Analysis of 14-Mev $n-p$ Scattering*

GEORGE SNOW
Brookhaven National Laboratory, Upton, New York
(Received March 20, 1952)

The total $n-p$ cross section at 14.10 ± 0.05 Mev is calculated for several choices of the $n-p$ singlet potential well parameters for each of the potential well shapes: square, exponential, and Yukawa. These calculations are compared with the experimental result for the cross section at this energy, $\sigma_{\text{total}}=0.689\pm0.005$ barn. Limits on the values of the effective range, the intrinsic range, and the depth of the singlet potential are obtained for each well shape

Contrary to the hypothesis of charge independence of nuclear forces, the $n-p$ singlet intrinsic range is found to be smaller than the $p-p$ singlet intrinsic range for all three well shapes. However, the large error in the $n-p$ singlet range precludes any conclusive statement to the effect that $n-p$ and $p-p$ intrinsic ranges are unequal.

I. INTRODUCTION

USING the theory of effective range,[1] Blatt and Jackson[2] (quoted as BJ) and Bethe[3] have analyzed the low energy $n-p$ scattering cross-section data. In the shape independent approximation, the total S wave cross section is determined by four parameters: a_t, a_s, r_{0t}, r_{0s}, the triplet and singlet scattering amplitudes, and the triplet and singlet effective ranges at zero energy. From the experimental measurements of the binding energy of the deuteron ϵ, the coherent $n-p$ scattering amplitude f, and the epithermal total scattering cross section σ_{free}, the three parameters a_t, a_s, and r_{0t} can be determined with good precision, the last being a function of the assumed well shape. BJ showed that the experimental uncertainties in the $n-p$ scattering cross sections at higher energies precluded at that time any precise information about r_{0s} or the well shape. More recently, Salpeter[4] has analyzed the more accurate new data pertinent to this problem. Using the measurements by Lampi, Freier, and Williams[5] of $\sigma_{\text{total}}(E)$ for various neutron energies between 0.8 and 5 Mev, he finds r_{0s} (with a probable error of ±20 percent) for each of the three well shapes; square, exponential, and Yukawa. The values of the singlet $n-p$ intrinsic range (defined in BJ) derived from these values of r_{0s} and the value of a_s, overlap for each well shape with the triplet $n-p$ intrinsic range and the singlet $p-p$ intrinsic range. With this data one cannot obtain any information that would favor one well shape over another.

The purpose of this note is to analyze the accurate measurement recently made by Poss *et al.*[6] of the total $n-p$ scattering cross section at 14 Mev, in order to obtain more information about r_{0s}. At this energy the total cross section is relatively insensitive to the uncertainties in the triplet parameters a_t and r_{0t} which arise from the experimental errors in the thermal energy measurements as well as from the uncertain strength of the tensor force. In this respect, 14 Mev is a good energy for determining r_{0s}. The main virtue of this energy is the ease with which one can obtain a strong nearly monochromatic beam of neutrons from the $T(d,n)He^4$ reaction. The disadvantages of such an energy lie in the necessity of evaluating the contributions of angular momentum states greater than zero, which for the P state involves the exchange characteristics of the potential as well as its shape, and in the neglect of relativistic velocity dependent corrections which are assumed to be small. One further point is that for the accuracy required in the calculation of the cross section, the S wave phase shift in the singlet state must be calculated exactly, rather than by using the shape dependent expansion for $k\cot\delta$ given by (BJ), since the expansion parameter (kr_{0s}) is greater than 1 at 14 Mev. Nevertheless, the form of the cross section obtained from the effective range theory is very useful in computing the changes in σ resulting from small changes in the pertinent parameters.

In this paper we shall adopt the following experimental values: From the reaction cycle analysis of Li, Whaling, Fowler, and Lauritsen,[7] we have, for the binding energy of the deuteron,

$$\epsilon=2.225\pm0.002 \text{ Mev.} \tag{1}$$

We denote the fractional error in ϵ by ϵ_ϵ. For the coherent scattering amplitude f, we use the most recent value obtained by Burgy *et al.*,[8]

$$f=2(\tfrac{3}{4}a_t+\tfrac{1}{4}a_s)=-(3.78\pm0.02)\times10^{-13}\text{ cm.} \tag{2}$$

For the scattering cross section of slow neutrons by free protons, we use the value of Melkonian,[9]

$$\sigma_{\text{free}}=\pi(3a_t^2+a_s^2)=20.36\pm0.1\text{ barns.} \tag{3}$$

The fractional errors, ϵ_f, ϵ_σ, in f and σ_{free} are $\epsilon_f=\epsilon_\sigma$

* Research carried out under contract with the AEC.

[1] This theory was first surmised by Landau and Smorodinsky, J. Phys. Acad. Sci. (U.S.S.R.) 8, 154 (1944) and first derived by J. Schwinger (see reference 2). For a simple derivation see reference 3, or J. Schwinger, Phys. Rev. 78, 135 (1950).
[2] J. M. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949).
[3] H. A. Bethe, Phys. Rev. 76, 38 (1949).
[4] E. E. Salpeter, Phys. Rev. 82, 60 (1951).
[5] Lampi, Freier, and Williams, Phys. Rev. 80, 853 (1950).
[6] Poss, Salant, Snow, and Yuan, Phys. Rev. 85, 11 (1952).
[7] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. 83, 512 (1951).
[8] Burgy, Ringo, and Hughes, Phys. Rev. 84, 1160 (1951).
[9] E. Melkonian, Phys. Rev. 76, 1744 (1949).

TABLE I. n–p triplet potential well parameters, and the values and derivatives of the n–p triplet cross section at $E=14.10$ Mev ($k^2=0.1700\times10^{26}$ cm^{-2}). [$\alpha_t=0.1860\times10^{13}$ cm^{-1}, $\epsilon=2.225$ Mev, $\rho_t(0,-\epsilon)=1.702\times10^{-13}$ cm.]

Well shape	r_{0t} (10^{-13} cm)	b (10^{-13} cm)	s	P_t	σ_t (barns)	$\frac{k^2}{\sigma_t}\frac{\partial\sigma_t}{\partial k^2}$	$\frac{r_{0t}}{\sigma_t}\frac{\partial\sigma_t}{\partial r_{0t}}$	$\frac{\alpha_t}{\sigma_t}\frac{\partial\sigma_t}{\partial\alpha_t}$
Square	1.724	2.040	1.440	−0.040	0.5508	−0.931	0.065	−0.073
Exponential	1.687	2.346	1.416	+0.029	0.5474	−0.914	0.071	−0.101
Yukawa	1.637	2.913	1.419	+0.137	0.5413	−0.899	0.064	−0.137

$=0.005$. Solving for a_t and a_s we obtain

$$a_t=(5.378-2.05\epsilon_f+3.72\epsilon_\sigma)\times10^{-13}\text{ cm},\quad(4)$$

$$a_s=(-23.69-1.41\epsilon_f-11.2\epsilon_\sigma)\times10^{-13}\text{ cm}.\quad(5)$$

This yields

$$\alpha_t=1/a_t=0.1860(1\pm0.0039)\times10^{13}\text{ cm}^{-1},\quad(6)$$

$$\alpha_s=1/a_s=-0.04221(1\pm0.0024)\times10^{13}\text{ cm}^{-1}.\quad(7)$$

From our adopted value for ϵ, we obtain

$$\gamma=[M\epsilon/\hbar^2]^{\frac{1}{2}}=0.23166(1\pm0.0005)\times10^{13}\text{ cm}^{-1}.\quad(8)$$

Finally, from the experiment of Poss *et al.*,[6] we have

$$\sigma_{\text{total}}=0.689\pm0.005\text{ barn},\quad(9)$$

at

$$E=14.10\pm0.05\text{ Mev}.\quad(10)$$

We shall first discuss the contributions to the total cross section at 14.1 Mev of triplet and singlet S wave scattering and then of higher angular momenta scattering. Finally the total cross sections shall be obtained as functions of r_{0s} for each well shape and these shall be compared with experiment. In this way we shall obtain limits for r_{0s}, which depend primarily on the experimental error in the cross section [Eq. (9)] and on the uncertainty in the mean kinetic energy of the incident neutrons [Eq. (10)].

II. TRIPLET SCATTERING CROSS SECTION ($L=0$)

In the notation of Salpeter,[4] we can write the total ($L=0$) triplet scattering cross section, σ_t at neutron incident energy E, in the form

$$\sigma_t=3\pi/\{k^2+[\alpha_t-\tfrac{1}{2}\rho_t(0,E)k^2]^2\},\quad(11)$$

$$k^2=ME/2\hbar^2=1.206(E_{\text{Mev}})\times10^{24}\text{ cm}^2,\quad(12)$$

and

$$\rho_t(0,E)=r_{0t}-2P_tr_{0t}{}^3k^2+\cdots,\quad(13)$$

where P_t is a small shape dependent coefficient plotted as a function of (α_tr_{0t}) by BJ. We obtain r_{0t} from α_t and γ by using the two equations[3]

$$\alpha_t=\gamma-(\tfrac{1}{2})\gamma^2\rho_t(0,-\epsilon),\quad(14)$$

$$\rho_t(0,-\epsilon)\cong r_{0t}(1+2P_t\gamma^2r_{0t}{}^2).\quad(15)$$

$\rho_t(0,-\epsilon)$ is independent of the potential shape since it is determined by two experimentally derived numbers α_t and γ. We get from (14)

$$\rho_t(0,-\epsilon)=(1.702+2.64\epsilon_f-4.79\epsilon_\sigma+2.62\epsilon_\epsilon)\times10^{-13}\text{ cm},$$

$$\rho_t(0,-\epsilon)=1.702(1\pm0.017)\times10^{-13}\text{ cm}.\quad(16)$$

Using (15) and the values of P_t obtained from the graphs of BJ, r_{0t} is found for each potential well shape. These are listed in Table I together with the values of P_t. These values of r_{0t} have the same percentage uncertainty as $\rho_t(0,-\epsilon)$, i.e., 1.7 percent. For completeness, the corresponding values of the intrinsic range b and the well depth s (defined in BJ) are given for each potential well shape. σ_t, obtained from (9), is given in Table I for $E=14.1$ Mev along with the values of $(k^2/\sigma_t)(\partial\sigma_t/\partial k^2)$, $(\alpha_t/\sigma_t)(\partial\sigma_t/\partial\alpha_t)$, and $(r_{0t}/\sigma_t)(\partial\sigma_t/\partial r_{0t})$. These latter values are listed so that a fractional error in any of the parameters k^2, α_t, and r_{0t} can be converted immediately into a fractional error in σ_t; e.g.,

$$\Delta\sigma_t/\sigma_t=(r_{0t}/\sigma_t)(\partial\sigma_t/\partial r_{0t})(\Delta r_{0t}/r_{0t}).$$

It is clear from the smallness of the values in Table I of the derivative of σ_t with respect to r_{0t} and α_t that σ_t is very insensitive to errors in r_{0t} or α_t. This is because the term $(\alpha_t-\frac{1}{2}\rho_tk^2)^2$ is very small compared to k^2 ($<0.025k^2$). Hence, the two term expansion of (13) for $\rho_t(0,E)$ in powers of $(kr_{\sigma t})^2$ will yield an excellent approximation to σ_t even at this high energy. (This has been verified for the square well by an exact calculation of the triplet phase shift.) Furthermore, the presence of a substantial amount of tensor force will not alter the value of σ_t significantly, since the only effect of the tensor force on σ_t is to modify the shape dependent parameter P_t. The experimental percentage errors of 0.4 percent in α_t and 1.7 percent in r_{0t} yield uncertainties in σ_t of the order of 0.04 percent and 0.1 percent, respectively. These uncertainties are a factor of ten less than the present experimental uncertainty in the value of σ_{total} measured by Poss *et al.*,[6] and hence are not at all significant in the determination of r_{0s} from this measurement. At lower incident neutron energies, the percentage uncertainties in σ_t because of the uncertainties in α_t and r_{0t} are substantially greater. For example, at $E=4.5$ Mev, the probable errors of 0.4 percent in α_t and 1.7 percent in r_{0t}, yield probable errors in σ_t of 0.3 percent and 0.3 percent, respectively. Hence, an ideal experiment, in which $\sigma_{\text{total}}(E)$ and E were measured exactly, would allow a determination of r_{0s} with a probable error four times larger at 4.5 Mev than at 14.1 Mev. The point is that at energies close to 14 Mev, the triplet phase shift δ_t is near $\pi/2$, so that σ_t, which is proportional to $\sin^2\delta_t/k^2$, is a very slowly varying function of δ_t. It is this same fact, as pointed out by Massey and Buckingham,[10] that makes the energy region near 14 Mev a particularly insensitive one for obtaining information about the n–p interaction in states of higher angular momenta from a measurement of the differential cross section $\sigma(\theta)$, since the interference terms between the $L=0$ wave and the higher L waves in $\sigma(\theta)$, which are proportional to $\cos\delta_t(L=0)$, are very small. On the other hand, σ_t at 14.1 Mev is about one and one-half times more sensitive to an error in E than at 4.5 Mev. Therefore, the mean neutron incident energy must be known very accurately. From (10), we have for the mean neutron energy $E=14.10\times(1\pm0.0035)$ Mev. For any energy within this small energy range, the simple linear extrapolation formula

$$\sigma_t[E=14.10(1\pm\delta)]=\sigma_t(14.10)\pm\delta(k^2/\sigma_t)(\partial\sigma_t/\partial k^2), \quad (17)$$

can be used to determine the theoretical triplet cross section σ_t.

III. SINGLET SCATTERING CROSS SECTION $(L=0)$

Analogously to Eq. (11) for σ_t, the total $(L=0)$, singlet scattering cross section σ_s may be written in the form

$$\sigma_s=\pi/\{k^2+[\alpha_s-\tfrac{1}{2}\rho_s(0,E)k^2]^2\}, \quad (18)$$

$$\rho_s(0,E)=r_{0s}-2P_s r_{0s}{}^3k^2+\cdots. \quad (19)$$

r_{0s} is known only within quite wide limits[4] and may be much larger than r_{0t}. We must consider cases for which $k^2r_{0s}{}^2>1$. For such cases it is not at all clear that the expansion for $\rho_s(0,E)$ in powers of $k^2r_{0s}{}^2$ is useful, or that the first two terms given in (19) will yield a good approximation to the true $\rho_s(0,E)$. Therefore, an exact integration of the Schrödinger equation was carried out to obtain the phase shift δ_s, and consequently σ_s, for three assumed values of r_{0s} and for each potential well shape; square, exponential, and Yukawa. From each assumed value of r_{0s} and the measured value of α_s, one obtains from the formulas of BJ the corresponding depth s and intrinsic range b for each potential well shape. For the square and exponential wells, the phase shift δ_s can be given in closed form[11] as a function of s, b, and k. To obtain δ_s for the Yukawa well,[12] direct numerical integrations were carried out for each pair of values (b, s), that is, for each choice of r_{0s}. The extremes of the values of r_{0s} were taken from Salpeter's[4] analysis of the data of Lampi *et al.*[5] At $E=14.10$ Mev, the exact cross sections σ_s(ex) obtained in this way are listed in Table II. The cross section σ_s(BJ) obtained from Eqs. (18) and (19) are also listed in Table II. The values of P_s for each r_{0s} are obtained from Table IV of BJ. We see that σ_s(BJ) is a very good approximation to σ_s(ex) for the square and exponential wells, but not quite so good for the Yukawa well. For completeness the values of b and s are also given in Table II. Equation (16) for σ_s(BJ) is still extremely useful in obtaining the variations of σ_s because of uncertainties in α_s and k^2. By differentiation, we obtain the values of $(\alpha_s/\sigma_s)(\partial\sigma_s/\partial\alpha_s)$, $(r_{0s}/\sigma_s)(\partial\sigma_s/\partial r_{0s})$, and $(k^2/\sigma_s)(\partial\sigma_s/\partial k^2)$ listed in Table II. The small values of $(\alpha_s/\sigma_s)(\partial\sigma_s/\partial\alpha_s)$ show that the uncertainty in α_s contributes a negligible uncertainty to σ_s ($\sim$0.02 percent). On the other hand, σ_s is quite sensitive to the value of the energy $\sim k^2$. Just as with σ_t, we shall use the linear extrapolation formula

$$\sigma_s[E=14.10(1\pm\delta)]=\sigma_s(14.10)\pm\delta(k^2/\sigma_s)(\partial\sigma_s/\partial k^2) \quad (20)$$

to determine σ_s(ex) for any energy near 14.10 Mev. To discuss the sensitivity of a measurement of σ_{total} with respect to the value of r_{0s}, it is convenient to define the quantity S, such that

$$\Delta r_{0s}/r_{0s}=S\Delta\sigma_{\text{total}}/\sigma_{\text{total}}. \quad (21)$$

Then

$$S=(\sigma_{\text{total}}/\sigma_s)[(r_{0s}/\sigma_s)(\partial\sigma_s/\partial r_{0s})]^{-1}. \quad (22)$$

From the values of $(r_{0s}/\sigma_s)(\partial\sigma_s/\partial r_{0s})$ listed in Table II it follows that $S=-8.6$, -12.0, -16.9 for square, exponential, and Yukawa wells, respectively. Hence, a

TABLE II. n–p singlet potential well parameters and the values and derivatives of the n–p singlet cross section at $E=14.10$ Mev for various values of the singlet effective range, $[\alpha_s=-0.04221\times10^{13}\text{ cm}^{-1}]$.

Well shape	r_{0s} (10^{-13} cm)	b 10^{-13} cm	s	σ_s(BJ) (barns)	σ_s(exact) (barns)	σ_s(exact) $-\sigma_s$(BJ)	$\frac{k^2}{\sigma_s}\frac{\partial\sigma_s}{\partial k^2}$	$\frac{\alpha_s}{\sigma_s}\frac{\partial\sigma_s}{\partial\alpha_s}$	$\frac{r_{0s}}{\sigma_s}\frac{\partial\sigma_s}{\partial r_{0s}}$
	3.15	3.00	0.907	0.1107	0.1100	−0.0007	−1.37	···	···
Square	2.66	2.55	0.920	0.1251	0.1248	−0.0003	−1.27	−0.096	−0.63
	2.15	2.06	0.934	0.1398	0.1400	+0.0002	−1.17	···	···
	3.05	2.83	0.919	0.1226	0.1223	−0.0003	−1.22	···	···
Exponential	2.67	2.50	0.928	0.1311	0.1311	0	−1.19	−0.093	−0.43
	2.05	1.94	0.943	0.1456	0.1452	−0.0004	−1.12	···	···
	2.95	2.69	0.934	0.1341	0.1314	−0.0027	−1.10	···	···
Yukawa	2.67	2.46	0.939	0.1382	0.1365	−0.0017	−1.10	−0.089	−0.29
	1.95	1.83	0.954	0.1508	0.1495	−0.0013	−1.08	···	···

[10] H. S. W. Massey and R. A. Buckingham, Proc. Roy. Soc. (London) **163**, 281 (1937).
[11] See, for example L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1948), Chap. 5.
[12] I wish to thank Professor G. Chew for informing me of a compact method of carrying out these numerical integrations.

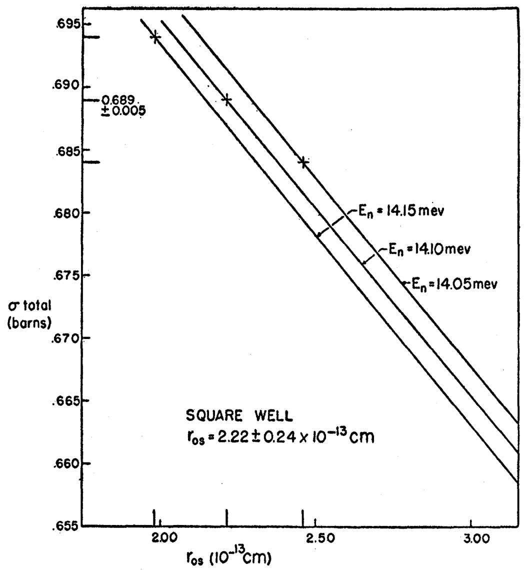

FIG. 1. The theoretical total n–p cross section σ_{total} for a square well, is plotted *versus* the effective singlet range r_{0s} for neutron energies $E_n = 14.05$, 14.10, 14.15 Mev. The limits for r_{0s} corresponding to the experimental result, (9), for σ_{total} are indicated.

1 percent error in σ_{total} would yield an 8.6 percent uncertainty in r_{0s} for a square well, and a 16.9 percent uncertainty in r_{0s} for a Yukawa well, if the energy and all other pertinent parameters were known exactly. At 14.1 Mev, the total cross section is only half as sensitive to r_{0s} for a Yukawa interaction as it is for a square well interaction. For comparison, at 4.5 Mev, S is -14.2, -15.6, -17.3 for square, exponential and Yukawa wells, respectively. Hence, a measurement of σ_{total} is substantially more sensitive to r_{0s} at 14.1 Mev than at 4.5 Mev for a short-tailed well, and of comparable sensitivity for a long-tailed well.

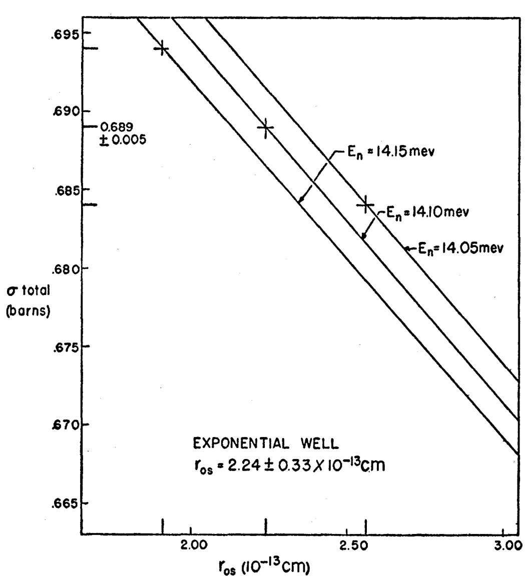

FIG. 2. Same as Fig. 1, for exponential well.

TABLE III. D wave contribution, σ_D, to the total n–p cross section at $E = 14.10$ Mev.

Well shape	σ_D (triplet) (b)	σ_D (singlet) (b)	σ_D (total) (b)
Square	2.8×10^{-6}	0.9×10^{-6}	3.7×10^{-6}
Exponential	7.0×10^{-4}	2.4×10^{-5}	7.2×10^{-4}
Yukawa	1.0×10^{-3}	1×10^{-4}	1.1×10^{-3}

IV. SCATTERING CROSS SECTION FOR $L=1,2$

The only further contributions to the total cross section that need be considered are those made by states of angular momenta $L=1$ and 2. To calculate the D wave contribution the Born approximation was used. The depth and width of the triplet and singlet potentials were obtained from the best values of r_{0t}, a_t, a_s and a mean value of r_{0s}. The results are listed in Table III. The contributions of the D wave to the total cross section for the exponential and Yukawa wells is large enough to be included, while for the square well, the contribution is negligible.

In order to calculate the P wave contributions, one must know the exchange character of the potential. Consider a potential of the form

$$V_{t,s} = \tfrac{1}{2}[(1+\Delta)+(1-\Delta)P_x]V_{t,s}, \qquad (23)$$

where P_x is the Majorana space exchange operator and $V_{t,s}$ are the triplet and singlet potentials that best fit the low energy data for the n–p system. From the fact that there is very little P wave evident in high energy n–p scattering, Christian and Hart[13] conclude that $0 \geq \Delta \geq -0.2$. (If $\Delta=0$, $V(L=1)=0$.) The P wave contribution to the total cross section has been calculated in Born approximation as a function of Δ. The Born approximation overestimates the total cross section for negative Δ, that is, for repulsive potentials. For the Yukawa well, an improved calculation using the Pais approximation[14] was made for $\Delta=-0.2$. These results are listed in Table IV. The maximum P wave contribution, for $\Delta=-0.2$, is less than 0.001 b. Therefore, it can be neglected in the theoretical cross section without introducing any appreciable uncertainty in the total cross section. Since σ_P and σ_D are very small, it is felt that more accurate estimates of these quantities including the effect of tensor forces would not appre-

TABLE IV. P wave contribution, σ_P, to the total n–p cross section at $E=14.10$ Mev.

	Born approximation				
Well shape	σ_P (triplet) (b)	σ_P (singlet) (b)	σ_P (total) (b)	σ_P (total) at $\Delta=-0.2$	Pais approx. σ_P (total) at $\Delta=-0.2$
Square	$0.0032\Delta^2$	$0.0013\Delta^2$	$0.0045\Delta^2$	0.0002	
Exponential	$0.0094\Delta^2$	$0.0017\Delta^2$	$0.011\Delta^2$	0.0004	
Yukawa	$0.023\Delta^2$	$0.003\Delta^2$	$0.026\Delta^2$	0.0010	0.00066

[13] Richard S. Christian and Edward W. Hart, Phys. Rev. **77**, 441 (1950).
[14] A. Pais, Proc. Cambridge Phil. Soc. **42**, 45 (1946).

ciably change our conclusions about the singlet effective range. It has already been pointed out that the tensor force will not appreciably effect the theoretical estimate of σ_{triplet} for $L=0$.

No attempt has been made to estimate any relativistic velocity dependent corrections to the theoretical cross section. We merely emply a wave number k corresponding to the relativistic relative momentum of the neutron and proton. This means that in Eq. (12), which defines k, we take E to be the relativistic kinetic energy of the neutron in the laboratory system.

V. RESULTS AND DISCUSSION

Summing the contributions of σ_t and σ_s for $L=0$ and 2 given in Tables I, II, and III, we obtain the theoretical values for σ_{total} at $E=14.10$ Mev as a function of r_{0s} for each potential well shape: square, exponential, and Yukawa.

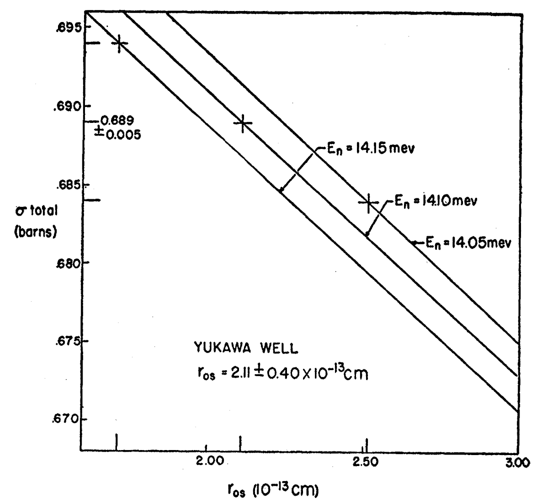

FIG. 3. Same as Fig. 1, for Yukawa well.

Using the derivatives of σ_t and σ_s with respect to energy, σ_{total} is also obtained at $E=14.05$ and 14.15 Mev, corresponding to the spread in the mean energy of the neutrons as given in (10). σ_{total} is plotted as a function of r_{0s} for $E=14.05$, 14.10, 14.15 Mev for square, exponential and Yukawa wells in Figs. 1, 2, and 3. The experimental total cross section, (9), then yields limits for r_{0s} for each potential well. As illustrated in these graphs, we adopt the extreme limits for r_{0s} obtained by comparing the upper limiting value of the cross section, 0.694 barn, with the σ_{total} curve at $E=14.15$ Mev, and the lower limiting value, 0.684 barn, with the σ_{total} curve at 14.05 Mev. We thus obtain for the three wells,

$$\begin{aligned}
&\text{Square} && r_{0s}=2.22\pm0.24\times10^{-13}\text{ cm},\\
&\text{Exponential} && r_{0s}=2.24\pm0.33\times10^{-13}\text{ cm},\\
&\text{Yukawa} && r_{0s}=2.11\pm0.40\times10^{-13}\text{ cm}.
\end{aligned}\tag{24}$$

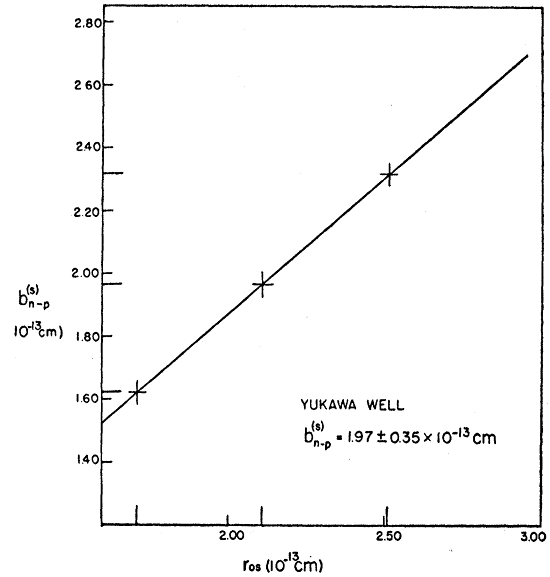

FIG. 4. The $n-p$ singlet intrinsic range $b_{n-p}^{(s)}$ is plotted *versus* the effective singlet range, r_{0s} for the Yukawa well, using $\alpha_s=-0.04221\times10^{13}$ cm^{-1}. The limits for $b_{n-p}^{(s)}$ corresponding to the experimental limits for r_{0s} are indicated.

About two-thirds of the uncertainty in r_{0s} arises from the probable error in the measurement of σ_{total} and one third from the neutron energy uncertainty. This result can be compared with the values of r_{0s} obtained by Salpeter,

$$r_{0s}=2.65(S),\ 2.55(E),\ 2.45(Y)\pm0.5\times10^{-13}\text{ cm},\tag{25}$$

for square, exponential, and Yukawa wells, respectively. The values of r_{0s} obtained here from the 14.10-Mev total $n-p$ cross section [Eq. (24)], are substantially

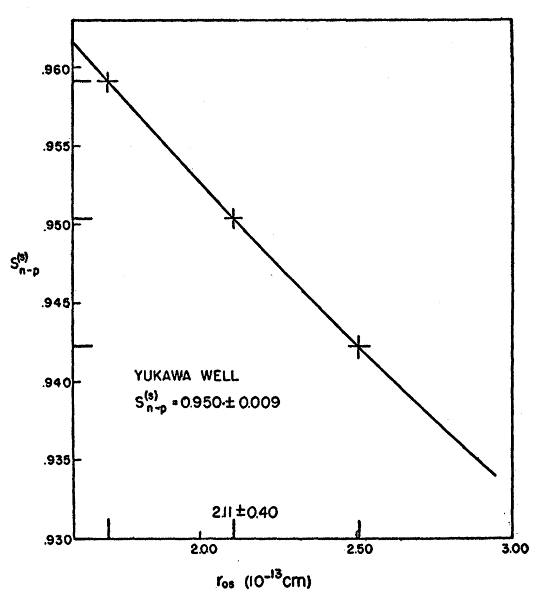

FIG. 5. Same as Fig. 4, except the singlet well depth $s_{n-p}^{(s)}$ instead of the intrinsic range $b_{n-p}^{(s)}$.

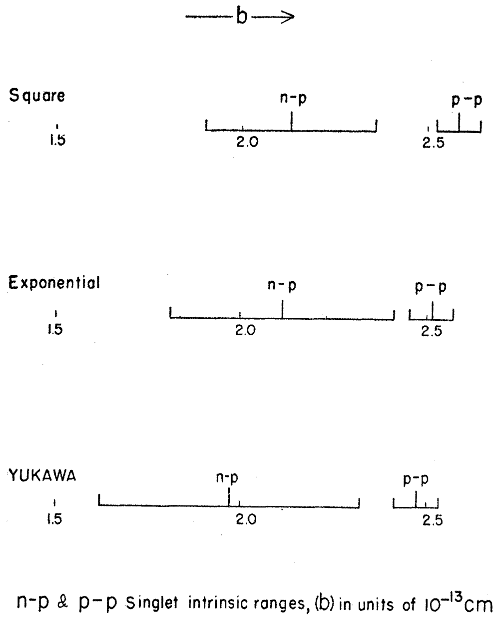

FIG. 6. Comparison for each potential shape of the p–p singlet intrinsic range with the n–p singlet intrinsic range obtained from the present analysis.

smaller but not inconsistent with the values of r_{0s} obtained from the lower energy measurements [Eq. (25)]. The mean value of r_{0s} depends only slightly on the potential well shape primarily because the shape dependent factor P_t plays only a small role in the determination of σ_t. With the reduced errors in r_{0s} obtained here, it is interesting to compare the parameters of the n–p and p–p singlet potential wells. Using the tables of BJ we have plotted the intrinsic range b and the depth s as a function of r_{0s} using the value of Eq. (7) for α_s. Figures 4 and 5 illustrate such graphs for the Yukawa well. The limits on r_{0s} given by Eq. (24) then determine limits for b and s. These values of b are listed in Table V along with the p–p singlet and n–p triplet intrinsic ranges. The p–p singlet intrinsic range is obtained from the analysis by Jackson and Blatt[15] of the very accurate p–p scattering data. The n–p triplet parameters are obtained from the values of r_{0t} and α_t given in Table I. Finally, the intrinsic ranges of Salpeter are tabulated.

The hypothesis of charge independence of nuclear forces would imply equality of the n–p and p–p singlet intrinsic ranges. From Table V we see that there is a definite indication that the n–p singlet intrinsic range $b_{n-p}^{(s)}$ is smaller than the p–p singlet intrinsic range,

TABLE V. Intrinsic ranges, b, of n–p and p–p potential wells, in units of 10^{-13} cm.

Well shape	n–p singlet from σ^{total} (14.1 Mev)	p–p singlet (Jackson and Blatt)	n–p triplet	n–p singlet (Salpeter)
Square	2.13±0.23	2.58±0.06	2.04±0.12	2.25±0.5
Exponential	2.11±0.30	2.51±0.06	2.35±0.12	2.4 ±0.5
Yukawa	1.97±0.35	2.47±0.06	2.91±0.12	2.55±0.5

$b_{p-p}^{(s)}$ for all three potential wells. Figure 6 makes this comparison of n–p and p–p ranges clearer. Obviously, the rather large error in $b_{n-p}^{(s)}$ precludes any conclusive statement to the effect that n–p and p–p intrinsic ranges are unequal. This indication of singlet range inequality is most pronounced for the square well. A comparison of the n–p singlet and n–p triplet intrinsic ranges shows that these ranges are different for the Yukawa well and that they overlap for the square and exponential wells. This difference for the Yukawa well, between n–p singlet and n–p triplet potential well widths, is perhaps not surprising since it was already established that the p–p singlet intrinsic range for the Yukawa well was smaller than the n–p triplet intrinsic range.

The values of the n–p singlet potential well depths obtained here are listed in Table VI along with the

TABLE VI. Well depths, s, of n–p and p–p potential wells.

Well shape	n–p singlet	p–p singlet	n–p triplet
Square	0.932±0.007	0.889±0.003	1.440
Exponential	0.938±0.008	0.907±0.003	1.416
Yukawa	0.950±0.009	0.922±0.003	1.419

values of s for the p–p singlet (Jackson and Blatt[15]) and the n–p triplet potentials. The present results indicate that $s_{(n-p)}^{(s)}$ is larger than $s_{(p-p)}^{(s)}$ for all potential wells. This discrepancy is smallest for the Yukawa well. This result reiterates with slightly more force the same result already obtained by Bethe[3] from a comparison of $a_s^{(n-p)}$ and $a_s^{(p-p)}$ assuming equal singlet n–p and p–p intrinsic ranges. Schwinger[16] has given a possible explanation of this small difference in potential well depth in terms of the different electromagnetic interactions that are present in the n–p and p–p systems.

I wish to thank Professor H. S. Snyder for some helpful discussions. The aid in numerical computation rendered by Miss Gertrude Nissenbaum and Miss Jean Snover is hereby gratefully acknowledged.

[15] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. 22, 77 (1950).

[16] J. Schwinger, Phys. Rev. 78, 135 (1950).