An Application of the Theory of the Effective Range to Meson-Nucleon Scattering*

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A theory of the effective range for P-state scattering is developed and the extent of its validity is investigated. It is found that a parameter of the theory can meaningfully be identified as an effective range and that the energy dependence of the effective range is very weak. The resulting formula for the energy dependence of the phase shift is shown to be exactly equivalent, if the relativistic energy is used, to a simple form of the Wigner-Eisenbud one-level formula for P-states in which the level shift is neglected. An application to meson-nucleon scattering is made which illustrates the use of the method and which suggests a high lying resonance in the scattering.

I. INTRODUCTION

EXPERIMENTS on the scattering of mesons by protons[1] have shown that a large fraction of the scattering is due to contributions from the partial waves of unit angular momentum. It has been pointed out that this result is a partial verification of the predictions of pseudoscalar meson theory with pseudovector coupling which predicts predominant P-state interactions.[2,3] In addition an attempt has been made by the author[3] to account for the details of the scattering by an analysis using the generalized single-level formulas of Wigner and Eisenbud[4] and assigning the level parameters on the basis of weak and strong coupling pseudoscalar meson theory with symmetric pseudovector coupling. The weak coupling theory gave the ratios of the level widths to the resonance energies and the strong coupling theory gave the resonance energies.

The applicability of the Wigner-Eisenbud formulas to scattering phenomena of this character, however, is possibly of doubtful validity because of the large level width (about 100 Mev) required to describe the data.[3] An examination of the Wigner-Eisenbud result [obtained from their Eq. (57)], which gives

$$\tan\delta_\lambda = \tfrac{1}{2}\Gamma_\lambda/(E_\lambda+\Delta_\lambda-E), \qquad (1)$$

shows that the approximate formula,

$$\tan\delta_\lambda \approx \tfrac{1}{2}\Gamma_\lambda/(E_\lambda-E), \qquad (2)$$

can be used other than in the vicinity of resonance only if the level shift is small relative to E_λ. In the notation of Wigner and Eisenbud, for P-states, we have

$$\Gamma_\lambda=(k^3a^2\gamma_\lambda^2)/(1+k^2a^2), \quad \Delta_\lambda=-\tfrac{1}{2}\Gamma_\lambda/(ka), \qquad (3)$$

where k is the momentum, a the radius of the internal region, and γ_λ^2 is a constant. Implicit in these formulas is also the smallness of the quantity ka. The condition for the validity of the approximate formula of Eq. (2) is therefore that

$$(k^2a\gamma_\lambda^2)/(1+k^2a^2)\ll 2E_\lambda, \qquad (4)$$

over the range for which Eq. (2) is expected to hold. For example, at a meson energy of 200 Mev in the laboratory system corresponding to $k^2=3\mu^2$ in the barycentric system, with $E_\lambda\approx\mu$, and with $a^2=1/(2\mu^2)$ as assumed in reference 3, the condition is that

$$\gamma_\lambda^2\ll 2.18. \qquad (5)$$

The assignment of γ_λ^2 made in analyzing the scattering data can be obtained by comparing Eq. (13) of reference 3 with Eq. (3) and that was $\gamma_\lambda^2=1.40$, so that the condition of Eq. (5) is only partially satisfied. Because of the possible lack of validity of formulas such as Eq. (2), it is of interest to show that these phenomena may be described from an entirely different viewpoint[5] making use of the theory of the effective range as developed by Bethe,[6] Chew and Goldberger,[7] and others,[8] which gives essentially identical results. Chew and Goldberger give a formula of general validity for arbitrary angular momentum states which can be specialized to our case of scattering in P-states; the result can, however, be derived directly in a simple manner which follows Bethe's treatment closely.

II. THEORY

We consider the Klein-Gordon equation for a meson moving in a potential V[9]

$$[\Delta-\mu^2+(E-V)^2]\psi=0. \qquad (6)$$

In this equation, we let $\psi=\Sigma_l P_l(\cos\theta)u_l(r)/r$, $k^2=E^2-\mu^2$, and find

$$(d^2/dr^2-l(l+1)/r^2+k^2-2EV+V^2)u_l=0. \qquad (7)$$

* Assisted by the joint program of the ONR and AEC.

1 Anderson, Fermi, Long, Martin, and Nagle, Phys. Rev. **85**, 934 (1952); Anderson, Fermi, Long, and Nagle, Phys. Rev. **85**, 936 (1952); Fermi, Anderson, and Nagle (to be published).

2 Nagle, Anderson, Fermi, Long, and Martin, Phys. Rev. **86**, 603 (1952).

3 K. A. Brueckner, Phys. Rev. **86**, 106 (1952).

4 E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

5 The usefulness of this method was pointed out to the author by Dr. Richard Christian in connection with the analysis of reactions involving light nuclei.

6 H. A. Bethe, Phys. Rev. **76**, 38 (1949).

7 G. F. Chew and M. L. Goldberger, Phys. Rev. **75**, 1637 (1949).

8 J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949); F. C. Barker and R. E. Peierls, Phys. Rev. **75**, 312 (1949).

9 The representation of the meson-nucleon interaction by a static potential can at best be an approximate description. The potential does not, however, appear in our final formulas so that the results are to some extent independent of this assumption.

If we define a new potential function,

$$2\mu v=2EV-V^2, \tag{8}$$

then Eq. (7) reduces to the ordinary Schrödinger equation except that the relativistic momentum appears. The potential v then is according to Eq. (8) energy dependent; however, in the applications which interest us, the potential V is considerably larger than the energy E and in addition the energy E varies only from about 170 Mev to 280 Mev for meson scattering with kinetic energies in the laboratory system varying from 50 to 200 Mev. We shall therefore consider v to be constant and shall expect small energy dependent corrections to the effective range from the variations in v.

Now specializing to the case $l=1$, we introduce the functions u_1 and ψ_1 corresponding to the momentum k_1, which satisfy Eq. (7) except that ψ_1 is the solution with $V=0$, u_1 will have the form outside the range of the potential

$$u_1=\text{constant}\times\left[\cos(k_1r+\delta_1)-\frac{\sin(k_1r+\delta_1)}{k_1r}\right], \tag{9}$$

which is also the general form of ψ_1 for all r. We fix the normalization by requiring that ψ_1 be equal to unity at ϵ much less than the range of the potential (we cannot take $\epsilon=0$ because of the singularity of ψ_1)[10] and that $u_1=\psi_1$ outside the range of the potential. Following Bethe's procedure, we now form the expression

$$\int_\epsilon^\infty\left\{\frac{d}{dr}\left(u_1\frac{du_2}{dr}-u_2\frac{du_1}{dr}\right)-\frac{d}{dr}\left(\psi_1\frac{d\psi_2}{dr}-\psi_2\frac{d\psi_1}{dr}\right)\right\}dr$$

$$=(k_1^2-k_2^2)\int_\epsilon^\infty(u_1u_2-\psi_1\psi_2)dr. \tag{10}$$

Making use of the equality of u_1 and ψ_1 at large distances, the vanishing of u at ϵ (to order $k\epsilon$), and the normalization of ψ, we find

$$\left(\frac{d\psi_1}{dr}-\frac{d\psi_2}{dr}\right)_{r=\epsilon}=(k_2^2-k_1^2)\int_\epsilon^\infty(u_1u_2-\psi_1\psi_2)dr. \tag{11}$$

The expression for ψ which satisfies the normalization at $r=\epsilon$ is

$$\psi(r)=\left[\cos(kr+\delta)-\frac{\sin(kr+\delta)}{kr}\right]$$

$$\times\left[\cos(k\epsilon+\delta)-\frac{\sin(k\epsilon+\delta)}{k\epsilon}\right]^{-1}. \tag{12}$$

Expanding this for small r gives

$$d\psi(\epsilon)/dr=-1/\epsilon+k^2\epsilon+k^3\epsilon^2\cot\delta+\mathcal{O}(k^4\epsilon^3). \tag{13}$$

Combining this result with Eq. (11) leads to the useful expression

$$k_1^3\cot\delta_1-k_2^3\cot\delta_2$$

$$=\frac{(k_2^2-k_1^2)}{\epsilon^2}\left[\int_\epsilon^\infty(u_1u_2-\psi_1\psi_2)dr+\epsilon\right], \tag{14}$$

in which the singular terms in $1/\epsilon$ have canceled. We now define the quantity

$$\frac{1}{\epsilon^2}\left[\int_\epsilon^\infty(u_1u_2-\psi_1\psi_2)dr+\epsilon\right]=[\rho(k_1,k_2)]^{-1}, \tag{15}$$

and write Eq. (14) in the form

$$k_1^3\cot\delta_1-k_2^3\cot\delta_2=(k_2^2-k_1^2)/\rho(k_1,k_2). \tag{16}$$

Equation (16) is the analog for P-states of Bethe's Eq. (12) for S-states.[6]

This result can be expressed in two particularly useful ways. Consider first the limit as $k_1\to 0$, $k_2=k$. We define

$$\lim_{k\to 0}k^3\cot\delta=a^{-3}, \tag{17}$$

where a is the "scattering length,"[11] and make use of the effective range approximation,

$$\rho(0,k)=r_0, \tag{18}$$

to introduce the "effective range" r_0. Equation (15) then takes the form

$$k^3\cot\delta=a^{-3}-k^2/r_0. \tag{19}$$

The approximate energy independence of $\rho(0,k)=r_0$ follows, as Bethe has shown, from the form of the integral of Eq. (14). The integrand is finite only within the range of the forces and, for a strong interaction, will be insensitive to variations in the energy. We also can obtain another useful form for Eq. (16) in the vicinity of the vanishing point of $\cot\delta$ which we assume to be at a momentum value k_0. Then we have

$$k^3\cot\delta=(k_0^2-k^2)/\rho(k,k_0)\cong(k_0^2-k^2)/r_0. \tag{20}$$

Equation (19) can be written in a form which clearly shows its relation to the Wigner-Eisenbud expression of Eq. (1). This is

$$\tan\delta=\frac{1}{2}\frac{(-2k^3r_0/2\mu)}{(r_0/2\mu a^3-\text{“}E\text{”})}, \tag{21}$$

where we have introduced "E"$=k^2/2\mu$ which reduces to the kinetic energy in the nonrelativistic limit. If we identify $2k^3r_0/2\mu=-\Gamma_\lambda$, $r_0/2\mu a^3=E_\lambda$, then Eq. (21) is identical with Eq. (2), except that E has been replaced by "E". The form of this result shows that the energy dependence and "resonance" behavior of the phase shifts given by the effective range theory are almost

[10] In all expressions which we derive, we shall implicitly understand that the limit as $\epsilon\to 0$ is to be taken.

[11] The parameter a is of course not a scattering length in the sense of the corresponding constant which appears in the description of S-state scattering.

identical with the predictions of the Breit-Wigner resonance theory.

We shall next consider the validity of the approximations involved in the use of the simple effective range formula of Eq. (18). We write

$$\frac{1}{\rho(k,0)}=\frac{1}{\epsilon^2}\left[\int_\epsilon^\infty (u_0{}^2-\psi_0{}^2)dr+\epsilon\right]$$
$$+\frac{1}{\epsilon^2}\int_\epsilon^\infty [u_0(u_k-u_0)-\psi_0(\psi_k-\psi_0)]dr$$
$$\equiv\frac{1}{\rho(0,0)}+\frac{1}{\epsilon^2}\int_\epsilon^\infty [u_0(u_k-u_0)-\psi_0(\psi_k-\psi_0)]dr. \quad (22)$$

The first term in this equation we have identified as the reciprocal of the effective range r_0; the second gives the energy dependent correction to the effective range. To show the connection between r_0 and the range R of the potential, we first write the expression of Eq. (12) for ψ correct to terms quadratic in k

$$\psi_k(r)=\epsilon\left[\frac{1}{r}+\frac{r^2}{3a^3}\right]+k^2\frac{\epsilon r}{3}\left[\frac{3}{2}-\frac{r}{r_0}-\frac{r^3}{10a^3}\right]$$
$$\equiv\epsilon(\alpha_0+k^2\alpha_1). \quad (23)$$

A correctly normalized expression for u, if $v(r)$ is a solution of Eq. (7), is

$$u(r)=v(r)\psi(R)/v(R), \quad (24)$$

where R is the range of the potential at which point u and ψ are defined to be equal. Using these results, we have

$$\frac{1}{r_0}=\int_\epsilon^\infty\left\{\left[\frac{v_0(r)}{v_0(R)}\alpha_0(R)\right]^2-\alpha^2{}_0(r)\right\}dr+\frac{1}{\epsilon}. \quad (25)$$

The singular terms in $\alpha_0{}^2(r)$ can be canceled if we write

$$\frac{1}{\epsilon}=\int_\epsilon^R r^{-2}dr+\frac{1}{R}.$$

We then have

$$\frac{1}{r_0}=\frac{1}{R}+\int_0^R\left\{\left[\frac{v_0(r)}{v_0(R)}\alpha_0(R)\right]^2-\frac{2}{3}\frac{r}{a^3}-\left(\frac{r^2}{3a^3}\right)^2\right\}dr. \quad (26)$$

The contribution from the integral will in general not be a large correction if a^3 is not small compared with the range R, since the integrand vanishes at $r=0$ and is equal to $1/R^2$ at $r=R$. We therefore see that r_0 is of the magnitude of R and can meaningfully be identified as an effective range.

To obtain the energy dependent correction term, we make use of the expansion for ψ and in addition write

$$u(k)=u_0+k^2v_1. \quad (27)$$

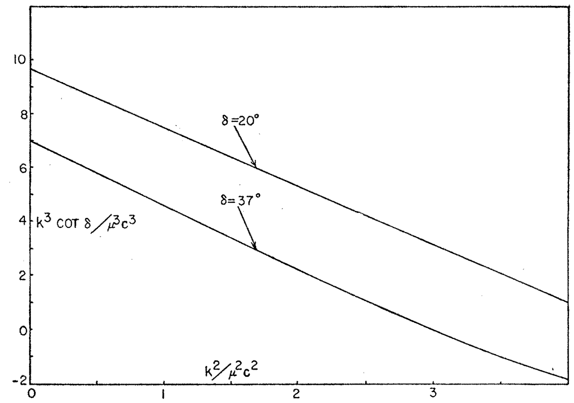

FIG. 1. Effective range plot for scattering from an attractive square well potential of range $\frac{1}{2}\hbar/\mu c$. The potential strengths have been adjusted to give a phase shift of 20° and of 37° at $k=1.30\mu c$. The effective range is approximately $0.44\hbar/\mu c$ for both well depths.

We then have

$$\frac{1}{\rho(k,0)}-\frac{1}{r_0}=k^2\int_0^\infty\left(\frac{u_0v_1}{\epsilon^2}-\alpha_0\alpha_1\right)dr. \quad (28)$$

To estimate the size of this correction, we notice that

$$\alpha_0\alpha_1=\tfrac{1}{3}(1+r^3/3a^3)(\tfrac{3}{2}-r/r_0-r^3/10a^3) \quad (29)$$

vanishes at approximately $r=(3/2)r_0$ if R^3/a^3 is not large. We also note that u_0v_1/ϵ^2 vanishes at $r=0$, and is equal to $\alpha_0\alpha_1$ at the range of the forces, so that the integrand vanishes at $r=R$. We therefore clearly set an upper limit on the integral if we evaluate

$$\int_0^R(-\alpha_0\alpha_1)dr\approx-\tfrac{1}{3}r_0. \quad (30)$$

If we now write

$$\rho(k,0)=r_0+Pk^2r_0{}^3, \quad (31)$$

where the numerical factor P gives the size of the energy-dependent correction, this argument leads us to include that P is somewhat less than $\frac{1}{3}$. This result shows that we can expect a high degree of validity for the approximation of treating $\rho(k,0)$ as a constant, as long as kr_0 is of the order of one or less, contrary to what we might expect on the basis of a purely dimensional argument. The results obtained by Bethe[6] and by Blatt and Jackson[8] in the analysis of nucleon scattering show that a similar situation holds for S-state interactions.

To show for a simple case the extent of the validity of the formula of Eq. (19), we have explicitly calculated the P-state scattering from an attractive square well potential of range one-half the meson compton wavelength. The results are given in Fig. 1, plotting $(k^3/\mu^3c^3)\cot\delta$ against k^2/μ^2c^2, where the well depth has been adjusted to give phase shifts of 20° and 37° at $k=1.3\mu c$. The "scattering length" a in these cases is approximately $a=0.48\hbar/\mu c$ and $0.95\hbar/\mu c$, respectively; the effective range is very nearly the same for both and

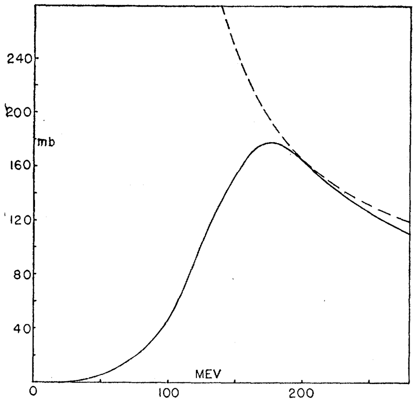

FIG. 2. $P_{\frac{3}{2}}$ scattering from the deeper square well potentials of Fig. 1. The laboratory energy is indicated. The dashed line gives the maximum possible $P_{\frac{3}{2}}$ scattering corresponding to a phase shift of 90°.

is $r_0=0.44\hbar/\mu c$. It is possible to obtain explicitly the first correction term to the effective range from these results. For the case of the deeper well, $\rho(k, 0)$ is approximately

$$\rho(k, 0)=r_0+0.18k^2r_0^3, \tag{32}$$

showing the smallness of the coefficient P of Eq. (22) in agreement with the conclusions reached above. Since in this case the effective range is about one-half the meson Compton wavelength, the correction to r_0 is negligible over the range of energies considered. It is apparent from these results that the approximation of treating $\rho(k, 0)$ as a constant gives an excellent description of the scattering, although at the higher energies, the ratio $r_0/\lambda\!\!\!^{-}$ increases to nearly one: It is also interesting that the scattering from the deeper potential, with $\delta=37°$ at $k=1.3\mu c$, passes through a "resonance" at $k^2=3.08\mu^2c^2$, corresponding to a laboratory energy of 202 Mev.[12] This is shown in Fig. 2 which gives the scattering from this potential or what is equivalent, with the phase shift given by the effective range formula of Eq. (19) with the parameters a and r_0 deduced from the effective range plot in Fig. (1). The scattering reaches a maximum below the resonance and falls off at higher energies, as the phase shift passes 90°, slightly more rapidly than $8\pi\lambda\!\!\!^{-2}$, the maximum $P_{\frac{3}{2}}$ cross section.

TABLE I. Evaluation of $k^3 \cot\delta$ for meson scattering. Columns 1 and 2 give the laboratory energy and total cross section, column 3, the meson momentum in the barycentric system, columns 4 and 5 the values of $k^3 \cot\delta$ uncorrected for and corrected for S-state scattering. The experimental points are from the work of Fermi *et al.* (see reference 1) except for the 52-Mev point which is due to R. P. Shutt and co-workers (private communication).

E(Mev)	σ(mb)	k^2/μ^2c^2	$k^3/(\mu^3c^3)\cot\delta$	$k^3/(\mu^3c^3)\cot\delta$
52	20 ± 4	0.60	2.95 ± 0.30	4.16 ± 0.42
82	50 ± 13	1.00	2.99 ± 0.37	3.36 ± 0.42
110	74 ± 5	1.41	3.67 ± 0.12	3.55 ± 0.12
118	91 ± 6	1.54	3.10 ± 0.10	3.29 ± 0.11
135	121 ± 19	1.78	2.71 ± 0.23	2.93 ± 0.24
135	152 ± 14	1.78	2.17 ± 0.10	2.35 ± 0.11

[12] It is interesting that the P-state scattering from any strongly attractive short-ranged potential will show this resonance behavior if the "scattering length" a is positive, since the phase shift passes through 90° at $k^2=r_0/a^3$.

III. APPLICATION TO MESON SCATTERING

To illustrate the method further, we apply it to an analysis of the scattering of positive pions in hydrogen. The cross sections are summarized in Table I together with the corresponding values of the meson momentum in the barycentric system. The phase shift δ can be calculated if it is assumed that the scattering is entirely due to the $P_{\frac{3}{2}}$ state,[13] since in this case the total cross section is given by

$$\sigma=8\pi\lambda\!\!\!^{-2}\sin^2\delta.$$

The values of $k^3 \cot\delta$ calculated on this basis are given in column 4 of Table I. Fermi[1] finds evidence for some even-odd wave interference at high energies with a magnitude for the even-wave contributions of 10 to 20 mb estimated on the assumption that it is entirely due to S-waves. The values of $k^3 \cot\delta$ obtained when the total cross section has been corrected for a even-state contribution of 10 millibarns are given in column 5 of Table I. These results are also given in Fig. 3, plotting $k^3 \cot\delta$ against k^2 which, according to Eq. (18), should give a straight line with vertical intercept at $k^2=0$ of $1/a^3$ and intercept at $k^3 \cot\delta=0$ of r_0/a^3. These results, although rather rough, indicate a "scattering length" a of about $9.67\hbar/\mu c$ and an effective range with a sign such that the phase shift will pass through 90° at energies considerably above 135 Mev. This is not at present in disagreement with the hypothesis of a resonance for the $J=\frac{3}{2}$ state in the vicinity of 200 Mev.

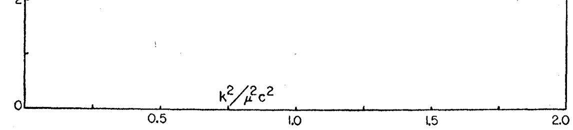

FIG. 3. Effective range plot for meson-nucleon scattering. The experimental points are from column 5 of Table I. The straight line is for a "scattering length" of $0.590\hbar/\mu c$ and effective range of $0.91\hbar/\mu c$. The extrapolated phase shift passes through 90° at $k=3.10\mu c$.

[13] Measurements by Fermi *et al.* and by Shutt *et al.* of the angular distribution of positive pions scattered in hydrogen strongly suggest that the $P_{\frac{3}{2}}$ scattering accounts for almost all of the total cross section. The author is indebted to E. Fermi and his collaborators and to R. P. Shunt and his collaborators for preprints of their papers in advance of publication.

These remarks can of course be made much more definite when further information on the scattering is available.

IV. CONCLUSIONS

It is found that in so far as a potential can be meaningfully used to describe a strong P-state interaction, an effective range theory is applicable. Therefore, as in S-state scattering, only two parameters can be determined, the "scattering length" which determines the low energy scattering, and the effective range which is a measure of the range of the potential. This theory is applicable over a considerable range of energies because of the smallness of the energy dependent corrections to the effective range. The theory is equivalent to a simple form of the Wigner-Eisenbud generalized single-level (neglecting the level shift and taking $ka \ll 1$) except that the relativistic energy must be used. As a consequence, the scattering will show the usual resonance features associated with the Breit-Wigner theory, and in fact, such resonance behavior seems typical of a sufficiently strong P-state interaction.

Application of the theory to meson scattering shows that over the range of energies for which experimental data is available, the effective range plot seems valid. If extrapolated to high energies, it suggests a resonance at an energy considerably higher than the highest energy at which experiments have been made of 135 Mev.

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Average Energy of Secondary Electrons in Anthracene Due to Gamma-Irradiation*

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The average energy of the electrons produced in anthracene scintillators by γ-rays in the energy range from 0.28 to 2.76 Mev is determined experimentally. The values obtained are in good agreement with theory. Above 1 Mev γ-ray energy, crystal size is shown to be of importance due to leakage of high energy electrons from the crystal surfaces.

FOR electrons incident on an anthracene crystal, it has been shown that the scintillation pulses are proportional to the electron energy in the range from 0.1 to about 3 Mev.[1,2] The same behavior can be assumed for secondary electrons produced inside the crystal by incident γ-rays, provided all the electron energy is expended within the scintillator. If the energy spectrum of the secondary electrons produced in anthracene is the same as that from the absorption of the same γ-rays in air, the crystal is said to be "air-equivalent." Since the average energy is a measure of the electron energy spectral distribution, we have determined theoretically and experimentally the average energy of the secondary electrons produced in anthracene by sources of various γ-ray energy.

For γ-rays of energy E_γ (in Mev) incident on any matter, it can be shown[3] that the average energy of the secondary electrons produced by the three γ-ray absorption processes is given by

$$E_{\mathrm{Av}}=\frac{\sigma_a E_\gamma+\tau(E_\gamma-W)+\kappa(E_\gamma-1.02)}{\sigma+\tau+\kappa},$$

where τ and κ are the linear photoelectric and pair-production absorption coefficients, $\sigma=(\sigma_a+\sigma_s)$ is the total linear Compton absorption coefficient in the usual Klein-Nishina notation, and W is the electronic binding energy. The values of E_{Av} thus computed for various monochromatic γ-ray energies are shown in Fig. 1 for air, anthracene, and sodium iodide as absorbing media. From Fig. 1 it can be seen that anthracene is "air-equivalent" in the energy range considered, while the inorganic NaI(Tl) scintillator deviates widely below 1 Mev, due to photoelectric absorption by the high-Z elements therein.

Experimentally, E_{Av} was determined in anthracene for the γ-rays from Hg^{203}, Cs^{134}, Co^{60}, Na^{24}, and a radium needle with 0.5-mm platinum filtration. Two different anthracene crystals were used: (a) a round crystal 0.6 cm thick and 3 cm in diameter, and (b) a rectangular crystal 2 cm thick and 1×2 cm in cross section. The efficiency of light collection was increased by covering top and side faces of the crystals with a $\frac{1}{4}$-mil aluminum foil. The crystals were joined to the RCA 5819 photomultiplier tube by a Lucite light pipe, with mineral oil

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[1] J. I. Hopkins, Phys. Rev. **77**, 406 (1950).
[2] J. I. Hopkins, Rev. Sci. Instr. **22**, 29 (1951).
[3] G. D. Prestwich and T. H. Colvin, unpublished S.M. thesis, Massachusetts Institute of Technology (1952).