$E(Z')$ an amount $-(Z_{\text{eff}}+Z'-Z)^2R/n^2$, and (using the virial theorem) to $e\varphi(Z)$ an amount $-2Z_{\text{eff}}R/n^2$. The total contribution to ΔE is just $(Z'-Z)^2R/n^2$. Thus each closed shell of principal quantum number n adds $2R(Z'-Z)^2=27.2(Z'-Z)^2$ ev to ΔE. For $|Z'-Z|=1$, $Z=10$, this gives $\Delta E=54$ ev, while (4a) and (4b) give 53 ev and 57 ev. For $Z=90$ the simple recipe gives $\Delta E=125$ ev, and (4a) and (4b) give 110 ev and 138 ev.

A formal treatment of the arguments given above is quite simple. Let P_n be the probability that the electronic system of the final atom will be left in a state of energy $E_{Z'n}$. If $n=0$ denotes the ground state, the mean excitation energy of the atom will be

$$\Delta E=\Sigma_n P_n(E_{Z'n}-E_{Z'0}).$$

For a completely nonadiabatic transition from initial to final atom,

$$P_n=(\psi_{Z'n},\psi_{Z0})|^2,$$

where the $\psi_{Z'n}$ are the wave functions of the states of the final atom, ψ_{Z0} is the wave function of the ground state of the initial atom, and

$$(\psi_{Z'n},\psi_{Z0})=\int\psi_{Z'n}{}^*\psi_{Z0}d\tau.$$

If the Schrödinger equation for the initial atom is

$$E_{Z0}\psi_{Z0}=H_Z\psi_{Z0},$$

that for the final atom is

$$E_{Z'n}\psi_{Z'n}=\left[H_Z-\sum_{i=1}^{Z}\frac{(Z'-Z)e^2}{r_i}\right]\psi_{Z'n},$$

and we can write

$$\Delta E=\Sigma_n(\psi_{Z0},\psi_{Z'n})(\psi_{Z'n},\psi_{Z0})(E_{Z'n}-E_{Z'0})$$
$$=\Sigma_n(\psi_{Z0},\psi_{Z'n})\left(\psi_{Z'n},\left[H_Z-\sum_{i=1}^{Z}\frac{(Z'-Z)e^2}{r_i}-E_{Z'0}\right]\psi_{Z0}\right).$$

The closure theorem can be used to evaluate the sum over n, and we obtain

$$\Delta E=-\left(\psi_{Z0},\sum_{i=1}^{Z}\frac{(Z'-Z)e^2}{r_i}\psi_{Z0}\right)-(E_{Z'0}-E_{Z0})$$
$$=(Z'-Z)e\varphi(Z)-(E_{Z'0}-E_{Z0}),\quad(5)$$

in agreement with (4). The quantity ΔE is, by its definition, positive. Thus if $E_{Z'0}-E_{Z0}$ is expanded in a Taylor series in powers of $Z'-Z$, the term linear in $Z'-Z$ in (5) must vanish. This can be regarded as a proof of (1).

[1] See, for example, G. Ambrosino and H. Piatier, Compt. rend. **232**, 400 (1951). The problem as it affects β-decay has been discussed by Herman M. Schwartz [Phys. Rev. **86**, 195 (1952)] with whose final conclusion we concur.
[2] L. L. Foldy, Phys. Rev. **83**, 397 (1951).
[3] If one neglects the excitation energy of the excited states of the residual ion, then one can use the completeness relations for the final states of the residual ion and show that the shape of the β-spectrum is not affected, apart from the usual effects of the screened Coulomb field.

Heat Conduction by the Unsaturated Helium II Film

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FOLLOWING the recently reported experiments on mass flow in absorbed He II films,[1] preliminary measurements have been made on the transport of heat by such films.

The method used is similar in principle to that of Bowers, Brewer, and Mendelssohn.[2] A copper-nickel tube 10 cm long, of 0.45-cm i.d. and 0.013-cm wall thickness, had copper chambers attached to top and bottom; the top chamber was provided with a heater; the bottom was in direct contact with the surrounding helium bath. The tube and top chamber were insulated by high vacuum ($<10^{-7}$ mm Hg). Carbon thermometers were located at the center and at both ends of the tube, so that temperature gradients along the tube could be determined. The thermometers were sensitive to 0.0002°.

The heat conductance of the system (without film contribution) was 15 μwatts/degree, so that quite small heat currents could be measured.

The preliminary measurements show:

1. *Saturated films* ($P/P_0=1$, where P is the pressure inside the system and P_0 is the vapor pressure of the bath) show critical heat input rates beyond which the ΔT between top and bottom of the tube rises beyond the experimental range. These critical heat inputs are proportional to the transfer rates of the saturated He II film (Mendelssohn and White), as has already been reported by Bowers *et al.*[2] However, below the critical heating rate there is always a finite ΔT established for all except quite low heat inputs. The ΔT for a given heat input depends on the amount of excess liquid in the system, but the critical heating rate does not.

2. *Unsaturated films* ($P/P_0<1$). For a film of given thickness, determined by P/P_0, the film contribution to the heat transport is zero (<0.1 μwatt) at all temperatures above a sharply defined temperature T_f. At this temperature, the film contribution appears abruptly, rising to relatively large values at lower temperatures, depending on the film thickness and the temperature. The temperature T_f is identical with the "onset" temperature for film flow reported by Long and Meyer (reference 1, method I). In Fig. 1

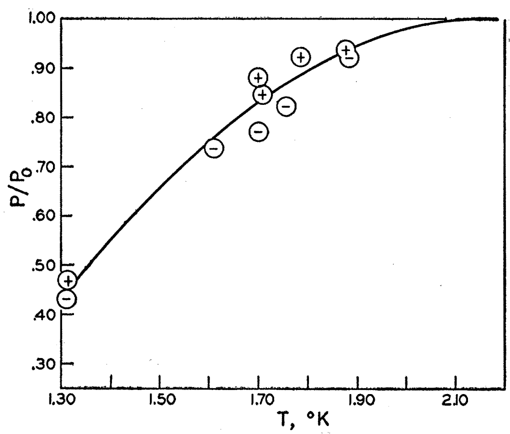

FIG. 1. T_f as function of saturation P/P_0. (T_f is the temperature above which at a certain saturation, the film does not show superfluidity.)

is shown a curve of these temperatures T_f plotted against the saturation P/P_0, as derived from the flow measurements of reference 1. The points shown are from the heat transport experiments. For all temperatures above T_f, no film flow and no high heat transport occur; at all temperatures below T_f, film flow occurs and contributes to the heat transport. No attempt was made to accurately re-determine the curve by the heat transport technique, since the flow method previously used is experimentally simpler and more precise.

There is also in the unsaturated films a finite ΔT established in the heat transport experiments. The heat flow is not a linear function of ΔT, but appears to follow roughly $(\Delta T)^{\frac{1}{3}}$, as in the bulk liquid case. The temperature distribution along the tube is such that the ΔT between center and bottom is finite, but much smaller than that between center and top. This suggests that in the heat transport cycle (which presumably involves film flow from cold to warm parts, evaporation, return flow of vapor through the wide tube, then condensation at the cold end) the vaporization process may be the rate-determining step. If, however, the finite ΔT measured in the lower half of the tube is indeed real, then perhaps the film transport cannot be regarded as a true superfluid process.

The heat transport by the film is being investigated in more detail.

[1] E. Long and L. Meyer, Phys. Rev. **85**, 1030 (1952).
[2] Bowers, Brewer, and Mendelssohn, Phil. Mag. **42**, 1445 (1951).