936).

[6] Von O. Ritter *et al.*, Z. Naturforsch. **6a**, 243 (1951).

photographs with 2900 meters of useful track length with primary energies between 100 and 400 kev about 821 events were found with $\epsilon \geqslant 10$ percent. The conclusion was reached that the agreement between theory and experiment was within the experimental uncertainty.

II. THEORY

The result obtained from the calculation relating to the collision of an electron with a positron based on the Dirac theory of the positron, where the positron is considered as an unoccupied state of negative energy would be different from that which would be obtained if the calculation were done considering the positron as an independent, positively charged particle in a state of positive energy whose behavior is described by the Dirac equation. The difference would be due to the effect of exchange between the electron we observe initially and the virtual electrons in states of negative energy.

The "ordinary" scattering process is that one in which one electron in a state of positive energy goes over into a new state of positive energy, while a virtual electron in a state of negative energy jumps into the unoccupied state of negative energy which is the initial positron, leaving its state unoccupied to be observed as the new state of the positron. The effect of exchange arises because the process may take place in an alternative way; namely, the electron in a positive energy state may jump into the unoccupied state of negative energy which is the initial positron (this is annihilation), while a virtual electron in a negative energy state jumps into a state of positive energy which is then the final electron, leaving a state of negative energy unoccupied which is the final positron (this is creation of a pair).

Bhabha considered the collision of an electron with a positron including the effect of exchange and, basing his work on that of Möller, found for the effective cross section for the scattering of the positron with fractional energy exchange in the range ϵ to $\epsilon+d\epsilon$ in the system in which the electron is initially at rest (laboratory system, L)

$$dQ=\frac{2\pi e^4}{m^2c^4}\frac{\gamma}{(\gamma-1)^2\epsilon^2}F(\gamma,\epsilon)d\epsilon,$$

where

$$F(\gamma,\epsilon)=\frac{1}{\gamma(\gamma+1)}\Big[\{1+2(\gamma-1)(1-\epsilon)+(\gamma-1)^2(1-\epsilon+\tfrac{1}{2}\epsilon^2)\}+\frac{(\gamma-1)^2\epsilon^2}{(\gamma+1)^2}\{3+2(\gamma-1)+(\gamma-1)^2(\tfrac{1}{2}-\epsilon+\epsilon^2)\}-\frac{(\gamma-1)\epsilon}{\gamma+1}\{3+4(\gamma-1)(1-\epsilon)+(\gamma-1)^2(1-\epsilon)^2\}\Big].$$

Here e is the charge on the electron, m is the mass of the electron, c is the velocity of light, γ is E/mc^2, E is the total energy of the positron, ϵ is the ratio of the kinetic energy of the final electron to the kinetic energy of the initial positron in system L. The first term in square brackets is the ordinary scattering term, the second term is the one due to the annihilation of the initial pair and the simultaneous creation of a new pair, and the third term then represents the interference between the direct scattering and the latter process. By using only the first term of $F(\gamma,\epsilon)$ in the expression for dQ we obtain what may be called the differential cross section for "ordinary" scattering, dQ_0, i.e., the scattering in the absence of exchange effects.

III. APPARATUS

A cloud chamber of conventional design was used. It had an inside diameter of 8.25 in. and a depth of 4.38 in. The chamber was filled with helium at an average pressure of 104.9 cm of Hg in the expanded position. The optimum mixture of 70 percent n-propyl alcohol and 30 percent water was used to provide the vapor. An expansion ratio of 1.09 ± 0.005 resulted.

The chamber operated automatically on a cycle of one minute and was controlled by a simple circuit which involved only two electronic tubes. Most of the operations were controlled by relays and microswitches, operated by cams on the timing motor. A sweeping field of 600 v was provided. This sweeping field was removed at the instant of expansion and replaced when the expansion valve closed. The chamber was operated in a magnetic field produced by a pair of large Helmholtz coils giving a field of 305 ± 4 gauss.

The light source for photography consisted of two General Electric FT-127 xenon-filled arcs. They were flashed by discharging 50 mfd, charged to 2000 v, through each of them. The now conventional cylindrical lens system was used to produce a parallel beam of light of width about 4.5 cm. Stereoscopic photographs were taken on Eastman Linagraph Ortho 35-mm film. This film is particularly sensitive to the blue light of the discharge tubes and is a good compromise of speed and graininess. Several photographs of typical events may be seen in Fig. 1.

The positron source consisted of approximately 2 milli-microcuries of Na^{22},[7] which has a half-life of 2.6 years and a maximum positron energy of 0.58 Mev. The source was received as NaCl dissolved in water. The source as used was prepared by evaporating a diluted portion of the original solution on a thin collodion film held by a thin wire ring. Other collodion films were then placed above and below the first film so that none of the active material would escape into the chamber. The source was mounted at the center of the chamber and an average of 6.4 positron tracks per expansion was obtained.

[7] The Na-22 used was obtained from the AEC, Isotopes Division, Oak Ridge, Tennessee.

IV. ANALYSIS OF PICTURES

After the films were developed, they were replaced in the camera and reprojected through the same optical system with which the pictures were taken. Normally all measurements on the tracks were made on the image of the direct view of the chamber, reprojected onto a ground-glass screen. To measure the radii of curvature of the tracks a set of circles of known radius was superimposed until one was found that corresponded as closely as possible with the portion of track under consideration.

Tracks out of the plane perpendicular to the magnetic field were reprojected to this plane. The measured radii were thus caused to be about 2.2 percent large for an angle $\alpha=10°$ between the track and the plane perpendicular to the magnetic field. It was possible to determine the angle α by a "fusion" [8] method. For long tracks it was sufficient, however, to look at the picture stereoscopically to see whether the angle α was large. Both fusion and stereoscopic viewing helped to settle questions relating to the shape and form of the tracks and to determine whether an apparent secondary started on a track.

Track lengths were measured in two ways. The higher energy tracks with a rather uniform curvature were measured by bending a flexible centimeter rule into the shape of the track. The shorter, more tortuous tracks were measured by bending a thin wire into the shape of the track and measuring its length after it was straightened out.

On every twentieth picture the curvature and length of all the tracks were recorded. From these measurements the energy spectrum of the track length available was determined. This knowledge was essential for the determination of the theoretical number of scattering events to be expected.

For every positron-electron scattering event all the available information was recorded. This included the curvature of the primary positron and the curvature and/or range of the secondary electron and positron.

In all these measurements certain criteria of track selection were imposed. These criteria were as objective as possible so that the number of decisions to be made was reduced to a minimum. The criteria were similar to those of Shearin and Pardue.[9]

A. The Range-Energy Curve for Electrons in Helium

The energies of the low energy positrons and the secondary electrons whose tracks end in the chamber were determined by the measurement of their ranges. As no published curve for range *vs* energy for electrons in helium for the energy region considered could be found, an attempt was made to determine such a curve.

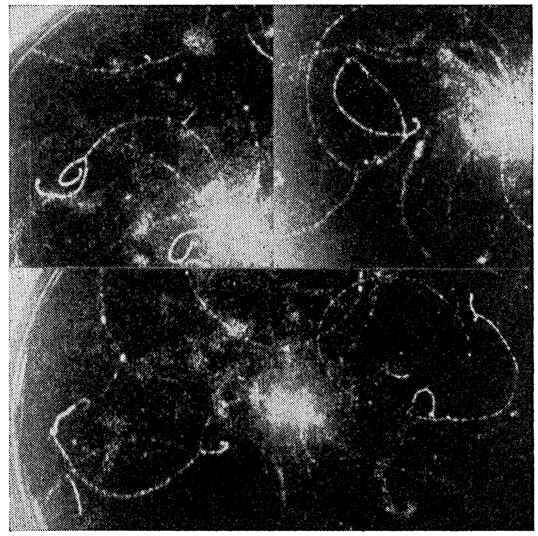

FIG. 1. Photographs of typical events. Upper left: $E_0^+=61$ kev, $\epsilon=0.6$. Upper right: $E_0^+=82$ kev, $\epsilon=0.6$. Lower left: $E_0^+=42$ kev, $\epsilon=0.7$. Lower right: $E_0^+=61$ kev, $\epsilon=0.3$.

A survey of the literature pertaining to the range-energy relation for electrons in air was made and a mean curve was drawn through what seemed the most reliable data. A similar survey of the literature was made to determine the range-energy relation for electrons in helium, but in this case less experimental data are available. The three sources of experimental data were the papers of Osgood,[10] Lehmann,[11] and O'Neill and Scott.[12] For energies between 1 and 100 kev no experimental results could be found. The values were calculated for this region by two methods. The stopping power of helium relative to air (using the above results for air), as it applies to electrons,[13,14] was used. In addition the expression of Tsien[13] for the range, involving the exponential integral, was used. Again, for helium, the best fit curve was drawn through the experimental and calculated points. In carrying through these calculations the average ionization potentials used were those given by Mano.[15]

To find the range-energy curve for the mixture in the chamber one must take into account the vapor and the residual air in the chamber. The average ionization potential and the average number of electrons per atom are then calculated. With these figures one may transpose from the range in air or helium to the range in the mixture in the chamber. It was found that in the scattering events the law of conservation of energy was well satisfied if the range-energy curve, found in the manner described above, was used.

[8] For a discussion of this method and other techniques see C. C. Jones and A. Ruark, Proc. Am. Phil. Soc. **82**, 253 (1940).
[9] P. E. Shearin and T. E. Pardue, Proc. Am. Phil. Soc. **85**, 243 (1942).
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[15] G. Mano, Ann. phys. **1**, 407 (1934).

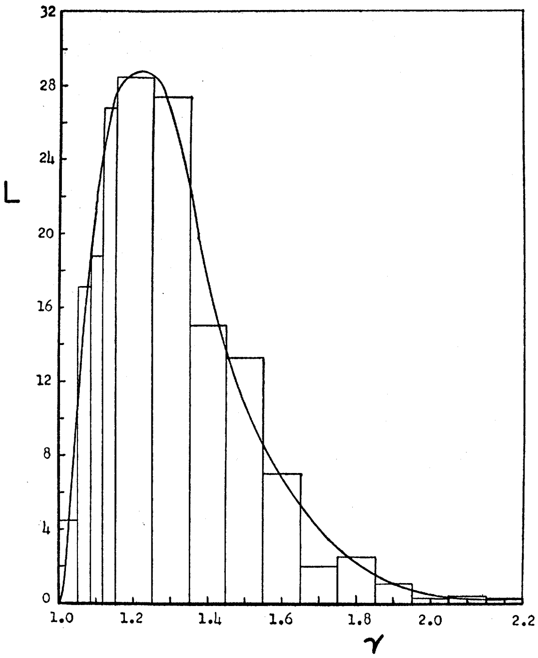


FIG. 2. Energy spectrum of the positrons used. L is track length in meters; γ is the positron energy in units of the electron rest energy. Multiply by 20.22 to find the total track length.

V. RESULTS

Several sources of error in the measurements should be mentioned. The error in the measurement of $H\rho$, including the error in H, varies from 3.5 to 5.8 percent. In addition, the error in ρ due to multiple scattering varies, according to Bethe's formulas,[16] from 4.4 to 6.8 percent. The resultant error in the energies determined by curvature measurements varies from 8.5 to 18 percent for energies between 600 and 50 kev.

The estimated error in the ranges, including error in the range-energy curve, error due to straggling, and error in the actual measurement of the ranges, varies from 12 to 14 percent. The corresponding error in the energies determined by range measurements is 6 to 7 percent.

The important thing about all these errors is not how they affect the energy value assigned to a given track, but rather how they affect the assignment of a value for the fractional energy exchange to the event in question.

A. Theoretical Number of Scattering Events

The number of events to be expected theoretically may be calculated by the theory of Bhabha discussed earlier. For the scattering of positrons by electrons the differential scattering cross section is given by dQ, which is defined as the number of collisions per second in the range ϵ to $\epsilon+d\epsilon$ per scattering center per unit current of incident particles with a given γ. But the current of incident particles with a given γ is $N_i v_i$, where N_i is the number of incident particles with a velocity v_i. Then, if n_s is the number of scattering centers (electrons) per unit volume, the probability of collision in range $d\epsilon$ for given γ is

$$N(\gamma, \epsilon)/N_i = n_s v_i \Delta t dQ.$$

But $v_i\Delta t = \Delta L$, where L is track length. Then we may write

$$N(\gamma, \epsilon) = n_s N_i(\gamma)\Delta L(\gamma) dQ = n_s(\Sigma \text{ lengths of tracks})_\gamma dQ.$$

Finally

$$N(\epsilon) = \sum_{\gamma=1}^{\gamma_{\max}} N(\gamma, \epsilon).$$

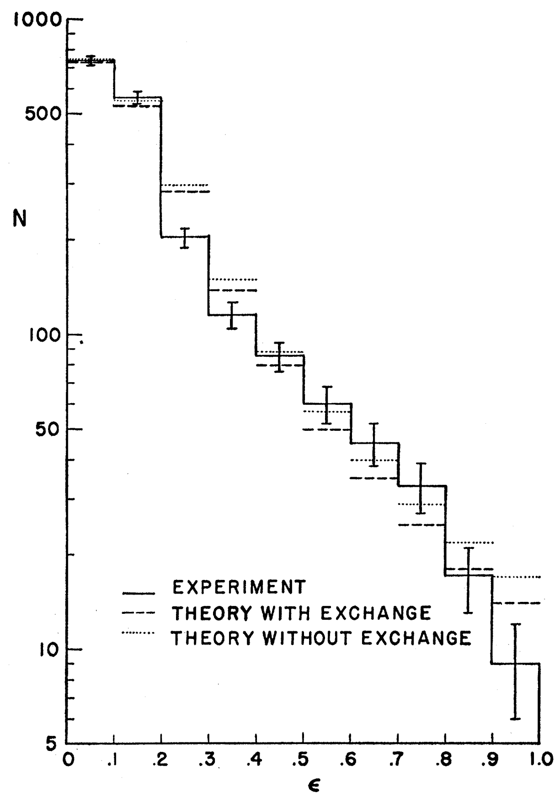


FIG. 3. The number, N, of positron-electron scattering events as a function of the fraction, ϵ, of the positron's energy that is transferred to the electron. These results are for the present experiment.

The above calculation has been carried through with γ of 1.1, 1.2, ⋯, 2.2, and ϵ of 0.05, 0.15, ⋯, 0.95. The total track length for each γ was determined by the energy spectrum measurements. These measurements were made on 219 out of the 4428 useful pictures. The results were plotted as a hodograph in Fig. 2. A smooth curve, representing the energy spectrum, was drawn through the blocks in the usual way. It should be noted that the maximum of this curve, falling at $\gamma=1.2$ which corresponds to a kinetic energy of 102 kev, is at lower energy than the maximum of the given energy spectrum for Na^{22} which is at 200 kev. This shift

[16] H. A. Bethe, Phys. Rev. **70**, 821 (1946).

toward lower energies is due to the fact that the long-range tracks pass out of the chamber or out of the illuminated region. By multiplying the values corresponding to each γ, read from the smooth curve, by the ratio of the total number of pictures to the number of spectrum pictures, namely 20.22, one finds the total track length for each γ.

B. Experimental Number of Scattering Events

After all the measurements were completed, the range and curvature measurements were converted to energy values and each scattering event classified as to primary energy, γ, and fractional energy exchange, ϵ. When more than one measurement of the energies were available, the usual practice was to average them. The results of these measurements are given in Table I along with the number of events to be expected theoretically and the sum of all events to date. These values are plotted in Figs. 3 and 4.

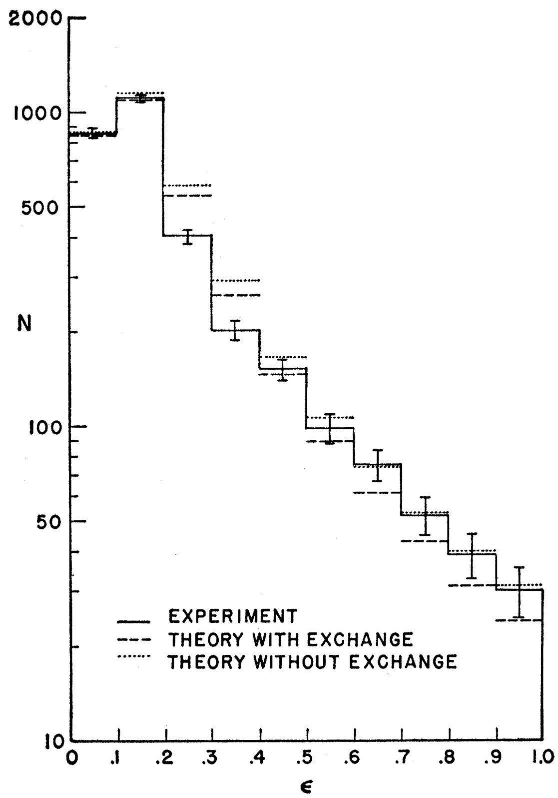


FIG. 4. The summation of the results of all experiments to date. N is the number of positron-electron scattering events, and ϵ is the fraction of the positron's energy that is transferred to the electron.

TABLE I. Comparison of the experimental number, $N_e(\epsilon)$, of positron-electron scattering events, with fractional energy exchange ϵ, with the number expected by the theory of Bhabha with exchange taken into account, $N_w(\epsilon)$, and without exchange, $N_o(\epsilon)$. 1. Present experiment. 2. Sum of results of Ho Zah-Wei, Ritter *et al.*, and present experiment.

$\epsilon=$	0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1
1. $N_e(\epsilon)$	736	562	203	115	85	60	45	33	17	9
$N_w(\epsilon)$	725	533	284	138	80	50	35	25	18	14
$N_o(\epsilon)$	738	553	299	149	88	57	40	29	22	17
2. $N_e(\epsilon)$	858	1106	402	202	152	98	75	52	39	30
$N_w(\epsilon)$	845	1097	541	261	147	89	61	43	31	24
$N_o(\epsilon)$	862	1141	581	291	167	106	74	53	40	31

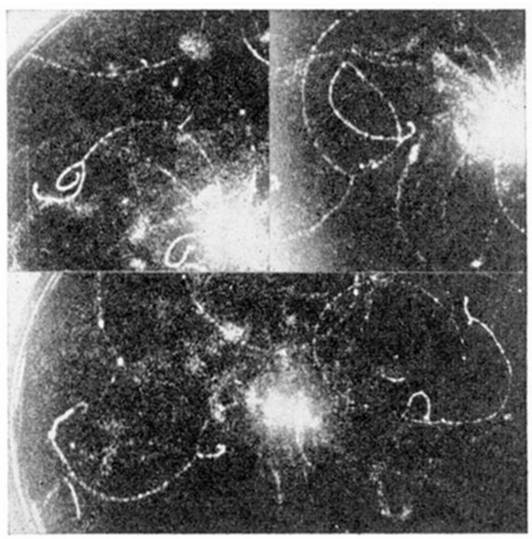
VI. CONCLUSION

In Fig. 3 it is seen that the agreement between theory and the results of this experiment is not close enough to determine whether the exchange effect should be included. In Fig. 4, however, it appears that the agreement between theory and experiment is best when the exchange effect is excluded. The lack of agreement between $\epsilon=0.2$ and 0.4 is unexplained.

One factor that might be the cause of some discrepancies is the following. The number of events to be expected is very sensitive to the track length of low energy. This is the steep portion of the energy spectrum curve, Fig. 2. Thus, a small error in the low energy track length measured could shift the theoretical values considerably.

In conclusion, it may be stated that the number of positron-electron scattering events observed for positron energies up to 600 kev is in good agreement with the theory of Bhabha. The discrepancies that do exist between theory and the 1129 events of this experiment are probably statistical. Furthermore, the number of events observed is not sufficient to discriminate between the theory including exchange and the theory without exchange. Experiments involving a larger number of events and with positrons of higher energy, $\gamma=4$, or more, are necessary to determine whether the exchange theory is completely correct.

It is a pleasure to acknowledge the guidance and encouragement of Dr. Paul E. Shearin and Dr. Nathan Rosen. This problem was first brought to my attention by Dr. Herman M. Schwartz.



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