tion of two neutrons to an odd neutron nucleus or of two protons to an odd proton nucleus pushes the moments towards the Schmidt curves, provided that l and j are unchanged, and of the fact that odd neutron nuclei tend to deviate less from the Schmidt curves than do odd proton nuclei with the same l and j and with comparable A.

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APPENDIX

The evaluation of the matrix elements of $M_1{}^z$ and $M_3{}^z$ is greatly facilitated by the use of the well-known theorem that in the diagonal matrix element of any vector $\mathbf{A}$ whose components satisfy certain commutation rules, $\mathbf{A}$ can be replaced by $[j(j+1)]^{-1}(\mathbf{A}\cdot\mathbf{J})\mathbf{J}$. In the case of $\mathbf{M}_3$, the theorem should be applied immediately. In the case of $\mathbf{M}_1$, the theorem should be applied after the spin of the outer particle has been eliminated, with j now replaced by l. The matrix element then involves a scalar operator and, as such, is independent of the projection of l. Equation (2) follows, finally, upon the use of the relation

$$\mathbf{r}_1\times\mathbf{r}_2\cdot\mathbf{L}_1P_l(\mu)=-i\hbar r_1r_2(1-\mu^2)P_l'(\mu).$$

Specialization to the shell model is then trivial. The shell model evaluation of $\langle M_1{}^z\rangle$ also follows from Appendix I of reference 9 by setting

$$I_{nln'l'}=\Big(\sum_{m=-l}^{l}m^2K_{nlmn'l'}\Big)\Big/\sum_{m=-l}^{l}m^2.$$

The derivation there is more general in that it includes cases in which there are three or more particles outside of the core.

There are certain formal advantages to replacing the factor $[\mathbf{r}_{\pi\nu}\cdot(\boldsymbol{\sigma}_\pi-\boldsymbol{\sigma}_\nu)]\mathbf{r}_{\pi\nu}$ of $\mathbf{M}_3$ by $[\mathbf{r}_{\pi\nu}\cdot(\boldsymbol{\sigma}_\pi-\boldsymbol{\sigma}_\nu)]\mathbf{r}_{\pi\nu}-\frac{1}{3}(\boldsymbol{\sigma}_\pi-\boldsymbol{\sigma}_\nu)r_{\pi\nu}{}^2$, which corresponds to a linear combination of $\mathbf{M}_2$ and $\mathbf{M}_3$, since this has simpler transformation properties under space rotation and Eq. (4) assumes a simpler form.

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Surface Production of Charged Mesons by Photons on Nuclei*

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An estimate has been made of the photoproduction cross section of charged mesons from nuclei, and in particular from "surface-nucleons," i.e., the weakly interacting nucleons which make up the less dense nucleon atmosphere surrounding the main body of a nucleus. Comparison with experiment indicates that the production of mesons from the core of a nucleus is appreciably suppressed, apart from the effects of the initial momentum distribution of the nucleus, and of meson absorption. It is found, for example, that apart from having the correct $A^{\frac{2}{3}}$ dependence for the $\pi^++\pi^-$ cross section, the surface production alone can account for large fractions of the observed yields, and because of differences in average binding between neutrons and protons in nuclei, gives π^-/π^+ ratios which have the same trends as a function of A as the observed ratios. A possible explanation of these results is that there occurs a large competing photodisintegration process as a result of meson exchange effects between strongly coupled nucleons in the interior of a nucleus.

1. INTRODUCTION

The main features of the experimental observations on the production of charged π-mesons from nuclei are the following:

(1) The yields are considerably less than from appropriate equal numbers of free nucleons,[1,2] and

(2) the sum of the π^+ and π^- cross sections exhibits very accurately an $A^{\frac{2}{3}}$ dependence.[1,3]

There are two well-known effects that are in the right direction for explaining these results. Firstly, because of the momentum distribution of nucleons in a nucleus, it is not energetically possible for all the protons or all the neutrons to participate in the production.[4] Secondly, some of the mesons which are produced will be absorbed before they escape from the nucleus. Each of these effects in general reduces the meson yields, and since the absorption of a meson produced in the interior of a nucleus is more probable than for one produced at the surface, the second tends to produce the observed $A^{\frac{2}{3}}$ dependence.[5]

However, on the basis of estimates[6] of the absorption mean free path for mesons in nuclear matter, obtained from the results of meson scattering experiments, the very good experimental $A^{\frac{2}{3}}$ dependence is difficult to understand as due merely to absorption of the mesons. This result might be taken to indicate, therefore, that there is in some way a further suppression of the meson

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[1] R. M. Littauer and D. Walker, Phys. Rev. **83**, 206 (1951); **86**, 838 (1952).
[2] J. Steinberger and A. Bishop, Phys. Rev. **78**, 494 (1950).
[3] R. F. Mozley, Phys. Rev. **80**, 493 (1950).
[4] M. Lax and H. Feshbach, Phys. Rev. **81**, 189 (1951).
[5] Brueckner, Serber, and Watson **84**, 258 (1951).
[6] J. Steinberger (private communication).

production in the interior of a nucleus, and that the actual production from surface nucleons is predominant in the first place. For such a conclusion to be justified, however, it must also be shown that the above two effects are not by themselves sufficient to explain the observed yields, and in particular that the observed cross sections, as well as characteristic features such as the π^- to π^+ ratios, also are compatible with predominant surface production.

In the present paper, the cross sections for charged meson production from surface nucleons is estimated, surface nucleons being defined as those nucleons which the photon catches outside the main core of a nucleus, i.e., with radial coordinates greater than the nuclear "radius" r_0, and which are subject to weak nuclear interaction. It is found that, apart from having the correct $A^{\frac{2}{3}}$ dependence, this effect can account also for large fractions (60–70 percent) of the observed cross sections. Moreover, the average ratio of neutron and proton numbers outside a nuclear core is dependent on the average binding of neutrons as compared with protons in the nucleus, and this introduces a variation with A in the ratio of the yields of negative and positive mesons which has the same trends as the experimental variation.

These results support the view, therefore, that meson production from surface nucleons is predominant, and that production from the interior of a nucleus is largely suppressed. This conclusion is also consistent with what is considered to be a lower estimate of the total production in the event of no such interior suppression (allowance being made for meson absorption by the final nucleus), this estimate being considerably greater than the observed yields. The results indicate, in fact, that the production from the core of a nucleus is suppressed in general by a factor $\gtrsim 3$ (assuming the suppression to occur only in the nuclear core), apart from the effects of the initial momentum distribution of the nucleons and of meson absorption.

Such suppression can, perhaps, be understood as the result of a large competing photodisintegration process due to meson exchange effects between strongly interacting nucleons within the nucleus. When a high energy photon interacts with a nucleon which is itself interacting with one or more neighbors in a nucleus and capable of exchanging energy with them, there exists the possibility that there will occur a direct photodisintegration of this interacting nucleon group via meson exchange.[7] This process would then compete with meson production, some of the mesons which would have escaped the influence of the parent nucleon had it been free being retained by one member of the interacting nucleon group, the excess energy being taken up in the form of a disintegration of the group. Of course, once a meson does escape from the influence of the parent nucleon and any strongly interacting neighbors, there is also the possibility of it being absorbed in other parts of the nucleus before it finally escapes altogether. This also produces a photodisintegration of the nucleus, but is just the absorption allowed for by use of the mean free path obtained from scattering experiments. The process mentioned above is best thought of as causing a damping of the actual meson production in the first place.

The probability of such a competing process will depend, among other things, on the extent to which nucleons in a nucleus are coupled by their interaction. In particular, for photons incident on nuclei, the process will be much more important on the average for nucleons in the core of a nucleus than for surface nucleons, and should therefore enhance the surface production of mesons relative to production from the interior. This is essentially the process recently proposed by Wilson,[8] and if assumed to account for the entire suppression mentioned above, must have a cross section comparable with that of the actual meson production.

In the particular case of the deuteron, the anomalously large cross sections observed for high energy photodisintegration[9–11] seem consistent with the above picture.[12,13] In this case the nucleons are interacting strongly for a small fraction of the time only, and any suppression of meson production will be quite small. Only a relatively small probability for the above competing process is, however, sufficient to explain the high photodisintegration cross section.

2. PHOTOMESON PRODUCTION FROM NUCLEI

We wish to calculate the cross section for production of charged mesons by photons on nuclei, in such a way that we can estimate the production from those nucleons which the photon catches beyond the boundary defined by the radius r_0 of the nucleus. By r_0 we will mean the radius of the central core of a nucleus, inside of which the density of nucleons is roughly constant, but outside of which the density falls off rapidly, any one nucleon being here subject to weak interaction with other members of the nucleus. We shall, in fact, consider a nucleon outside this boundary to be free, and to have an energy (in general negative) depending on the particular state of the nucleus it leaves behind.

In the following calculations we shall, for definiteness,

[7] This effect can be described by particular models, e.g., in terms of the role played by the nucleon isobaric state in photomeson production (see reference 11). In the case of a free nucleon, an intermediate excited state produced by a photon has but two modes of decay—either by re-emission of the photon, or by emission of a meson. For a nucleon interacting with one or more neighbors, however, an intermediate excited state has a third possible mode of decay, *viz.* that in which a meson in the excited state of the first nucleon transfers to the ground state of a second nucleon, the excess energy going into kinetic energy of separation of the nucleon group.

[8] R. R. Wilson, Phys. Rev. **86**, 125 (1952).
[9] T. Benedict and W. Woodward, Phys. Rev. **86**, 629 (1952).
[10] S. Kikuchi, Phys. Rev. **85**, 753 (1952).
[11] W. Gilbert and J. Rose, Phys. Rev. **85**, 766 (1952).
[12] N. Austern, Bull. Am. Phys. Soc. **27**, No. 3, 32 (1952).
[13] R. H. Huddlestone and T. V. Lepore, Bull. Amer. Phys. Soc. **27**, No. 3, 31 (1952).

consider the production of π^+ mesons, that of π^- mesons being completely analogous.

(a) Free Particle Production

We first write the free particle cross section in the form required later for comparison (using throughout the notation employed by Lax and Feshbach[4]).

The matrix element for this process can always be written

$$\langle n|T|p\rangle, \tag{1}$$

$|n\rangle$ representing the final neutron state, $|p\rangle$ the initial proton state, and T being an operator which has the form

$$T=e^{i\mathbf{q}\cdot\mathbf{r}}\tau^+(\mathbf{K}\cdot\boldsymbol{\sigma}+L).$$

Here $\mathbf{r}$ is the space coordinate of the nucleon, $\mathbf{q}$ the difference $(\boldsymbol{\nu}-\boldsymbol{\mu})$ between the photon momentum $\boldsymbol{\nu}$ and meson momentum $\boldsymbol{\mu}$, τ^+ the isotopic spin operator which converts the proton into a neutron, and $\boldsymbol{\sigma}$ the spin operator, while $\mathbf{K}$ and L are matrices that depend, in general, on the photon and meson momenta and polarizations as well as the nucleon momenta. Thus the free proton cross section, in units $\hbar=c=$ meson mass $=$ unity, and for a particular meson momentum (interval $d\boldsymbol{\mu}$) becomes

$$\sigma_f=(2\pi)^{-2}d\boldsymbol{\mu}\int d\mathbf{n}(K^2+L^2)\delta(\mathbf{n}+\mathbf{q})\delta(n_0+\mu_0-\nu_0), \tag{2}$$

where $\mathbf{n}$ is the neutron momentum, n_0 the neutron kinetic energy, $\mu_0=(1+\mu^2)^{\frac{1}{2}}$ the meson energy, and $\nu_0=\nu$ the photon energy. Here we have considered the initial proton to be at rest, and are writing the nucleon energies nonrelativistically.

We will later be making comparisons with experimental results obtained with a bremsstrahlung spectrum, say $f(\nu_0)d\nu_0$. On multiplying (2) by this distribution, therefore, and carrying out the integration over $\mathbf{n}$ and ν_0, we obtain

$$\sigma_f=(2\pi)^{-2}d\boldsymbol{\mu}[(K^2+L^2)f]_{\mathbf{n}=-\mathbf{q},\nu_0=n_0+\mu_0}. \tag{3}$$

The solution of the simultaneous equations $\mathbf{n}=-\mathbf{q}$, and $\nu_0=n_0+\mu_0$ for a given μ tell us, of course, the recoil momentum and energy of the neutron and the particular photon energy responsible for the production.

(b) Nuclear Production Cross Section

The cross section for a nuclear target is

$$\sigma=(2\pi)^{-2}d\boldsymbol{\mu}\int f(\nu_0)d\nu_0|\sum_f\psi_f,\sum_\lambda T_\lambda\psi_i|^2\delta(E_f-E_i), \tag{4}$$

where ψ_i and ψ_f are the initial and final nuclear states, E_i and E_f the initial and final energies of the complete system, and T_λ operates on the coordinates of particle λ. The sum over λ goes over all particles in the nucleus.

To compute the matrix-element $(\psi_f, \sum_\lambda T_\lambda\psi_i)$ we expand the antisymmetrical wave function ψ_i of the initial nucleus with respect to states of the remaining nucleus when one proton (say the αth) has been extracted, and certain single-particle wave functions for the αth proton, i.e.,

$$\psi_i=\sum_{stm_s\mu_\alpha}a^\alpha{}_{st,j_sm_s\mu_\alpha}v_{s,j_sm_s}(\xi_\alpha)g_t(\mathbf{r}_\alpha)\chi(\mu_\alpha), \tag{5}$$

where ξ_α represents all coordinates of the initial nucleus apart from those of the αth proton, and the $v_{s,j_sm_s}(\xi_\alpha)$ are the antisymmetrical wave functions of the states s of the nucleus with the αth proton absent. These states have total spins designated by j_s and orientations m_s. The spatial coordinates of the αth proton are given by r_α, and its spin orientation is μ_α, χ_α being the spin function.

The $g_t(\mathbf{r}_\alpha)$ are wave functions of states t of the proton in some potential field $V(r_\alpha)$, and the expansion (5) is valid for any $V(r_\alpha)$ for which the solutions of the corresponding wave equation (the $g_t(\mathbf{r}_\alpha)$) form a complete and orthogonal set. Eventually we will like to think of $V(r_\alpha)$ as being a good average of the interaction by a proton in the nucleus, and to assume that $V\rightarrow 0$ for $r_\alpha > r_0$ where we identify r_0 with the radius of the nucleus. We assume all the functions in (5) are normalized to unity so that

$$\sum_{stm_s\mu_\alpha}|a^\alpha{}_{st,j_sm_s\mu_\alpha}|^2=1.$$

Such an expansion as (5) is always possible, the antisymmetrization of ψ_i exhibiting itself in certain properties of the a's. It is, of course, equally possible to perform the expansion in terms of wave functions $v_s(\xi_{\alpha'})$ and $g_t(\mathbf{r}_{\alpha'})$ in which the coordinates of a different proton (the α'th) have been chosen for the single particle wave functions. And because of the asymmetry of ψ_i, we will have that

$$a^{\alpha'}{}_{st,j_sm_s\mu}=\pm a^\alpha{}_{st,j_sm_s\mu},$$

the sign being determined by the number of pairs of particle coordinates which must be interchanged in the new expansion in order to put it in a form identical with (5). We may, therefore, choose whichever of the Z such expansions is most convenient; in computing the matrix element $(\psi_f, \sum_\lambda T_\lambda\psi_i)$ we will use the expansion involving a^α with the term T_α acting on proton α, paying attention to the appropriate sign when later converting one a^α into another.

We intend calculating the meson production from the "outside" region of a nucleus; and when the incident photon catches a nucleon outside the nucleus, producing a meson, this nucleon which is essentially free will recoil with a certain momentum $\mathbf{n}$, its final wave function being given by a plane wave $\exp(i\mathbf{n}\cdot\mathbf{r}_\alpha)$. The final wave function ψ_f (f designating a state s of the remaining nucleus with spin orientation m_s, and a spin orientation μ_f of the recoil particle, as well as a particular momentum $\mathbf{n}$) we will take therefore to be the normalized function

$$\psi_f=(2\pi)^{-\frac{3}{2}}Z^{-\frac{1}{2}}[v_{s,j_sm_s}(\xi_\alpha)\exp(i\mathbf{n}\cdot\mathbf{r}_\alpha)\chi(\mu_\alpha)]^{(A)},\quad(6)$$

the notation $[\quad]^{(A)}$ meaning the product within the bracket is to be made antisymmetrical. The normalization factor $Z^{-\frac{1}{2}}$ is not exact; however, we will find that, for cases to which we apply the results, the recoil momentum ($\mathbf{n}$) is much higher than the momentum of a nucleon in any of the states s of importance of the remaining nucleus. The nondiagonal overlap integrals will therefore be very small.

Substituting Eqs. (5) and (6) for the wave functions ψ_i and ψ_f, respectively, and neglecting nondiagonal overlap integrals (the "two-particle" contributions of Lax and Feshbach[4]) the matrix element becomes

$$(\psi_f,\sum_\lambda T_\lambda\psi_i)=(2\pi)^{-\frac{3}{2}}Z^{-\frac{1}{2}}\sum_{\mu_i t}\chi(\mu_f)(\mathbf{K}\cdot\boldsymbol{\sigma}+L)\chi(\mu_i)\sum_{\alpha=1}^{Z}\pm a^\alpha{}_{st,j_sm_s\mu_i}\int\exp[i\mathbf{r}_\alpha\cdot(\mathbf{q}+\mathbf{n})]g_t(\mathbf{r}_\alpha)d\mathbf{r}_\alpha.\quad(7)$$

The signs of the a^α's are, however, such as to give all terms under the summation the same sign. Equation (6) becomes therefore

$$(\psi_f,\sum_\lambda T_\lambda\psi_i)=\pm(2\pi)^{-\frac{3}{2}}Z^{\frac{1}{2}}\sum_{\mu_i t}\chi(\mu_f)(\mathbf{K}\cdot\boldsymbol{\sigma}+L)\chi(\mu_i)\times a_{st,j_sm_s\mu_i}\int\exp[i\mathbf{r}\cdot(\mathbf{q}+\mathbf{n})]g_t(\mathbf{r})d\mathbf{r},\quad(8)$$

where the superscript α can now be dropped.

On substituting (8) in (4) and performing the summation over final states and integrating over ν_0, we obtain

$$\sigma=(2\pi)^{-5}Zd\boldsymbol{\mu}\sum_{sm_s\mu}\int d\mathbf{n}[(K^2+L^2)f]_{\nu_0=n_0+\mu_0+\epsilon_s}\times\left|\sum_t a_{st,j_sm_s\mu}\int\exp[i\mathbf{r}\cdot(\mathbf{q}+\mathbf{n})]g_t(\mathbf{r})d\mathbf{r}\right|^2,\quad(9)$$

where ϵ_s is the binding energy of a proton in the initial nucleus when the remaining nucleons are in the state s.

It is of interest to note that if the integration over $\mathbf{r}$ in (9) is taken over all space, then (9) would give the complete production cross section under the approximation of neglecting the interaction of the emerging neutron, as well as the meson, with the residual nucleus (if there were no damping factor of the type discussed in the introduction). This is the procedure of Lax and Feshbach,[4] who further assume that the most important of the residual states s have a small energy spread. Under these circumstances Eq. (9) becomes immediately

$$\sigma=Z(2\pi)^{-2}d\boldsymbol{\mu}\int d\mathbf{n}[(K^2+L^2)f]_{\nu_0=n_0+\mu_0+\epsilon}\rho(\mathbf{n}+\mathbf{q}),\quad(10)$$

where $\rho(\mathbf{k})$ is the normalized distribution of momenta $\mathbf{k}$ in the initial nucleus, and where ϵ is the average value of ϵ_s, i.e., is the average binding energy of a proton in the initial nucleus. This is precisely the result given by Lax and Feshbach, but actually derived by them only on the basis of the Hartree model of nuclear structure.

The integrand of (10) has a maximum when $\mathbf{n}\simeq-\mathbf{q}$, i.e., at or near the Compton line for free production, and if the factor $[(K^2+L^2)f]_{\nu_0=n_0+\mu_0+\epsilon}$ is substantially constant over the region where $\rho(\mathbf{n}+\mathbf{q})$ is large, the cross section becomes of course merely $Z\sigma_f$. However, because of the large spread in the momenta available to nucleons in a nucleus, the width of $\rho(\mathbf{n}+\mathbf{q})$ is quite large and the cross section is usually reduced due to the product $[(K^2+L^2)f]$ becoming zero over part of the significant range of integration of $\mathbf{n}$ (corresponding ν_0 above the cutoff of the bremsstrahlung spectrum). An exception to this, which will be important in the case to which we apply the results, will be discussed shortly.

For computing the surface effect we take the integral over $\mathbf{r}$ in (9) only over the region $r>r_0$ (the nuclear radius). In this event Eq. (9) becomes

$$\sigma=Z(2\pi)^{-2}\tau d\boldsymbol{\mu}\times\int d\mathbf{n}[(K^2+L^2)f]_{\nu_0=n_0+\mu_0+\epsilon}\rho'(\mathbf{n}+\mathbf{q},r_0),\quad(11)$$

where $\rho'(\mathbf{k},r_0)$ is the normalized distribution of momenta for protons outside the core of the initial nucleus, and where we have again represented the shift in the energy equation from that for free production by the average value ϵ. In (11) also, τ is the fraction of the time which a proton in the initial nucleus spends outside the core of radius r_0, i.e.,

$$\tau=\sum_{sm_s\mu}\int_{r>r_0}d\mathbf{r}\left|\sum_t a_{st,j_sm_s\mu}g_t(\mathbf{r})\right|^2=\int_{r>r_0}d\mathbf{r}\left|\sum_t A_t g_t(\mathbf{r})\right|^2=\int_{r>r_0}D(\mathbf{r})d\mathbf{r},\quad(12)$$

where $D(\mathbf{r})$ is the normalized density distribution of protons in the initial nucleus.

The integrand of (11) also has a maximum when $\mathbf{n}\simeq-\mathbf{q}$, but here the spread about this point is appreciably less than in (10), due to the fact that a nucleon beyond the range of interaction with other nucleons in general has a low momentum, and the entire spread of momenta given in (10) by $\rho(\mathbf{n}+\mathbf{q})$ is not available to these nucleons. This may be seen from the fact that, provided our potential $V(r)=0$ for $r\geqslant r_0$, we can write

$$g_t(r)=\sum_{lm}b_{t,lm}\frac{K_{l+\frac{1}{2}}(\kappa_t r)}{r^{\frac{1}{2}}}Y_{lm}(\theta\varphi)=\sum_{lm}b_{t,lm}'\frac{e^{-\kappa_t r}}{r}Y_{lm}(\theta\varphi)\sum_{n=0}^{l}\frac{(l+n)!}{n!(l-n)!(2\kappa_t r)^n},\quad(13)$$

where $K_{l+\frac{1}{2}}$ is a Bessel function of imaginary argument. The spread in ρ' is determined essentially by something like the average value of κ_t (i.e., $\sum_t |A_t|^2\kappa_t$), corresponding to an energy equal to the average binding energy ϵ of a proton in the initial nucleus ($\simeq 8$ Mev) rather than approximately 20 Mev as for the general distribution in (10).

In cases where the product $[(K^2+L^2)f]_{\nu_0=n_0+\mu_0+\epsilon}$ is substantially constant over the significant spread of ρ', (11) becomes, as in the similar case for Eq. (10), merely

$$\sigma = \sigma_f Z\tau. \tag{14}$$

Also, for the same reason as in the case of Eq. (10), the surface cross section will usually be somewhat lower than this, although the reduction is less than for Eq. (10) because of the smaller spread in momenta in this case. However, for the conditions appropriate for a comparison, as will be made in the next section, with the experimental results of Littauer and Walker,[1] Eq. (14) should be a quite good approximation to the surface cross section, and may even slightly underestimate it. These experimental results were obtained by observing mesons with energies in the interval 50–80 Mev at an angle of 135° to the photon direction, and which were produced by a 310-Mev maximum energy bremsstrahlung spectrum from the Cornell synchrotron. In Fig. 1 is plotted the product $(K^2+L^2)_{\mathbf{n}=-\mathbf{q},\nu_0=n_0+\mu_0}$ against ν_0 and the corresponding meson kinetic energy at this angle, making use of the (corrected) cross section for meson production from hydrogen as obtained by Bishop, Steinberger, and Cook,[14] as well as the known bremsstrahlung spectrum. It is seen that free-particle production of mesons in the range 50–80 Mev utilizes photons in the energy range 265 to 330 Mev. (The corresponding recoil energies lie between 75 and 110 Mev.) In particular, the meson energy 65 Mev corresponds to a photon energy of 298 Mev, which is in the region where the $[(K^2+L^2)f]$ curve is dropping rapidly to zero. Hence the free-particle yield of Littauer and Walker, to which all their other results are referred, contains contributions from mesons essentially in the energy region 50–65 Mev only.

For production from a nucleus there is, as just discussed, no such unique photon energy corresponding to a given meson energy. For each meson energy there is a spread in photon energies about (approximately) the free-particle value which, in the case of surface production for these energies, has a width ~ 50 Mev. Thus the probability per nucleon of producing a meson with energy between 50–65 Mev from a nuclear surface is less than the corresponding free particle probability, since the photon energy spread will continue above the cutoff of the $[(K^2+L^2)f]$ curve. On the other hand, such loss of yield for mesons in the lower half of the accepted energy region will be largely counterbalanced by the corresponding increase in yield for mesons in

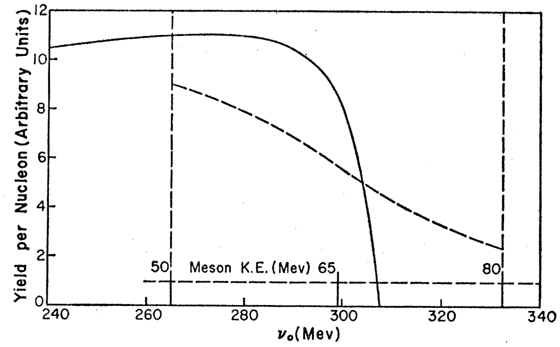

FIG. 1. Energy dependences for the yields of charged mesons produced from nucleons by photons from a bremsstrahlung spectrum. The full curve is for free-particle production as a function of the photon energy and the corresponding meson kinetic energy. The dotted curve gives the corresponding yield per nucleon from surface nucleons as a function of meson kinetic energy only. The ordinates at 50- and 80-Mev meson kinetic energy are the energy limits of the mesons observed in the experiments of Littauer and Walker (see reference 1).

the upper half of this region, particularly since the cross section carries a weighting factor μ^2 from the volume element in the meson momentum space.

As an example, suppose we take ρ' of the form

$$\rho' = \kappa_{\mathrm{Av}}/[\pi^2(\kappa_{\mathrm{Av}}^2+k^2)],$$

where κ_{Av} corresponds to an energy of 8 Mev. Then from Eq. (11) we find (taking also $\epsilon = 8$ Mev in the energy equation) that the surface yield per nucleon as a function of meson energy is as given by the dotted curve in Fig. 1. The total area under this curve between the limits 50<meson K.E.<80 Mev is very similar to that under the free-particle curve. Thus Eq. (14) should be a good approximation to the surface cross section for the interval of meson energies considered.

Arguments similar to the above also indicate that Eq. (10), which gives the total production cross section in the event of there being no interior suppression, should give $\sigma \simeq Z\sigma_f$ for the above experimental conditions (although there is some small reduction in this case due to the larger spread of momenta available). Equation (10), however, does not take into account the interaction of the recoil particle or of the meson with the final nucleus, and although small for surface production, the effects of these may well be important for interior production. Indeed, if the average interaction of the recoil nucleon with the final nucleus is represented by an attractive well of depth 30 Mev, the effect of this is to lower the most probable photon energy corresponding to a given meson energy by about 50 Mev from the free-particle value (see appendix). This will greatly enhance the production, since more of the required photon energies are now available in the bremsstrahlung spectrum, and moreover the magnitude of the product $(K^2+L^2)f$ at the appropriate energies will be appreciably greater. The latter effect is due to the fact that the beginning of the rise of the (K^2+L^2) function[14] (threshold) occurs for lower photon energies,

[14] Bishop, Steinberger, and Cook, Phys. Rev. **80**, 291 (1950).

i.e., larger f values, so that the above product attains higher values than in the free-particle case. On the other hand, if the average interaction of the meson with the final nucleus is represented by an attractive well of depth 30 Mev (see Sec. 3), this raises the above most probable photon energy by about 30 Mev. The over-all effect of these interactions is, however, to lower the photon energies utilized (on the above model by 20 Mev), and hence to produce a corresponding increase in the cross section over that given by Eq. (10).

It is estimated, in fact, that if there is no suppression of meson production in the interior of a nucleus, the total yields under the conditions of the above experiments, before allowing for meson absorption, should certainly satisfy

$$\sigma \gtrsim Z\sigma_f, \tag{15}$$

and this inequality will be sufficient for any later discussion. The above lower limit would actually be higher if the well depth to be taken for the meson attractive interaction were appreciably less than 30 Mev, and very much so if this interaction were in part repulsive.[15]

It is also worth noting that, although the average interaction experienced by a nucleon or meson in the nuclear surface is expected to be much smaller than in the interior, the effect of any such interaction should be to increase the surface yield over the value given by (14).

To estimate τ in (14) we first note from (12) and (13) that, for most of the orbital angular momentum states and binding energies of importance in nuclei,

$$\int_{r_0}^{\infty} D(\mathbf{r})dr \simeq \sum_{tt'} \frac{A_t A_{t'}^*[g_t(\mathbf{r})g_{t'}^*(\mathbf{r})]_{r=r_0}}{\kappa_t+\kappa_{t'}}. \tag{16}$$

Now when we choose our generating function $V(r)$ to be the best possible such potential to represent the average interaction experienced by a proton in the initial nucleus (and therefore $\rightarrow 0$ at $r=r_0$) we expect the important coefficients A_t to have corresponding values of κ_t which represent energies congregating in the region of the actual binding energies for protons in the nucleus, i.e., with an average value ~ 8 Mev.[16] If we replace $\kappa_t+\kappa_{t'}$ approximately by $2\kappa_{\mathrm{Av}}=2(2m\epsilon)^{\frac{1}{2}}$ therefore, we obtain

$$\int_{r_0}^{\infty} D(\mathbf{r})dr = \frac{\rho(r)_{r_0}}{2\kappa_{\mathrm{Av}}}. \tag{17}$$

To obtain τ from (17) we must assume something about the density within the central core; if, for example, for a given orientation of $\mathbf{r}$, $D(\mathbf{r})$ is taken to be constant for $0 \leqslant r \leqslant r_0$, we obtain

$$\tau = 1/(1+\tfrac{2}{3}\kappa_{\mathrm{Av}} r_0), \tag{18}$$

and this is the estimate we use.

In the case of π^- production the same formulas are obtained, of course, with Z replaced by N, the κ_{Av} of (18) being now given by the average binding energy of a neutron in the initial nucleus rather than a proton.

3. COMPARISON WITH EXPERIMENT AND DISCUSSION

Since $\kappa_{\mathrm{Av}} r_0$ is in general substantially greater than unity, it is seen, as was to be expected of course, that the sum of the π^- and π^+ surface cross sections has an $A/r_0 = A^{\frac{2}{3}}$ proportionality.

The magnitude of the $\pi^+ + \pi^-$ surface yield for different elements, obtained from (14) and (18), is plotted in Fig. 2 and compared with the experimental results of Littauer and Walker.[1] The κ_{Av} used was obtained directly from the average binding energy of a proton or a neutron in the appropriate initial nucleus, allowance being made for the Coulomb differences between the two cases. The radius r_0 was taken to be $1.2\times 10^{-13}A^{\frac{1}{3}}$ cm. On the nuclear model which assumes constant density out to a certain radius r_0, and zero beyond, the more usual value of r_0 is $1.45\times 10^{-13}A^{\frac{1}{3}}$ cm. However, when the diffuse surface region in which the density falls off in a distance $\sim\frac{1}{2}\kappa_{\mathrm{Av}}$ is considered, the experimental results on neutron scattering indicate a radius of the core appreciably less than this,[17] and also the mass differences between mirror nuclei (interpreted as due mainly to differences in Coulomb energy) give approximately the value 1.2 for the coefficient.

The results in Fig. 2 have all been normalized to unity for the production of π^+ mesons from hydrogen (for the theoretical values the free-particle π^-/π^+ ratio being taken as 1.2).[1] Also, the theoretical values have already been reduced from those given directly by Eq. (14), on the average by $\sim$40–50 percent, to allow for the absorption of some of the surface meaons as they pass through the nucleus before escaping. For estimating this the mean free path for absorption of charged mesons in a nuclear core was taken to be 9.5×10^{-13} cm, and the interaction of a meson with the nucleus represented by an attractive well whose shape is that of the density distribution of nucleons in the nucleus (approximated by a well of trapezoidal shape), and whose central depth is approximately 30 Mev. This is the model employed by Steinberger,[6] and the above mean free path and well depth are the results obtained by him, in interpreting experimental results of Byfield, Kessler and Lederman,[18] and of Sachs and Steinberger[19] concerning the inelastic cross section and elastic scattering of 60-Mev mesons. [This is about the average

[15] R. LeLevier, Bull. Am. Phys. Soc. 27, No. 3, 41 (1952).

[16] In the case of the independent particle model of nuclear structure, the sum over t has Z terms, and the number of different values of κ_t is equal to the number of proton subshells filled or partially filled in the nucleus. The limits of the spread in κ_t are given by the binding energy of a proton in the lowest subshell and that in the highest, the average value κ_{Av} being given by the average of the κ_t's, each one being weighted by the number of protons in the appropriate subshell.

[17] R. Jastrow and J. E. Roberts, Phys. Rev. **85**, 757 (1952).

[18] Byfield, Kessler, and Lederman, Phys. Rev. **86**, 17 (1952).

[19] A. Sachs and J. Steinberger, Phys. Rev. **82**, 958 (1951).

energy of the mesons observed by Littauer and Walker (see Fig. 1).]

It is seen that quite large fractions of the observed yields can be accounted for by surface production. Moreover, it is of interest to note that, because of differences in the average binding energies of neutrons as compared to protons in a nucleus, and thus in the number of neutrons as compared to protons in the nuclear surfaces [Eq. (18)], there is a variation in the ratio of the individual π^- to π^+ surface yields which has the same trends as the experimental ratios. In the case of Be^9, for example, if the small binding energy difference between this nucleus and Be^8 be attributed to the additional neutron, the effect of such a lightly bound neutron which spends a large amount of its time on the outside of the nucleus is to increase the π^- surface yield relative to the π^+ by more than just the ratio N/Z of the total neutron and proton numbers. For symmetrical light nuclei such as C^{12}, the average binding energies and therefore the π^- and π^+ surface yields are about equal, but for heavier such nuclei the Coulomb interaction causes the protons to be less bound than the neutrons. There is, therefore, for symmetrical nuclei, a gradual decrease of the surface π^-/π^+ ratio as A is increased up to Ca^{40}, the last stable such nucleus. For nuclei in this range which have an additional odd neutron, the ratio will be above the general trend just mentioned. For heavy elements in which the neutrons appreciably exceed the protons in number, the average binding energies of the two again become about equal, and the π^-/π^+ surface ratios become merely the ratio of total neutron and proton numbers in the nucleus.

These trends are all in agreement with the experimental observations, although the magnitudes of the variations are not as large as the experimental ones. To take the two extremes, for example, one would expect on the above basis a π^-/π^+ ratio of something like 1.6—1.7 for Be^9 compared with the observed value 2.2, and for Ca^{40} something like 0.8 compared with the observed value of 0.6. However with much of the variation thus accounted for, there are some further effects which, although in themselves probably quite small, should bring the magnitudes of the variations more closely into line with experiment when added to the above. These may be summarized as follows:

(a) The average binding of a neutron or proton enters not only into the expression for τ [Eq. (18)] but also in the energy equation $\nu_0 = n_0 + \mu_0 + \epsilon$ of (11). The larger ϵ the higher the photon energies necessary to produce mesons of a given energy, and the differences between neutron and proton binding energies will in this way, therefore, result in slightly different π^- and π^+ yields. The resultant deviations from unity of the π^-/π^+ ratios will certainly be in the right direction, but will be quite small.

(b) There are essentially different lower limits on the recoil nucleon energies for π^- and π^+ production, due

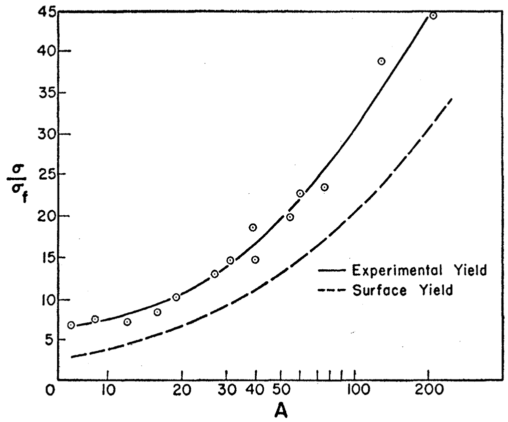

FIG. 2. The yields of charged mesons, produced by photons incident on nuclei, as a function of A. The full curve is drawn through the experimental points of Littauer and Walker (see reference 1) and the dotted curve gives the yields to be expected for surface production.

to differences in binding energies of the ground states of the two final nuclei.[20] The fact, however, that the most probable recoil energies are $\sim$100 Mev leads to a very small probability of forming the final nuclei in low-lying states (the reason for being able to neglect "two-particle" contributions when obtaining the cross section). Any deviations from unity of the π^-/π^+ ratio resulting from the foregoing must therefore be quite small, although again they will be in the right direction (see the correlation between the binding energy differences and the π^-/π^+ ratios given by Littauer and Walker[1]). Estimates by Bethe and Hayakawa[21] indicate, in fact, that for production from the whole nucleus, these deviations will amount to less than 10–20 percent, and this will be less for surface production.

(c) Although in deriving (11) we assumed no interaction for nucleons outside the nuclear radius, there will in general be some weak interaction which has the effect of lowering the photon energies utilized. Any difference in this average interaction for neutrons and protons may therefore cause differences in the π^- and π^+ yields in the same way as do differences in the binding energies (a). Similarly, differences in the average interactions of π^- and π^+ mesons with surface nucleons may produce differences in the yields, and in particular the Coulomb interaction will tend to increase the π^+ yield relative to the π^-. We again expect these to be rather small effects for the energies under consideration,[22] except perhaps for the different Coulomb interactions just mentioned which might be expected to

[20] G. F. Chew and J. Steinberger, Phys. Rev. **78**, 497 (1950).

[21] H. A. Bethe and S. Hayakawa (private communication).

[22] Of course if the upper limit of the photon spectrum were appreciably lower than 310 Mev (say 250 Mev), then for production of the same energy mesons (about 65 Mev) the deviations from unity in the π^-/π^+ ratios by effects (a) and (c) would be much greater than for the energies we are considering. Here the total yields would be very much smaller in the first place, and the differences in the π^- and π^+ yields produced by these effects would be relatively much more important.

play a more important role for heavy elements. The observed π^-/π^+ ratios for heavy elements appear, however, to be little influenced by this.

It seems therefore that the main features of the experimental results of Littauer and Walker are compatible with surface production alone, and that the interior production must contribute to a small extent only in these results and must therefore be quite heavily damped. In fact, on referring to (15) and estimating the absorption of mesons produced in the interior of a nucleus by means of the model previously mentioned, we conclude that the production from the interior of a nucleus must be suppressed by a factor $\gtrsim 3$.

Obviously the view that no nucleons outside the boundary r_0 interact with other nucleons and that the above suppression of meson production must occur only for nucleons within a certain core, is an idealization. It is true, nevertheless, that nucleons near the outside of a nucleus are subject, on the average, to much weaker interaction than those well within a nucleus. Thus the above results indicate that there is a suppression of meson production, and that this suppression occurs mainly in the case of production from nucleons which are subject to relatively strong nuclear interaction. This is compatible with the idea that the damping is due to the competing process discussed in the Introduction, of a photodisintegration via meson exchange between strongly coupled nucleons. This process should, if this be the case, have a cross section comparable with that of the actual meson production.

In conclusion I would like to thank Professors H. A. Bethe and R. R. Wilson, and Dr. N. Austern and Dr. E. E. Salpeter for interesting and informative discussions, and also Dr. R. M. Littauer for details of experimental results prior to their publication.

APPENDIX

1. For free-particle production, the conservation of energy and momentum equations are

$$\nu_0 = n_0 + \mu_0, \quad \boldsymbol{\nu} = \mathbf{n} + \boldsymbol{\mu}.$$

The solution for ν_0 from these equations is

$$\nu_0 = \tfrac{1}{2}(2mc^2 + \cos\vartheta\,\mu_0 v/c) \times\left[1 - \left\{1 - \frac{4(2mc^2\mu_0 + \mu_0^2 v^2/c^2)}{(2mc^2 + \cos\vartheta\,\mu_0 v/c)^2}\right\}^{\frac{1}{2}}\right],$$

where mc^2 is the rest energy of a nucleon, v the velocity of the meson, and ϑ the angle between the meson and photon directions. For $\mu_0 = 215$ Mev (K.E. $= 65$ Mev) and $\vartheta = 135°$, for example, this yields $\nu_0 = 298$ Mev.

2. Production from a nucleus

We consider production from a nucleon which leaves the rest of the (A-1) nucleus in a state s. The energy of this nucleus is $-\epsilon_s$, whatever its kinetic energy, and for the high recoil energies pertaining to the cases of interest to us, the energy of the rest of the nucleus remains unchanged by the production process. Using the average value $-\epsilon$ for the energy of the initial nucleon, the conservation of energy condition is therefore

$$\nu_0 - \epsilon = n_0' + \mu_0',$$

where n_0' and μ_0' are the energies of the recoil nucleon and meson after they escape from the nucleus.

The average or most probable momentum of the initial nucleon is $\mathbf{k} \simeq 0$ and the most probable photon energy can therefore be obtained from the additional equation

$$\boldsymbol{\nu} = \mathbf{n} + \boldsymbol{\mu}$$

as in the free-particle case. Here $\mathbf{n}$ and $\boldsymbol{\mu}$ are the recoil nucleon and meson momenta inside the final nucleus (corresponding energies n_0 and μ_0). If there is no interaction of these particles with the final nucleus, n_0' and $\mu_0' = n_0$ and μ_0. If, however, the interaction of the recoil nucleon be represented by an attractive well of depth V_n, and that of the meson by an attractive well of depth V_μ, the energy equation becomes

$$\nu_0 = n_0 + \mu_0 + \epsilon - V_n - V_\mu.$$

The solution giving the most probable photon energy is now

$$\nu_0 = \tfrac{1}{2}(2mc^2 + \cos\vartheta\,\mu_0 v/c) \times\left[1 - \left\{1 - \frac{4(2mc^2(\mu_0 + \epsilon - V_n - V_\mu) + \mu_0^2 v^2/c^2)}{(2mc^2 + \cos\vartheta\,\mu_0 v/c)^2}\right\}^{\frac{1}{2}}\right].$$

For comparison with the free-particle case, we take, for example, $\mu_0' = \mu_0 - V_0 = 215$ (external K.E. $= 65$ Mev) and $\vartheta = 135°$ (and put $\epsilon = 0$). Then

(a) for $V_n = 30$ Mev and $V_\mu = 0$, $\nu_0 \simeq 248$ Mev;
(b) for $V_n = 0$, $V_\mu = 30$ Mev, $\nu_0 \simeq 328$ Mev;
(c) for $V_n = 30$ Mev, $V_\mu = 30$ Mev, $\nu_0 \simeq 278$ Mev.

Thus the over-all effect of the nucleon and meson interactions with the final nucleus is to lower the photon energies utilized.