low and high energy ends of the spectrum. These sections have, however, been measured by other techniques by Bonner *et al.*[6] and by Watt.[7] Watt also gives[7] an empirical formula which fits satisfactorily the spectrum obtained as a composite of these three independent measurements.

In developing the instrumental techniques applied to this measurement I have been generously supported and assisted by others of the Argonne National Laboratory, as indicated in the report on the Ranger[2,3] design.

[6] Bonner, Ferrell, and Rinehart, Phys. Rev. **87**, 1032 (1952).
[7] B. E. Watt, Phys. Rev. **87**, 1037 (1952).

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Energy Spectrum of Neutrons from Thermal Fission of U^{235}

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A proton recoil counter has been used to determine the neutron spectrum, in the energy range 3.3–17 Mev, of a beam produced by irradiating 95 percent U^{235} (metal) in the central experimental hole of the Los Alamos Homogeneous Reactor. Most of the fissions were induced by slow neutrons. The data are combined with those obtained by D. Hill and by T. W. Bonner, R. A. Ferrell and M. C. Rinehart; the composite spectrum so obtained extends from 0.075 to 17 Mev. Fits with two general formulas are discussed.

INTRODUCTION

THE present experiment was designed specifically to obtain the high energy portion of the fission neutron spectrum, as previous investigators had measured the spectrum up to about 6 Mev.

It was known that the intensity of the very high energy neutrons would be low, so the apparatus was designed to operate stably for long periods and to have a low background count in the presence of the intense (but well collimated) beam of neutrons and gamma-rays available from a U^{235} sample placed in the central experimental hole ("glory hole") of the Los Alamos Homogeneous Reactor.[1] A more detailed report of the present experiment is available as Los Alamos Declassified Report No. 935.

APPARATUS

The neutron beam was produced by inserting samples of U^{235} (95 percent U^{235}, 5 percent U^{238}) in the "glory hole" of the water boiler, where they were exposed to a thermal neutron flux of 3×10^{11} per square centimeter per second. Two such sources were used: (1) a 28.5-g disk of U^{235} 0.168 in. thick mounted on a graphite rod, and (2) 54.1 g of U^{235} as an assembly of 31 disks 0.010 in. thick equally spaced along an 18-in. aluminum tube. Figure 1 shows the placement of the latter of these sources as well as the method of collimating the neutron beam.

It was necessary to consider the effect of neutron scattering in the walls of the collimating tube on the neutron energy spectrum of the emergent beam. A very crude calculation considering only single isotropic scattering indicated that less than one percent of this beam had been scattered in the walls. Since the most objectionable feature of such scattering is distortion of the spectrum through degradation, and since an energy loss of 3 Mev would put neutrons observed in this experiment into a group about ten times as strong as the parent group, it is believed that wall scattering is negligible in this experiment.

The distribution in energy of the fast neutrons was measured by means of the spectrometer shown in Fig. 2. Recoil protons ejected from any one of four polyethylene foils were detected by observing coincidences between counts of three proportional counters. In order to reduce the counting rates in the three counters and thereby the background produced by accidental coincidences, the counters were located outside the neutron beam as shown in Figs. 1 and 2. To increase the stability of the counter characteristics, no moving parts were placed in the counters. The distribution in energy of the neutrons was deduced from the distribution in range of the recoiling protons. Any desired combination of seven aluminum absorbers could be introduced between the polyethylene foils and the counters. An integral proton range distribution was obtained by counting all the

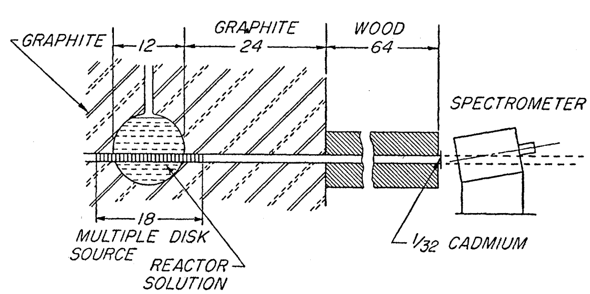

FIG. 1. Diagram of the experimental set-up to show the arrangement of materials surrounding the beam. Numbers give dimensions in inches.

[1] Rev. Sci. Instr. **22**, 489 (1951).

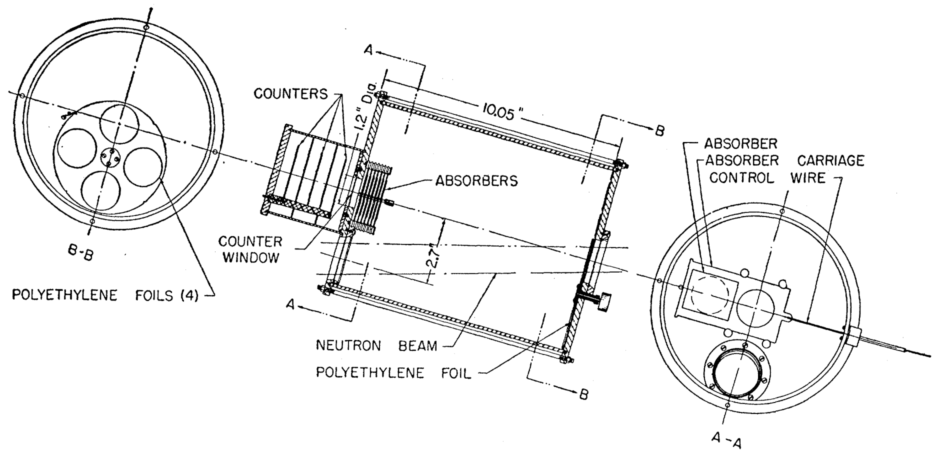

FIG. 2. Cross sections of the proton recoil spectrometer used to investigate the high energy portion of the fission neutron spectrum.

protons with energy greater than the minimum required to penetrate the particular combination of absorbers and produce a coincidence count. The differential proton range distribution and hence the neutron energy distribution was obtained by taking differences between the coincidence counts obtained with different absorbers.

The thickness of the aluminum absorbers ranged from 6.4 to 216 mg/cm^2 and that of the polyethylene foils from 2.06 to 71.9 mg/cm^2. Particular combinations of absorbers and polyethylene foils were chosen to give the desired minimum proton energy and energy resolution.

In order to avoid coincidence counts produced by electrons, it is necessary to bias the counters against particles of low specific ionization. Since the specific ionization of protons decreases with increasing proton energy, this bias will determine a maximum proton energy which will result in a coincidence count. When the absorber thickness is changed, different parts of the range of protons of a given energy will lie in the counter, and thus the maximum proton energy for which a coincidence count will occur depends on the absorber thickness. Since the differential proton range distribution is obtained under the assumption that all protons which have sufficient range to penetrate through all three counters will produce a coincidence, care must be taken to avoid the loss of an appreciable number of protons of low specific ionization. In the present experiment the proton energy distribution decreased rapidly enough with energy so that this loss was inappreciable. In all cases the operating characteristics of the spectrometer were chosen so that there was at least a separation of 6 Mev between the minimum and maximum proton energies capable of producing a coincidence count. For the observed spectrum less than one percent of the protons would be lost because of their low specific ionization. By varying the bias on the counters, a check of this number was made.

CALCULATIONS OF PROTON ENERGY

The minimum possible path within the counters necessary to produce a triple coincidence was 5.1 cm, the maximum possible path was 7.6 cm, and the mean value was 6.3 cm. In computing the range equivalent of the counters, the value 6.3 cm was used; the maximum error was 1 cm. At the operating pressure of 22 in. of mercury and with the 7.91 mg/cm^2 Duralumin counter window, the minimum proton range capable of producing a triple coincidence was 15 ± 1 mg/cm^2, corresponding to an energy of 2.45 ± 0.1 Mev. At the higher energies, the maximum error is less (0.05 Mev at 5 Mev). From the characteristics of the counter, it is believed that the probable error is less than 0.05 Mev at the lowest energy, and correspondingly lower at the higher energies.

From the atomic stopping powers it was computed that 1 mg/cm^2 of $(CH_2)_x$ was equivalent to 1.40 mg/cm^2 of aluminum. From the thicknesses of the polyethylene foils and aluminum absorbers and the equivalent thickness of the counters and their window, the average minimum range of a proton giving a triple coincidence was computed. The range was then converted to proton energy by the range-energy relation computed by Smith.[2] The energy difference between particular combinations of absorbers was also computed from the same data, but could be used only when the polyethylene foil and the absorber combination were not simultaneously changed.

CALCULATION OF THE NEUTRON DISTRIBUTION FROM THE OBSERVED COUNTING RATES

It is convenient to define the following quantities: C_p = proton counting rate = number of protons/min penetrating the counters; E_p = minimum average proton energy in Mev required for a count (lower bound of the integral proton count); E_n = neutron energy in Mev; P = "Water Boiler" power level, kilowatts (proportional to source strength); σ_s = total proton scattering cross section, barns; the scattering taken to be isotropic in c.m. system; $(t\rho)$ = polyethylene foil thickness, mg/cm^2; and $N(E_n)$ = neutron distribution function, neutrons/cm^2-sec-Mev.

[2] J. H. Smith, Phys. Rev. **71**, 32 (1947).

The neutron-proton scattering cross sections used were obtained from the theoretical curves of Bohm and Richman,[3] and Goldstein.[4] The curve developed by Bohm and Richman fits the data of Bailey *et al.*[5] quite well in the energy range 0.4 to 6 Mev. The curve developed by Goldstein fits the data of Sherr,[6] and Sleator[7] in the energy range 9 to 23 Mev. However, "reasonable extrapolations" into the gap cannot be made to join. A smooth curve was constructed by drawing a rather arbitrary join meeting Bohm and Richman's curve at 4 Mev and Goldstein's curve at about 14 Mev. Acceptable joins could be made which differed by 10 percent, so systematic errors of the order of 5 percent at 8 Mev are to be expected.

The relation between the neutron flux and the proton counting rate, and the relation between the neutron and proton energies, were derived from the geometry of the spectrometer.

The resulting formulas were

$$\langle N(E_n)\rangle_{\mathrm{Av}} = 3.04\times 10^6 \Delta(C_p/Pt\rho)/\sigma_s\Delta E_p, \tag{1}$$

$$0.93E_n = E_p, \tag{2}$$

where $\langle N(E_n)\rangle_{\mathrm{Av}}$ is the average of $N(E_n)$ over the energy interval corresponding to the recoil proton energy interval ΔE_p. As a compromise between statistical accuracy and resolving power, the values of ΔE_p were chosen to be about 1 Mev.

RESULTS

The first spectrum observed was that of a beam produced by the 28.5-g disk of U^{235} 0.168 in. thick mounted on a graphite rod and placed in the center of the water boiler. The spectrum obtained could be fitted with the empirical equation:

$$N(E_n) = 4.05\times 10^6 Pe^{-E_n/1.16}, \quad 3\leqslant E_n \leqslant 12 \text{ Mev}. \tag{3}$$

It became evident that the data could not yield the desired fission spectrum because (1) the intensity was too low to reach 15 Mev and (2) the number of neutrons produced in the reactor's solution and subsequently scattered into the beam by the walls of the collimating tube and the graphite rod backing constituted about a third of the total beam.

TABLE I. The "open hole" neutron distribution produced by the reactor without the U^{235} sample in position.

E_n Neutron energy in Mev	$N(E)\times 10^{-4}$ Neutrons/Mev-kw-sec
3.06	6.1 ±0.3
3.87	2.93±0.1
4.73	1.50±0.17
5.68	1.02±0.05
6.86	0.48±0.03
7.86	0.13±0.04
8.80	0.11±0.02
9.90	0.05±0.015

TABLE II. The spectrum of the neutron beam produced by the U^{235} disk source. The values of E_p and ΔE_p are the lower bound and widths of the proton energy intervals used. The fission spectrum was obtained by subtracting the intensities given by Eq. (4).

E_p Mev	ΔE_p Mev	E_n Mev	σ_s barns	$N(E_n)\times 10^{-4}$ neutrons Mev-kw-sec	$N(E_n)\times 10^{-4}$ fission spectrum neutrons/ Mev-kw-sec
2.58	1.07	3.30	2.18	119.5 ±1.5	114.3 ±1.5
3.65	1.08	4.46	1.75	56.3 ±0.8	54.0 ±0.8
4.73	1.21	5.68	1.48	24.0 ±0.3	23.0 ±0.3
5.91	1.00	6.86	1.29	9.7 ±0.2	9.26 ±0.2
6.91	0.85	7.86	1.15	4.4 ±0.2	4.18 ±0.2
7.78	1.22	8.97	1.03	2.2 ±0.1	2.1 ±0.1
9.00	0.71	10.04	0.93	0.8 ±0.05	0.75 ±0.05
9.71	1.21	11.04	0.85	0.42 ±0.02	0.40 ±0.02
10.91	0.85	12.16	0.78	0.20 ±0.02	0.18 ±0.02
11.75	1.06	13.17	0.72	0.10 ±0.02	0.094±0.02
12.83	0.98	14.29	0.67	0.03 ±0.01	0.027±0.01
13.49	0.98	15.00	0.64	0.016±0.003	0.014±0.003
14.46	1.01	16.05	0.59	0.016±0.003	0.015±0.003
15.48	1.15	17.22	0.54	0.001±0.003	0.001±0.003

A new source was designed to reduce the amount of material capable of scattering neutrons from the reactor into the beam and to raise the beam intensity as much as possible. The design chosen comprised 31 disks of U^{235}, each 0.010 in. thick and equally spaced along an 18-in. aluminum tube. The neutrons produced in the disk farthest from the spectrometer passed through 0.3 in. of uranium, so the distortion produced by inelastic scattering and absorption in the source was negligible.

The spectrum of the beam produced when the "glory hole" was empty was measured and the result is listed in Table I. The data can be fitted reasonably well with the equation

$$N(E_n) = 5.5\times 10^5 Pe^{-E_n/1.3}, \quad 3\leqslant E_n \leqslant 10 \text{ Mev}. \tag{4}$$

The spectrum produced by the fissions in the disks alone was obtained by subtracting the values given by Eq. (4) from the values obtained for the beam from the multiple disk source; Eq. (4) was assumed to hold throughout the energy range 3–17 Mev. Since the shape of the open hole spectrum is essentially the same as the spectrum of the beam from the multiple disk source, and constitutes only 4 percent of the total beam, it is believed that errors introduced by the subtraction are negligible. The final results are presented in Table II.

DISCUSSION

It is interesting to compare the observed spectrum with the predictions of the theoretical picture of the fission process. For this purpose, it is useful to combine the data of D. Hill and the data of T. W. Bonner *et al.* (given in the preceding papers) with the present data. In the following discussion the subscript n has been dropped from the neutron energy, E_n.

The first formula tried for fit was that developed by

[3] D. Bohm and C. Richman, Phys. Rev. **71**, 570 (1947).
[4] Louis Goldstein, unpublished Los Alamos Report No. 702 (1948).
[5] Bailey, Bennett, Bergstralh, Nuckolls, Richards, and Williams, Phys. Rev. **70**, 583 (1946).
[6] R. Sherr, Phys. Rev **68**, 240 (1945).
[7] W. Sleator, Jr., Phys. Rev. **72**, 207 (1947).

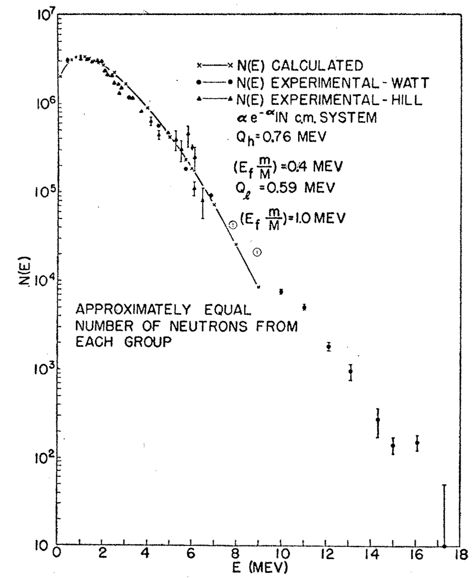

FIG. 3. Energy spectrum of fission neutrons. The calculated curve is based on Feather's assumptions. The subscripts l and h refer to the light and heavy fission fragments, respectively, and $\alpha = E'/Q$. It can be seen that above about 7 Mev the calculated curve deviates markedly from the experimental points.

Feather.[8] His three basic assumptions were (1) isotropic emission in the center-of-mass system of fragment and neutron; (2) neutron distribution in the c.m. system proportional to $E'e^{-E'/Q}$, where Q is an energy corresponding to the "temperature" of the fragment and E' is the neutron energy in the c.m. system; (3) fragment velocity at the time of neutron emission corresponding to the full kinetic energy.

The second assumption is based on the expected distribution of particles from a liquid drop model of the fragment. The third is based on the expectation that the neutron leaves the fragment in a time of the order 10^{-15} sec after the fragments separate; from the range energy relation deduced for such fragments, it is calculated that the fragment loses a negligible fraction of its original energy in that time, though several collisions may have occurred.

Attempts to find values for the two constants (Q and the product $E_f m/M$, where E_f is the kinetic energy of the fragment at the time of neutron emission and m and M are the masses of the neutron and fragment, respectively) appearing in the equation assuming one average fragment were unsuccessful. The curve computed by adding the spectra of two fragments, one having the average energy and mass of the light group and the other the average energy and mass of the heavy group, is shown in comparison with the composite curve in Fig. 3. The fit was regarded as unsatisfactory. Since rather laborious calculations are necessary to determine the spectrum given by Feather's formula, no attempt was made to add the spectra of a large number of fragments.

It is interesting to note that a simple formula giving quite acceptable fits is obtained by assuming a Maxwellian distribution ($E^{\frac{1}{2}}e^{-E/Q}$) in place of assumption (2) above. The resulting formula is

$$N(E) = \text{const} \times e^{-E/Q} \sinh[2Q^{-1}(EE_f m/M)^{\frac{1}{2}}]. \quad (5)$$

Several early reports on the fission spectrum mention this formula but none give the originator. It seems likely that it was derived by several investigators and spread by private communications. Assuming only one fragment, acceptable fits are obtainable with several sets of the constants Q and $(E_f m/M)$ which are to some extent interrelated. Partly because of the simplicity of the resulting equation, the values $Q = 1.00$ Mev and $E_f m/M = 0.5$ Mev were chosen as best representing the data. The formula is then

$$N(E) = 4.75 \times 10^6 \sinh(2E)^{\frac{1}{2}} e^{-E}. \quad (6)$$

Equation (6) and the present data together with those of Hill are plotted in Fig. 4. The results of Bonner *et al.* which cover the energy range 0.075 Mev to 0.6 Mev are not plotted in Fig. 4. However, their data are also represented by Eq. (6) which thus provide a good approximation to the fission neutron energy spectrum from 0.075 Mev to 17 Mev.

In order to see if Eq. (5) could be made to give an acceptable fit when the spectra from several groups were added, two such terms were used in which the products $(E_f m/M)$ were set equal to the averages for the heavy and light groups; as a second condition, the number of neutrons from each group were set equal, and as a third,

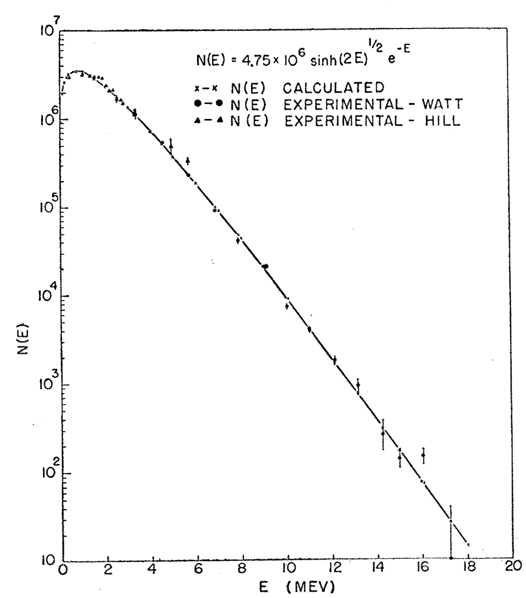

FIG. 4. Energy spectrum of fission neutrons. The calculated curve is based on the assumption of a Maxwellian distribution in the center-of-mass system for the neutrons emitted by a fission fragment.

[8] Norman Feather, unpublished report BM-148.

the values of constants (Q) were set equal. The absolute value of Q is then the only remaining arbitrary constant (except the normalization constant needed to fit any particular set of data) and was determined from the slope of the data in the region around 9 Mev. The resulting equation is

$$N(E)=1.088\times10^6\sinh(1.92E^{\frac{1}{2}})e^{-E/0.875}+2.937\times10^6\sinh(1.28E^{\frac{1}{2}})e^{-E/0.875}. \quad (7)$$

Both Eqs. (6) and (7) show curvature on a logarithmic plot, while the data fall more nearly on a straight line. The deviation of the data is of the same order of magnitude as the systematic error expected from the arbitrary join in the curves used for the (n,p) scattering cross section, and so cannot be taken as real. To explore the implications of a real deviation like that observed and to find an equation giving a fit passing more nearly through all points, the conditions of equal number from each group and equal Q for each group were abandoned.

The curvature of all the calculated spectra can be removed by assuming different Q's for the neutron emitting fragments, and several cases were calculated. Equations were found which passed within the standard deviation of each point, but the increased complexity seems hardly justified with the present data and the simplified assumptions leading to Eq. (6). However, the improvement of the fit seems to indicate that Feather's formula can be made to fit the data by assuming many fragments having different velocities, masses, and excitations.

Several of the equations developed were computed by Mr. Dura W. Sweeney whose help is gratefully acknowledged. I am also indebted to many other members of this laboratory for helpful discussions and criticisms.

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Conservation of Isotopic Spin in Nuclear Reactions*

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The effects of the charge independence of nuclear forces on nuclear reaction experiments is discussed. It is pointed out that charge independence establishes relationships between cross sections for some reactions and results in forbidding certain other reactions.

It is well known[1,2] that if the forces between two nucleons are independent of their charge states and depend only on their space and spin coordinates, the wave function of a system of nucleons will be invariant under certain charge transformations. This symmetry is usually described in terms of a constant of motion, the isotopic spin. Important consequences of this invariance of nuclear structure with respect to rotation in isotopic spin space have been discussed by Wigner[3,4] and others.

There is no very conclusive evidence to establish the charge independence of nuclear forces. While the similarity of energy levels of mirror nuclei strongly suggests the equivalence of neutron-neutron forces and proton-proton forces,[5] there is not yet much information on the equality of the neutron-proton interaction and the forces between like nucleons. In particular, no definite conclusions have been derived from nucleon-nucleon scattering. Although the low energy scattering experiments are not in contradiction to a description in terms of charge independent forces,[6] it is not clear that this is true of higher energy measurements.[7]

It appears to have been largely overlooked in recent work that information on the charge independence of nuclear forces may be obtained from observations on nuclear reactions. Charge independence has observable consequences in such reactions; in particular, it results in forbidding certain transitions which are allowed solely from considerations of spin and parity.

For the purpose of this discussion the third component T_3 of the isotopic spin T of a nucleus is defined, as usual, as the number of neutrons minus the number of protons in the nucleus divided by two. Systems having the same isotopic spin but different T_3 components form a set of multiplicity $2T+1$, which differ in energy only through Coulomb forces. On the assumption that it is a good quantum number, the value of T for any nucleus is easily determined by an examination of the binding energies of nuclei.[8,9] Generally the binding energy of light nuclei depends strongly upon T. Since there is close competition between low states with different values of T only in the

* Work supported by the AEC and the Wisconsin Alumni Research Foundation.
[1] E. Wigner, Phys. Rev. **51**, 106 (1937).
[2] N. Kemmer, Proc. Cambridge Phil. Soc. **34**, 354 (1938).
[3] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937).
[4] E. Wigner, Phys. Rev. **56**, 519 (1939).
[5] See, e.g., V. R. Johnson, Phys. Rev. **86**, 302 (1952).
[6] J. Schwinger, Phys. Rev. **78**, 135 (1950).
[7] R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1950).
[8] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).
[9] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 291 (1950).