the values of constants (Q) were set equal. The absolute value of Q is then the only remaining arbitrary constant (except the normalization constant needed to fit any particular set of data) and was determined from the slope of the data in the region around 9 Mev. The resulting equation is

$$N(E)=1.088\times 10^6 \sinh(1.92E^{\frac{1}{2}})e^{-E/0.875}+2.937\times 10^6 \sinh(1.28E^{\frac{1}{2}})e^{-E/0.875}. \quad (7)$$

Both Eqs. (6) and (7) show curvature on a logarithmic plot, while the data fall more nearly on a straight line. The deviation of the data is of the same order of magnitude as the systematic error expected from the arbitrary join in the curves used for the (n,p) scattering cross section, and so cannot be taken as real. To explore the implications of a real deviation like that observed and to find an equation giving a fit passing more nearly through all points, the conditions of equal number from each group and equal Q for each group were abandoned.

The curvature of all the calculated spectra can be removed by assuming different Q's for the neutron emitting fragments, and several cases were calculated. Equations were found which passed within the standard deviation of each point, but the increased complexity seems hardly justified with the present data and the simplified assumptions leading to Eq. (6). However, the improvement of the fit seems to indicate that Feather's formula can be made to fit the data by assuming many fragments having different velocities, masses, and excitations.

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Conservation of Isotopic Spin in Nuclear Reactions*

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The effects of the charge independence of nuclear forces on nuclear reaction experiments is discussed. It is pointed out that charge independence establishes relationships between cross sections for some reactions and results in forbidding certain other reactions.

It is well known[1,2] that if the forces between two nucleons are independent of their charge states and depend only on their space and spin coordinates, the wave function of a system of nucleons will be invariant under certain charge transformations. This symmetry is usually described in terms of a constant of motion, the isotopic spin. Important consequences of this invariance of nuclear structure with respect to rotation in isotopic spin space have been discussed by Wigner[3,4] and others.

There is no very conclusive evidence to establish the charge independence of nuclear forces. While the similarity of energy levels of mirror nuclei strongly suggests the equivalence of neutron-neutron forces and proton-proton forces,[5] there is not yet much information on the equality of the neutron-proton interaction and the forces between like nucleons. In particular, no definite conclusions have been derived from nucleon-nucleon scattering. Although the low energy scattering experiments are not in contradiction to a description in terms of charge independent forces,[6] it is not clear that this is true of higher energy measurements.[7]

It appears to have been largely overlooked in recent work that information on the charge independence of nuclear forces may be obtained from observations on nuclear reactions. Charge independence has observable consequences in such reactions; in particular, it results in forbidding certain transitions which are allowed solely from considerations of spin and parity.

For the purpose of this discussion the third component T_3 of the isotopic spin T of a nucleus is defined, as usual, as the number of neutrons minus the number of protons in the nucleus divided by two. Systems having the same isotopic spin but different T_3 components form a set of multiplicity $2T+1$, which differ in energy only through Coulomb forces. On the assumption that it is a good quantum number, the value of T for any nucleus is easily determined by an examination of the binding energies of nuclei.[8,9] Generally the binding energy of light nuclei depends strongly upon T. Since there is close competition between low states with different values of T only in the

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1 E. Wigner, Phys. Rev. **51**, 106 (1937).
2 N. Kemmer, Proc. Cambridge Phil. Soc. **34**, 354 (1938).
3 E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
4 E. Wigner, Phys. Rev. **56**, 519 (1939).
5 See, e.g., V. R. Johnson, Phys. Rev. **86**, 302 (1952).
6 J. Schwinger, Phys. Rev. **78**, 135 (1950).
7 R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1950).
8 E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).
9 Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).

$4n-2$ group of nuclei, most of the interesting applications of charge symmetry selection rules take place in reactions involving these systems. A typical example of such a system is the C^{14}, N^{14}, O^{14} triad. The isotopic spin function representing the ground state of N^{14} can be written as φ'^0_0, of C^{14}, φ_1^1, and of O^{14}, φ_1^{-1}, where the superscript represents the value of the T_3 component and the subscript the value of T. The φ_1^0 state will be an excited state of N^{14} displaced in energy from the O^{14} and C^{14} ground states by Coulomb forces and the neutron-proton mass difference. This appears to be the 2.3-Mev level of N^{14}.

The transformation properties of isotopic spin are the same as for angular momentum. Since the assumption of charge independence is equivalent to requiring the conservation of total isotopic spin and conservation of charge insures the conservation of the T_3 component, isotopic spin selection rules are identical with those of angular momentum. An example of a reaction forbidden because of isotopic spin selection rules is the $O^{16}(d, \alpha)$ reaction to the 2.3-Mev state of N^{14}. Since the isotopic spin of O^{16}, the deuteron, and the alpha-particle are all zero, while the N^{14} level has an isotopic spin of one, the transition will not be allowed. Although this reaction has been investigated by several groups[10–12] under a variety of experimental conditions it has not been observed.

There are a number of other reactions which should be forbidden by analogous considerations. The $C^{12}(d, \alpha)$ reaction to the 1.7-Mev state of B^{10} should not be observed, as this level appears to be the φ_1^0 state of the Be^{10}, B^{10}, C^{10} triad of total isotopic angular momentum one. The $C^{12}(d, \alpha)$ reaction has not been investigated at energies which would excite the 1.7-Mev level. For similar reasons the $Ne^{20}(d, \alpha)$ reaction to states in F^{18} which have corresponding levels in O^{18} should be forbidden, as these transitions would also violate the conservation of isotopic spin. Middleton and Tai investigated[13] the $Ne^{20}(d, \alpha)$ reaction and found a prominent group of alpha-particles which leave the F^{18} nucleus in a 1.05-Mev excited state. This is near the energy where one would expect the isotopic spin-one level corresponding to the ground state of O^{18}. According to the shell model F^{18} consists of a closed P-shell plus a neutron and a proton. There is close competition between $S_{\frac{1}{2}}$ and $D_{\frac{5}{2}}$ states in this region. Since the beta-decay of F^{18} to O^{18} is allowed, the ground state of F^{18} probably belongs to the spin one, isotopic spin zero, triplet $(S_{\frac{1}{2}})^2$ configuration. The spin zero, singlet $(S_{\frac{1}{2}})^2$ configuration with isotopic spin one, corresponding to the ground state of O^{18}, would be somewhat higher and might be close to the lowest $(D_{\frac{5}{2}})^2$ level, probably with spin five and isotopic spin zero. It therefore seems possible that this reaction does not violate charge independence but that the 1.05-Mev state reached by this reaction is an isotopic spin-zero level very close in energy to the isotopic spin one level corresponding to the ground state of O^{18}.

The inelastic scattering of deuterons and alpha-particles by Li^6, B^{10}, and N^{14} will be affected by charge independence selection rules. Since the ground states of these nuclei, and the deuteron or alpha-particle, all have an isotopic spin of zero, the low-lying isotopic spin-one levels of these nuclei should not be excited appreciably in the reactions.

It must be emphasized that, since the transition probability depends upon the square of the matrix element, reaction experiments are not a sensitive test of charge independence. A reaction cross section leading to a state consisting of a mixture of 10 percent of a wave function to which the transition were allowed and 90 percent of a wave function to which the transition were forbidden would result in a cross section only of the order of one percent of a completely allowed reaction. Conversely, the selection rules should hold reasonably well even if nuclear forces are only approximately charge independent. It is therefore necessary to place rather small upper limits on the cross sections for forbidden reactions in order to restrict very severely the dependence of nuclear forces on the nuclear charge. Coulomb forces will couple states of different isotopic spin but their influence on the wave functions of light nuclei is probably small. However, one cannot then expect to detect differences in the specifically nuclear forces of the magnitude of the Coulomb forces.†

It is sometimes easier experimentally to place a small limit on level widths rather than on reaction cross sections. If elastic scattering is the most probable process taking place, the scattering cross section at resonance is reasonably large and practically independent of the magnitude of the reaction matrix element. The width of the level is proportional to the square of the matrix element. There exist levels[14] in B^{10} between 4.5 and 6.5 Mev above the ground state which can break up through heavy particle emission only to the isotopic spin-zero combinations of Li^6 plus an alpha-particle, or Be^8 plus a deuteron. If the specifically nuclear forces are charge independent, such transitions from isotopic spin one states will only occur through the difference in neutron and proton wave functions caused by Coulomb forces. This effect should

[10] A. Ashmore and J. F. Raffle, Proc. Phys. Soc. (London) **A64**, 754 (1950).
[11] Burrows, Powell, and Rotblat, Proc. Roy. Soc. (London) **A209**, 478 (1951).
[12] Craig, Donahue, and Jones (to be published); Van de Graff, Sperduto, Buechner, and Enge, Phys. Rev. **86**, 966 (1952).
[13] R. Middleton and C. T. Tai, Proc. Phys. Soc. (London) **A64**, 801 (1951).

† *Note added in proof:*—I wish to thank Dr. N. M. Kroll for pointing out to me that selection rules will obtain in self-conjugate nuclei from the less severe condition of charge symmetry of nuclear forces. [See also in this connection Lynne E. H. Trainor, Phys. Rev. **85**, 962 (1952).] The evidence for charge independence in reactions involving these systems then depends upon the conjoint existence of selection rules affecting isotopic spin-one states, and the existence and energy equivalence of isobaric members of the charge triplet.
[14] Fay Ajzenberg, Phys. Rev. **82**, 43 (1951).

be small, and isotopic spin-one states should have very small widths. It might be possible to observe such states and measure their widths by scattering alpha-particles from Li^6.

Isotopic spin-one states of Li^6 could be investigated in a similar manner by scattering deuterons from helium. The lowest isotopic spin-one level is available energetically but, as Fowler[15] has pointed out, probably has spin zero and even parity and cannot therefore decay into a deuteron and an alpha-particle. There should, however, exist a level of Li^6 of even parity, isotopic spin one, probably of total angular momentum two, about 6 Mev above the ground state, corresponding to the first excited state of He^6. This state is forbidden by charge conservation from breaking down to a deuteron and an alpha-particle and should, therefore, have a very small width. It is possible energetically for this state to break up into He^5 plus a proton. If this width is much larger than the scattering width the resonance will be damped out and not easily observed by bombarding alpha-particles with deuterons.

The effects of charge independence selection rules should be discernible in other interactions involving states in a compound nucleus. The resonant scattering of protons[16] from Be^9 at 1.087-Mev bombarding energy has been interpreted[17] in terms of the formation of a compound state of spin zero and odd parity in the compound nucleus B^{10}. Although lower levels decay by both alpha-particle and deuteron emission this state does not appear to do so, though these transitions are not unfavored by angular momentum considerations. This behavior might be explained by presuming the isotopic spin of the state to be one, as in that case transitions to the isotopic spin-zero combinations of Li^6 and an alpha-particle or Be^8 and a deuteron would be forbidden.

If the forces between nucleons are independent of their charge, relationships will be established between transition probabilities to states of different charge belonging to the same isotopic spin multiplet. An example of this is the $Be^9(d, p)$ and $Be^9(d, n)$ reactions to the ground state of Be^{10} and the 1.7-Mev state of B^{10}, respectively. These states are the φ_1^1 and φ_1^0 components of a charge multiplet. Writing the initial charge wave function, representing the system consisting of Be^9 plus the deuteron, as $I_{\frac{1}{2}}^{\frac{1}{2}}$, and the nuclear isotopic spin functions as $\tau_{\frac{1}{2}}^{\frac{1}{2}}$ for the neutron and $\tau_{\frac{1}{2}}^{-\frac{1}{2}}$ for the proton, we can expand I in terms of φ and τ:

$$I_{\frac{1}{2}}^{\frac{1}{2}} = (\sqrt{2}\varphi_1^1\tau_{\frac{1}{2}}^{-\frac{1}{2}} - \varphi_1^0\tau_{\frac{1}{2}}^{\frac{1}{2}})/\sqrt{3}.$$

The ratio of the transition probabilities to Be^{10} and B^{10} will then be $(I_{\frac{1}{2}}^{\frac{1}{2}}|\varphi_1^1\tau_{\frac{1}{2}}^{-\frac{1}{2}})^2/(I_{\frac{1}{2}}^{\frac{1}{2}}|\varphi_1^0\tau_{\frac{1}{2}}^{\frac{1}{2}})^2$, which is equal to two, multiplied by the relative volumes of phase space available to the final systems. Uncertainties in the magnitude of Coulomb effects are sufficiently large so that this type of reaction probably cannot be used as a sensitive check on charge independence. However, the cross section relations may be useful in determining which excited states in systems like N^{14}, B^{10}, and F^{18}, correspond to the levels of isotopic spin one found in their isobaric neighbors.

I wish to thank Professor J. M. Luttinger, who suggested the possible importance of charge independence in nuclear reactions, for his interest in this discussion.

[15] Quoted by R. B. Day and R. L. Walker, Phys. Rev. **85**, 582 (1952).

[16] Thomas, Rubin, Fowler, and Lauritsen, Phys. Rev. **75**, 1612 (1949).

[17] R. Cohen, Ph.D. Thesis, Cal. Inst. Tech. (1949).