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¹ G. T. Seaborg, Phys. Rev. **85**, 157 (1952).

² E. Segrè, Phys. Rev. **86**, 21 (1952).

³ W. J. Whitehouse and W. Galbraith, Nature **169**, 494 (1952).

The Gauge Invariance Problem

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IT appears to be generally believed¹ that the recent formulation² of electrodynamics in the interaction representation is gauge invariant. This is not true, as will be shown below. Since a statement which is contrary to accepted notions is being made, it is well to state precisely the point in the theory where the lack of gauge invariance arises. The definition of the basis vectors of the representation in terms of a separation of the interaction-free electron-positron field into positive and negative frequency parts does not give a gauge invariant representation.

As no development of quantum electrodynamics is here intended, the formulas used to demonstrate the above point will be taken from the literature.

In the interaction representation, according to Schwinger, paper I, the effect of a gauge transformation of the second kind,

$$A_{\mu}(x) = A'_{\mu}(x) - \partial\Lambda(x)/\partial x_{\mu},$$

is compensated for in the equations of motion by the canonical transformation

$$\Psi(\sigma) = \exp[-iG(\sigma)]\Psi'(\sigma), \quad (\text{I, 2.40})$$

with

$$G(\sigma) = \frac{1}{\hbar c^2} \int_{\sigma} j_{\mu}(x)\Lambda(x)d\sigma_{\mu}. \quad (\text{I, 2.41})$$

The meaning of the canonical transformation (I, 2.40) is: If a certain physical state is represented by the state vector $\Psi(\sigma)$ when the potentials are $A_{\mu}(x)$, then the same physical state is represented by the new state vector $\Psi'(\sigma)$ when the potentials are $A'_{\mu}(x)$.

In the interaction representation the electron-positron spinor field satisfies the equation

$$(\gamma_{\mu}\partial/\partial x_{\mu} + \kappa_0)\psi = 0. \quad (\text{I, 2.16})$$

Further, in Schwinger, II, the spinor field $\psi(x)$ was decomposed into positive and negative frequency parts $\psi^{+}(x)$, $\psi^{-}(x)$ with definitions given by (II, 1.47) and (II, 1.48), respectively.

$$\psi^{+}(x) = \frac{1}{2\pi i} \int_{C_{+}} \psi(x - \epsilon\tau)(d\tau/\tau), \quad (\text{II, 1.47})$$

$$\psi^{-}(x) = \frac{1}{2\pi i} \int_{C_{-}} \psi(x + \epsilon\tau)(d\tau/\tau), \quad (\text{II, 1.48})$$

with the contour of integration extending from $-\infty$ to $+\infty$ and passing below the singularity at $\tau=0$ with ϵ a time-like four-vector with a positive time component. The "vacuum" state was then defined by the conditions

$$\psi^{+}(x)\Psi_0 = 0. \quad (\text{II, 1.63})$$

$$\overline{\psi^{-}}(x)\Psi_0 = 0. \quad (\text{II, 1.64})$$

Although Schwinger didn't do so explicitly, a one-electron state would then be defined by³

$$\overline{\psi^{+}}(x)\Psi_0 = \Psi(x),$$

with a corresponding extension to multiparticle states. A complete collection of vectors of the types enumerated above then constitutes a system of basis vectors for the representation. We now note that this system of basis vectors is chosen independently of the gauge. As has already been seen, the same physical state is represented by different vectors in different gauges; thus, the same vectors must represent different physical states in different gauges. The conclusion is therefore immediate that the representation given above is not gauge invariant. Even though the basis vectors are chosen in a gauge invariant manner these fixed vectors represent different physical states in different gauges and also, as is better known, on different space-like surfaces.

There is no implication intended here that electrodynamics is intrinsically gauge dependent, but only that this particular scheme for introducing electrons and positrons into the theory is gauge dependent.

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¹ The author obtained this impression from private conversations with a number of theoretical physicists. The author also had the same delusion until quite recently.

² S. Tomonaga, Prog. Theoret. Phys. **1**, 27 (1946); J. S. Schwinger, I, Phys. Rev. **74**, 1439 (1948); J. S. Schwinger, II, Phys. Rev. **75**, 651 (1949).

³ There is a difficulty here with the normalization of $\Psi(x)$ which can easily be avoided by taking a weighted average of $\psi(x)$ over a region of space time.

Angular Distribution of Photoprotons from Carbon*

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UTILIZING the techniques recently reported¹ for determining the angular distributions of photoprotons from targets of copper, cobalt, and nickel, we have measured the photoproton angular distribution from a 38 mg/cm² target of C¹² bombarded by bremsstrahlung of 23-Mev maximum energy. Results are given in Fig. 1.

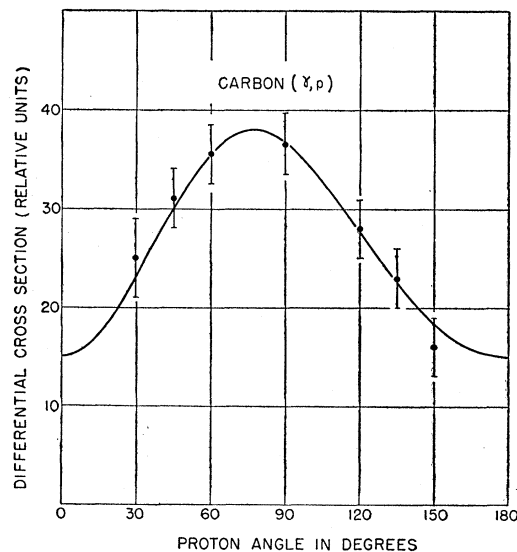


FIG. 1. Angular distribution of protons from a 38-mg/cm² target of carbon bombarded with bremsstrahlung of 23-Mev maximum energy.

The distribution shows, as did those of copper and cobalt, a large asymmetric component peaked in the forward direction. For carbon the asymmetric component represents a greater portion of the total protons ejected and the forward shift of the peak from 90° is about 10 degrees.² Nonetheless, the distribution can be fitted by an expression of the form

$$1 + (a \sin\theta + b \sin\theta \cos\theta)^2,$$

representing dipole and quadrupole interference terms. This is the same expression obtained for copper and cobalt. The solid line drawn through the experimental points of Fig. 1 is taken from the expression

$$(\sin\theta + 0.25 \sin\theta \cos\theta)^2$$

for the asymmetric component. Thus the amount of quadrupole absorption at these energies in carbon, about 1 percent in intensity, is less than that in copper and cobalt by a factor of 4.

The above analysis is considerably strengthened in the case of the $C^{12}(\gamma, p)B^{11}$ reaction in that the spins and parities of the initial and final states are known. From energetics considerations, only two levels of the residual B^{11} nucleus are involved, the ground level and the first excited level with 2.14 Mev of excitation. Transitions to the level at 4.46 Mev would yield protons of insufficient energy to reach the detectors. Transitions to the ground level of B^{11} , most probably a $p_{3/2}$ level,³ could satisfy all conditions of conservation of angular momentum and parity for dipole-quadrupole interference and for the observed symmetric component. Transitions to the first excited level of B^{11} would also satisfy these conditions with a spin of $\frac{1}{2}$ or $\frac{3}{2}$ for this level.

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¹ Mann, Halpern, and Rothman, Phys. Rev. **87**, 146 (1952).

² Angular uncertainties in the settings of the detector are restricted to ± 2 degrees.

³ Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1930).

Absorption Coefficients of Gamma-Rays

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RECENTLY published measurements of the absorption coefficients of gamma-rays in the energy range from 0.3 to 2.8 Mev have, on the whole, confirmed the values calculated from theory. However, in the case of the radiation from Co^{60} (mean energy 1.25 Mev) Davisson and Evans¹ reported values of the total absorption coefficients for lead and tantalum which were 2 percent and 5 percent less than the theoretical values; while Shimizu, Hanai, and Okamoto² reported values which were between 3 percent and 5.5 percent less than the theoretical values for 6 elements with atomic numbers between 73 and 81.

I have recently measured absorption coefficients for C, Al, Cu, Mo, Ag, W, Pb, U, and H_2O , using the isotopes Hg^{203} , Cr^{51} , Ru^{103} , Rb^{86} , Co^{60} , and K^{42} as sources of gamma-radiation. The arrangement of apparatus and the method of measurement were very similar to those described by Shimizu, Hanai, and Okamoto. Corrections were applied for the effect of bremsstrahlung and Compton scatter from the source, and for Compton scatter and Rayleigh scatter from the absorber. The standard deviations in the corrected values of the absorption coefficients were between 0.5 percent and 1 percent in most cases, but were only 0.3 percent for the measurements made with Co^{60} .

The theoretical values of the absorption coefficients were calculated in the usual way, from the Klein-Nishina formula for the Compton scatter coefficient, from Hulme *et al.*³ for the photoelectric coefficient, and from Bethe and Heitler⁴ for the pair production coefficient.

The measured and calculated values of the total electronic absorption coefficients are given in Table I, together with the standard deviations in the measured values due to experimental errors, and the possible errors in the theoretical values due to uncertainties in the values of the quantum energy. The values have all been multiplied by 10^{25} .

The measured values for elements of low atomic number agree, within the limits of the experimental errors, with the theoretical values, thus verifying the Klein-Nishina formula. On the other hand, the measured values for W, Pb, and U at the higher energies are significantly smaller than the theoretical values. There are no anomalous values, however. If the Klein-Nishina formula for

TABLE I. Measured and calculated values of $\mu \times 10^{25}$, with the standard deviations. The experimental value for each absorber is listed first, with the theoretical value directly below. The γ -ray energies in the headings are in Mev. The numbers following \pm are the standard deviations in the last two decimal places quoted.

	Hg ²⁰³	Cr ⁵¹	Ru ¹⁰³	Rb ⁸⁶	Co ⁶⁰ 1.1715 1.3316	K ⁴²
Abs.	0.279 \pm 02	0.325 \pm 05	0.496 \pm 02	1.076 \pm 03	1.3316 \pm 10	1.51 \pm 01
H ₂ O	3.580 \pm 80 3.631 \pm 05	3.520 \pm 25 3.429 \pm 11	2.920 \pm 21 2.902 \pm 04	2.065 \pm 19 2.037 \pm 03	1.890 \pm 08 1.885 \pm 02	1.701 \pm 14 1.713 \pm 06
C	3.720 \pm 40 3.631 \pm 05	3.476 \pm 25 3.429 \pm 11	2.879 \pm 16 2.902 \pm 04	2.032 \pm 13 2.037 \pm 03	1.874 \pm 06 1.885 \pm 02	1.697 \pm 16 1.713 \pm 06
Al	3.693 \pm 81 3.660 \pm 05	3.473 \pm 19 3.446 \pm 11	2.876 \pm 21 2.908 \pm 04	2.052 \pm 13 2.038 \pm 03	1.893 \pm 05 1.887 \pm 02	1.714 \pm 17 1.717 \pm 06
Cu	4.213 \pm 63 4.151 \pm 10	3.819 \pm 28 3.773 \pm 18	2.972 \pm 23 3.013 \pm 05	2.064 \pm 16 2.055 \pm 03	1.898 \pm 06 1.901 \pm 02	1.717 \pm 13 1.733 \pm 06
Mo	3.268 \pm 19 3.297 \pm 07	2.105 \pm 20 2.105 \pm 03	1.945 \pm 06 1.940 \pm 02	...
Ag	6.234 \pm 78 6.383 \pm 33	5.331 \pm 45 5.239 \pm 48	3.454 \pm 20 3.476 \pm 08	2.154 \pm 13 2.138 \pm 04	1.959 \pm 05 1.966 \pm 02	1.766 \pm 14 1.788 \pm 06
W	5.336 \pm 29 5.490 \pm 22	2.483 \pm 22 2.521 \pm 05	2.245 \pm 07 2.264 \pm 03	...
Pb	19.05 \pm 28 18.94 \pm 16	14.06 \pm 08 13.76 \pm 22	6.345 \pm 26 6.491 \pm 29	2.704 \pm 14 2.728 \pm 06	2.403 \pm 06 2.425 \pm 03	2.120 \pm 16 2.143 \pm 11
U	...	18.10 \pm 14 18.15 \pm 30	8.026 \pm 32 8.114 \pm 40	3.027 \pm 23 3.061 \pm 07	2.641 \pm 09 2.688 \pm 03	2.303 \pm 12 2.343 \pm 12

Compton scatter and the Bethe and Heitler values of the pair production coefficient are assumed to be correct, the results indicate that the calculations of Hulme *et al.* for the photoelectric coefficient are in error by the amounts given in Table II. These

TABLE II. Values of $[(\sigma_{\text{expt}} - \sigma_{\text{theor}}) / \sigma_{\text{theor}}] \times 100$, with standard deviations.

E(Mev)	0.279	0.325	0.496	1.076	1.1715 1.3316	1.51
% Error	-1.5 \pm 1.8	+1.9 \pm 2.1	-3.2 \pm 0.8	-3.5 \pm 1.5	-5.0 \pm 0.8	-6.8 \pm 2.3

amounts are the weighted means of the differences between the experimental and the theoretical values of the photoelectric coefficient for all the absorbers at each quantum energy.

A fuller account of this work is in preparation.

¹ C. M. Davisson and R. D. Evans, Phys. Rev. **81**, 406 (1951).

² Shimizu, Hanai, and Okamoto, Phys. Rev. **85**, 290 (1952).

³ Hulme, McDougall, Buckingham, and Fowler, Proc. Roy. Soc. (London) **A149**, 131 (1935).

⁴ H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).

Investigations of Complex Structure in Alpha-Emission with Nuclear Emulsions*

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A STUDY has been made, utilizing the method of photographic emulsions, of the conversion electrons accompanying the complex alpha-group structure in a number of nuclides. These conversion electrons, coincident in origin with the alpha-particles, were identified in Ilford G-5 nuclear emulsions. Energies of the electrons were approximated from their range and this, together with postulation of the shell of conversion (or binding energy) of the electrons, allowed calculation of the gamma-ray energy or nuclear energy level separation. The percentage of alpha-particles having such conversion electrons associated with them represent a lower limit to the alpha-decay leading to the